Correlation: Theory and Architectures

EVGA VLBI School ~ Espoo, Finland

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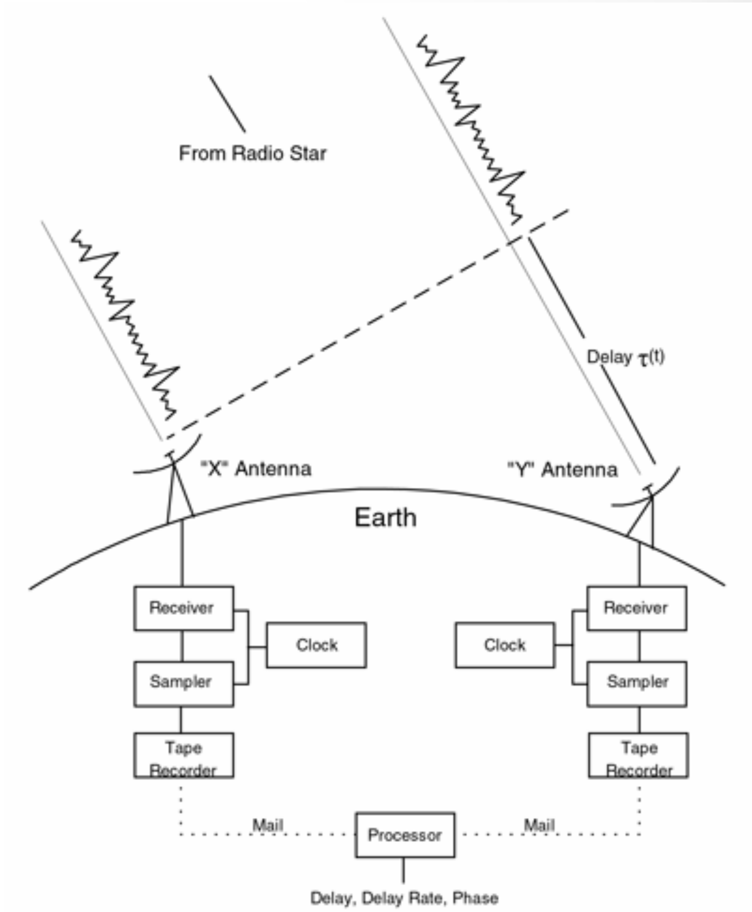
Outline

- exposition of the problem
- basic nature of correlation
- XF vs FX
- elements common to both
- hardware correlation / mk4
- software correlation / difx
- software correlation hardware
- correction factors
- VLBI2010

Nature of the Problem

- Extremely weak signal from natural radio source at great distance
- Principal parameters to be extracted:
    - delay
    - delay rate
    - correlation amplitude
    - interferometric phase

} complex visibility

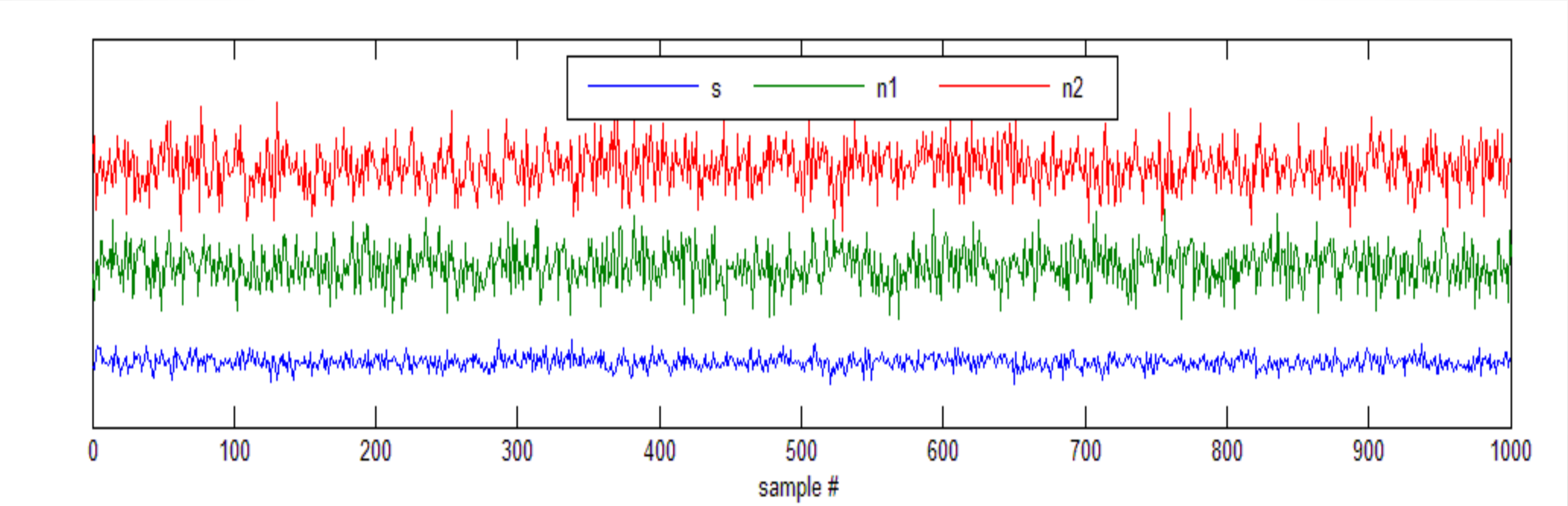


Correlation

- Signal is totally unknown (white noise)
- Receiver noise contributes much more power
  - e.g. SEFD at Westford ~ 5000Jy
  - geodetic sources ~ 1 Jy
- Typical snr is 1:1000 for a single sample
- By combining 10^{10} samples, we can make the snr of the average 10^{5} times larger, or 100:1

Cross-correlation of weak signals

- Let:
  - $s(t)$ be a weak astronomical signal
  - $n_1(t)$ and $n_2(t)$ be noise signals at sites 1 & 2
  - assuming zero-mean Gaussian random variables



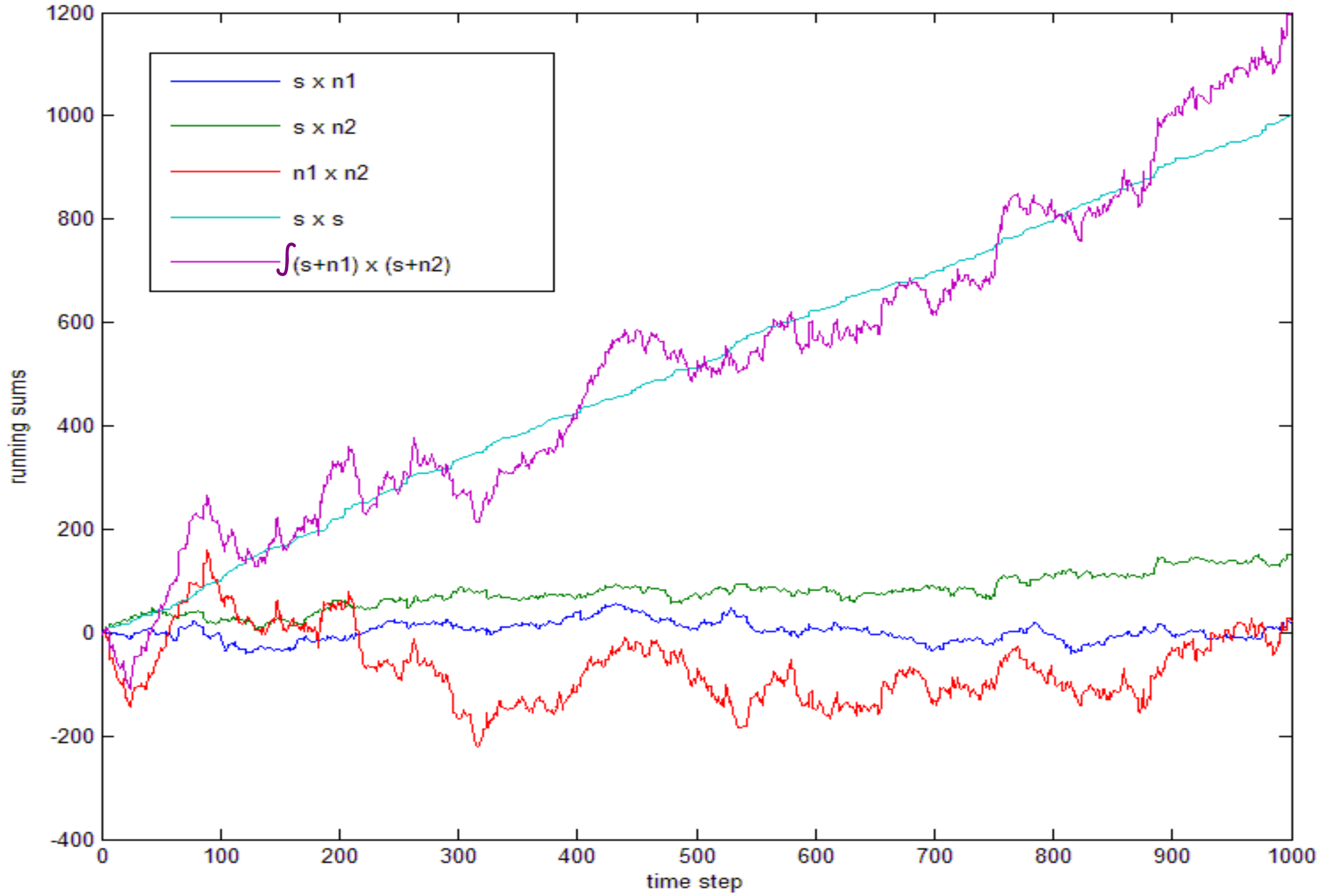
Cross-corr. of weak signals (cont' d)

- Product of signals is:

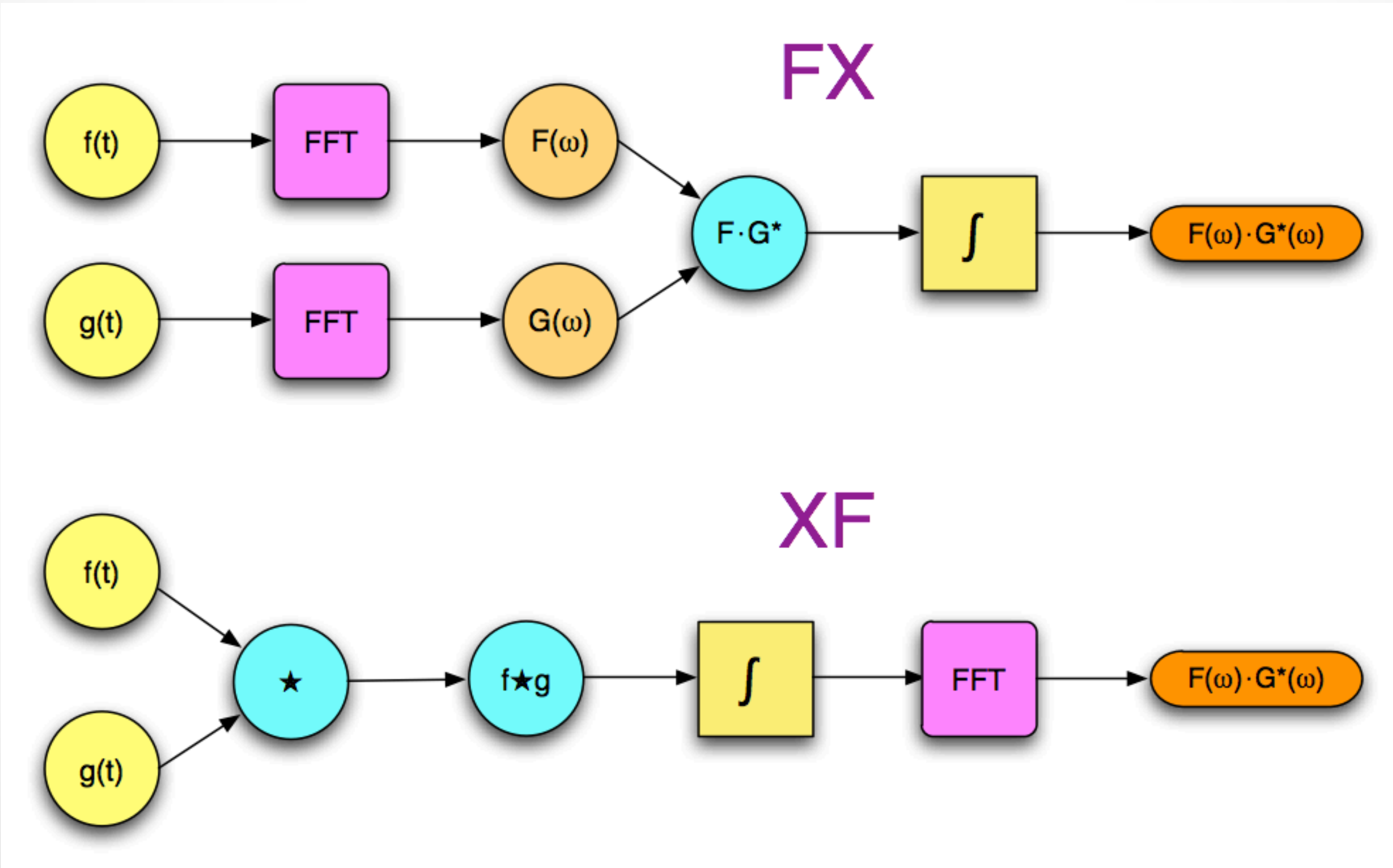
$$(s + n_1)\ (s + n_2) = s^2 + n_1 s + n_2 s + n_1 n_2$$

Only the 1st term has a non-zero time average!

Correlation components



FX/XF Equivalence



Relative (dis)advantages

- XF
  - conceptually simple
  - more easily implemented in digital hardware
    - working with small word sizes
    - allows FFT to be done post-integration
  - able to edit out single bad samples
- FX
  - cost for spectral channels grows only as $\log_2$ (N) vs. linear for XF
  - early word growth
  - little loss due to delay quantization

Elements common to both

- model: delay compensation, fringe rotation
- data sources
- fringe detection & observable parameter estimation
- technical details
  - discrete samples
  - integration
  - correlator beam

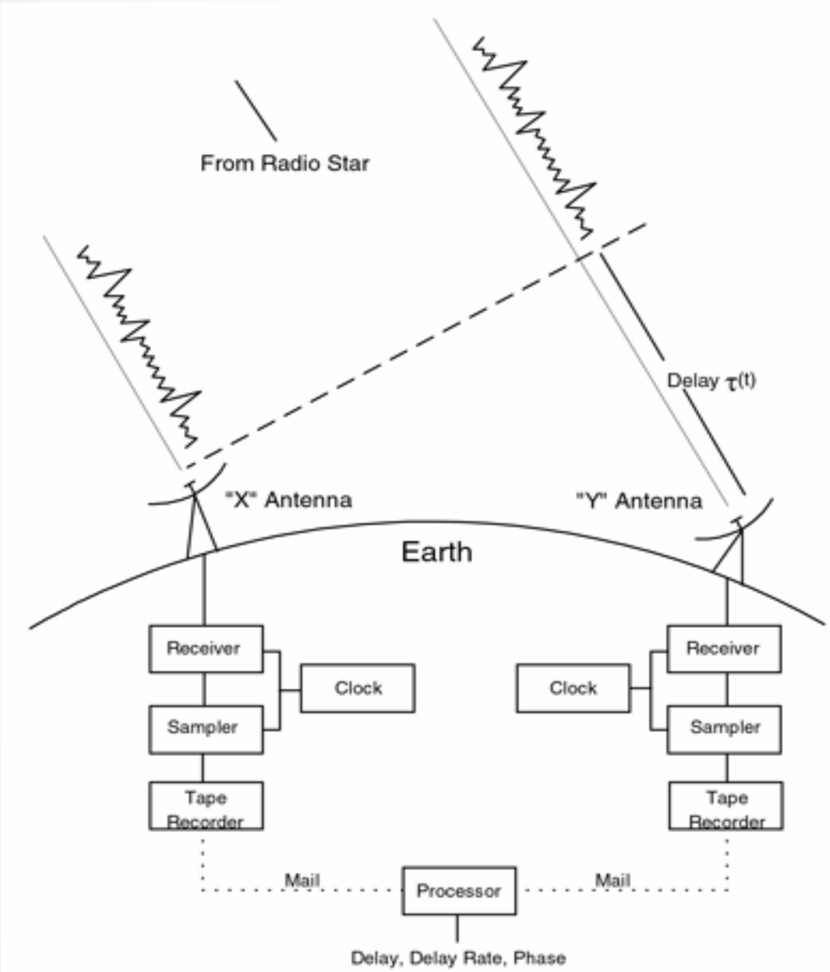
Model Insensitivity

- Fundamental principle in all correlators is that model doesn't have to be perfect
- Use of total quantities ensures that model sensitivity is very low

$$O = O - C + C$$
$$= O - (C + \delta C) + (C + \delta C)$$

- Model has to be good enough to have delay and rate within correlator window
- Typical model calculated by a high-precision external package designed for that purpose
    - e.g. DiFX uses Calc 9.0
    - keeps fit residuals small and smooth

Delay and delay rate



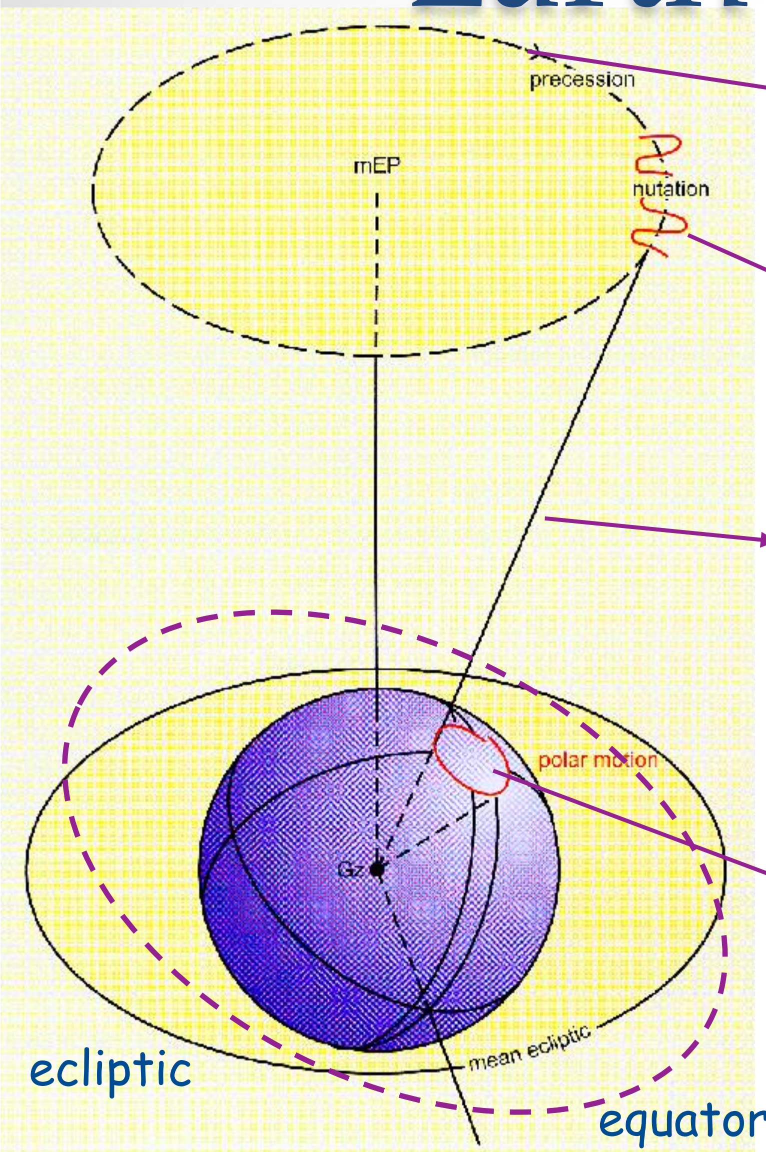
- maximum delay is Earth radius : 21 ms
- maximum rate is $\Omega_e \times D_e$ = 0.73 μ s/s

- e.g. for 32 MHz channel
  - max delay ~ 1.4e6 samples
  - max shift rate ~ 47 samples / s

- max fringe rate @ 10 GHz

  = $\Omega_e \times D_e \times f_{rf}$ = 7.3 KHz

Earth Orientation



precession: 50 asec/yr

nutation: largest term ~20″ with 18.6 y period

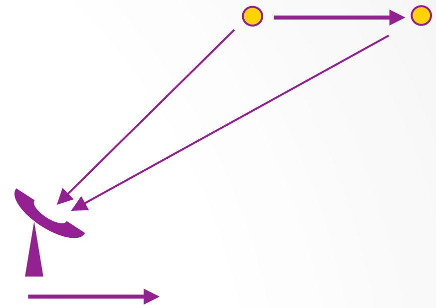
phase angle about rotation axis: due to sloppy Earth UT1 can change of 10's of ms on various time scales

polar motion (aka Chandler Wobble): quasi-circular with ~ 10 m radius annual & 433 day periodicities

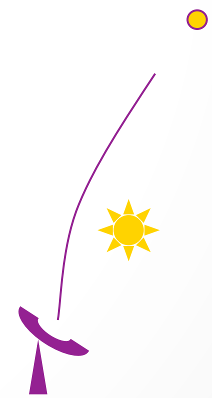
BKG Sonderheft "Earth Rotation" (1998)

“Physics” effects

- Annual aberration
  - due to Earth orbital motion
  - 20” amplitude
  - annual period



- Gravitational bending/delay
  - 1.75” grazing sun
  - 4 mas at 90 deg from Sun
  - effect of general relativity



Polar motion from 1962 to 2000

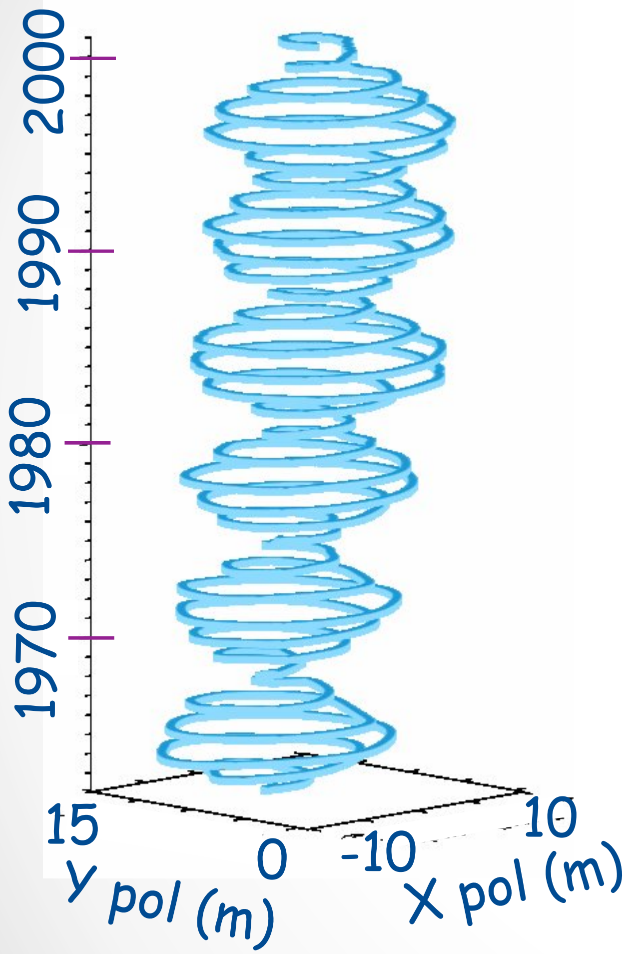
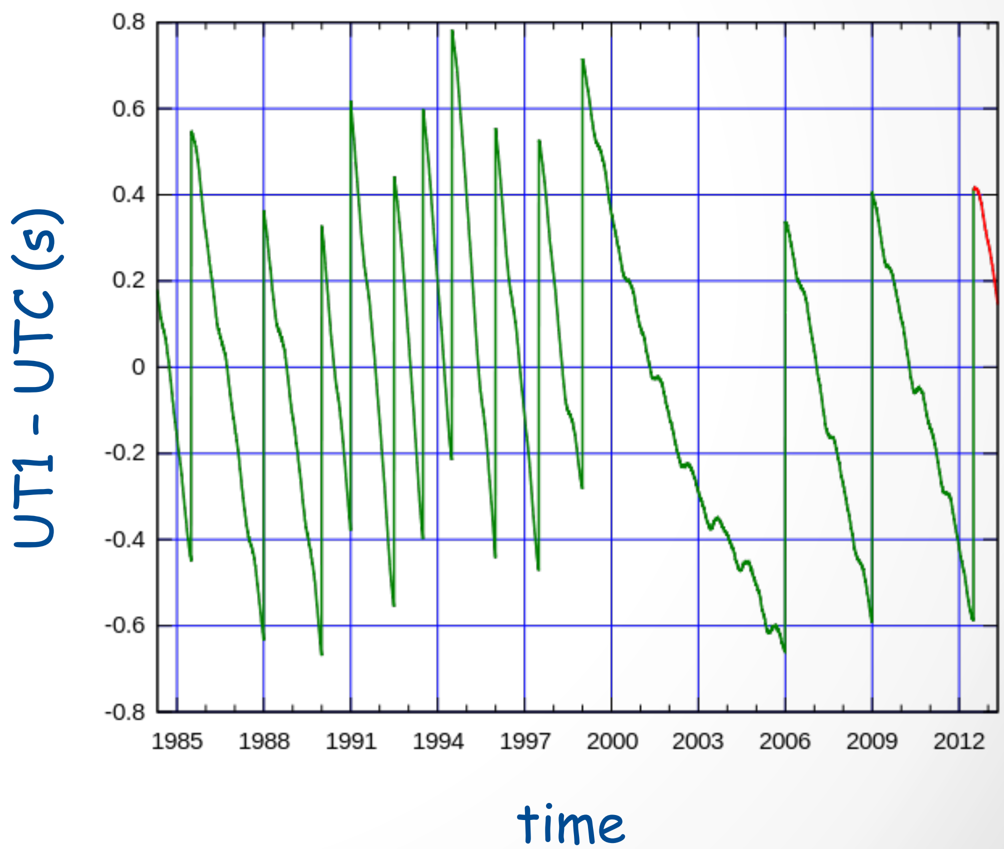


Image: courtesy H. Schmitz-Huebsch, DGFI Munich

UT1 from 1985 to 2013



Propagation Medium Effects

- Intergalactic medium
- Interstellar medium
- Interplanetary medium
- Ionosphere: ~ 2m at 2 GHz (also Faraday rotation)
- Wet troposphere: 0-30 cm at zenith
- Dry troposphere: 2.3 m at zenith

Only tropospheric effects are included in corr. models

Other Modeled Effects

- Linear clock drift
  - $\leq 10^{-12}$ (sec / sec)

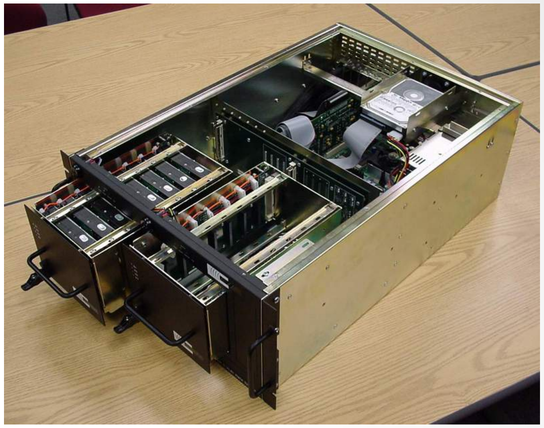
- Solid earth tides & ocean and atmosphere loading ~ 30 cm

Neglected Effects

- clock errors
  - non-linear H maser drift
  - ns level on t scale of a day
- source structure
  - ~ 5 cm over years
  - due to movement of components within the source
- antenna position
  - ~ 1cm thermal deformation (quasi-diurnal)
  - structural deformations due to weight
  - local ground deformation
  - crustal plate motion

Data sources

- Mark 5A
  - direct replacement for tapes
  - needs SUs
  - able to playback Mark 5B recorded modules
- Mark 5A+
  - able to playback Mark 5B recorded modules
- Mark 5B
  - no external de-formatter
  - max record/playback rate 1 Gb/s
- Mark 5B+
  - double speed recording M5B 2 Gb/s



- Mark 5C
  - records mk5b or vdif format
  - recording speed 4 Gb/s
- Mark 6
  - linux file system
  - recording speed 16 Gb/s
- Linux files
  - e.g. SAN

Fringe detection & parameter estimation

- output of correlator is an array of
  - FX: complex visibilities
  - XF: complex lags
- as a function of
  - time
  - frequency channel
  - frequency sub-channel (FX) or lag index (XF)
  - polarization
  - baseline
- post-processing software (e.g. fourfit) finds delay and rate that maximize the coherent summed correlation amplitude

Fringe Rotation

The natural fringe rate is : $\omega_{\mathrm{LO}} \frac{d\tau}{dt}$ *differential Doppler shift*

where: $$\frac{d\tau}{dt} = -\frac{1}{c} \omega_e \, u \, \cos\delta$$

δ declination of the source

ω_e angular velocity of Earth (7.2×10^{-5} rad/s)

u E-W component of baseline projection perpendicular to the line of sight

Geodetic VLBI fringe rates are of order 10 kHz

Fringe Rotation – a subtlety

- If done at the original RF, a delay model by itself would produce the correct Doppler shift
- Since we process at baseband, we need to have separate delay and phase models

Fringe Rotation Implementation

- Usually implemented by multiplying data stream by a complex “rotator” – sine & cosine waves of the appropriate frequency

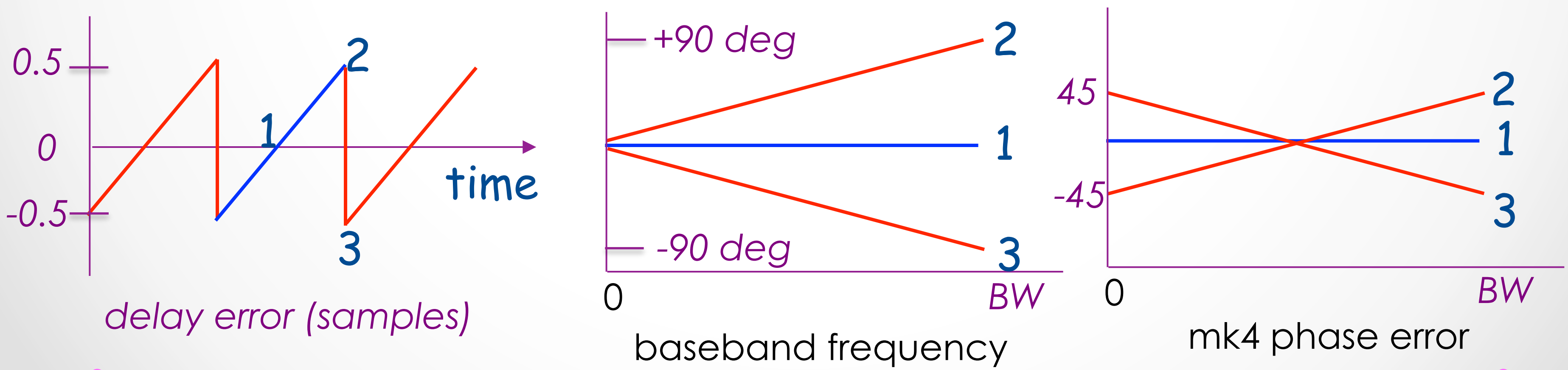
- Essentially lossless when done in floating point, but hardware correlators approximate the sine wave, often by a 3-level waveform

- Lost power (~3% for a 3-level sinusoid) is distributed into harmonics of fringe frequency, and average out

Technical Details

discrete samples

- data can only be aligned to within +/- 0.5 sample of the true delay
- the delay error induces a phase slope across each frequency channel, of at most 90 deg
- mk4 hardware is designed to keep phase error at midband 0 deg, so range is +/- 45 deg with minimal coherence loss (3.5%)



Technical Details

discrete samples

- in a software correlator, such as difx, this loss is much less
- data are corrected post-FFT with a phase ramp
- phase error is then only across the FFT channel width

Technical Details

integration

- residual visibilities (and lag values) change slowly, allowing their summing over time to reduce data output rates
- as model errors grow, so do the rates of change of the residual quantities
- in order to keep coherence loss low, the fringe phase shouldn't change by more than 0.5 rad during an AP
- for a fringe rate error of $\Delta\omega$ Hz, this implies that $\Delta T \leq 1 / (4\pi\Delta\omega)$

Technical Details

correlator beam

- primary antenna beam typically large:

  $\lambda / D_A \sim$ a few minutes of arc at X-band

- the interferometer fringe spacing is much smaller:

  $\lambda / B \sim$ milliarcseconds at X-band

Technical Details

correlator beam

- spacing & orientation of fringes continually changes
- fringe rotator stops fringe pattern at phase center
- residual pattern varies with time, more so farther from the phase center
- integration period then determines region of sky where phase stays coherent

Technical Details

correlator beam – effect of T_{AP}

$$R \approx 1 - \frac{\left[\dfrac{\pi\, \omega_e\, t\, r}{\theta}\right]^2}{6}$$

- R = reduced visibility amplitude loss
- ω_e = Earth angular velocity (rad/s)
- t = tInt
- r = distance from the phase centre (rad)
- θ = λ/d beamwidth (rad)

Technical Details

correlator beam – delay axis

- *decoherence due to channel width*
- *generally not parallel to delay rate dimension*
- *due to finite # of lags (or vis.)*

$$\frac{A(\tau)}{A(\tau_0)} \sim 1 - \frac{(\pi \Delta\nu\tau)^2}{6}$$

- A = amplitude
- $A(\tau_0)$ = ampl. at phase centre
- $A(\tau)$ = ampl. At wrong delay
- $\Delta\nu$ = bandwidth per spectral channel
- τ = group delay

Operational Correlators

DiFX @ Bonn

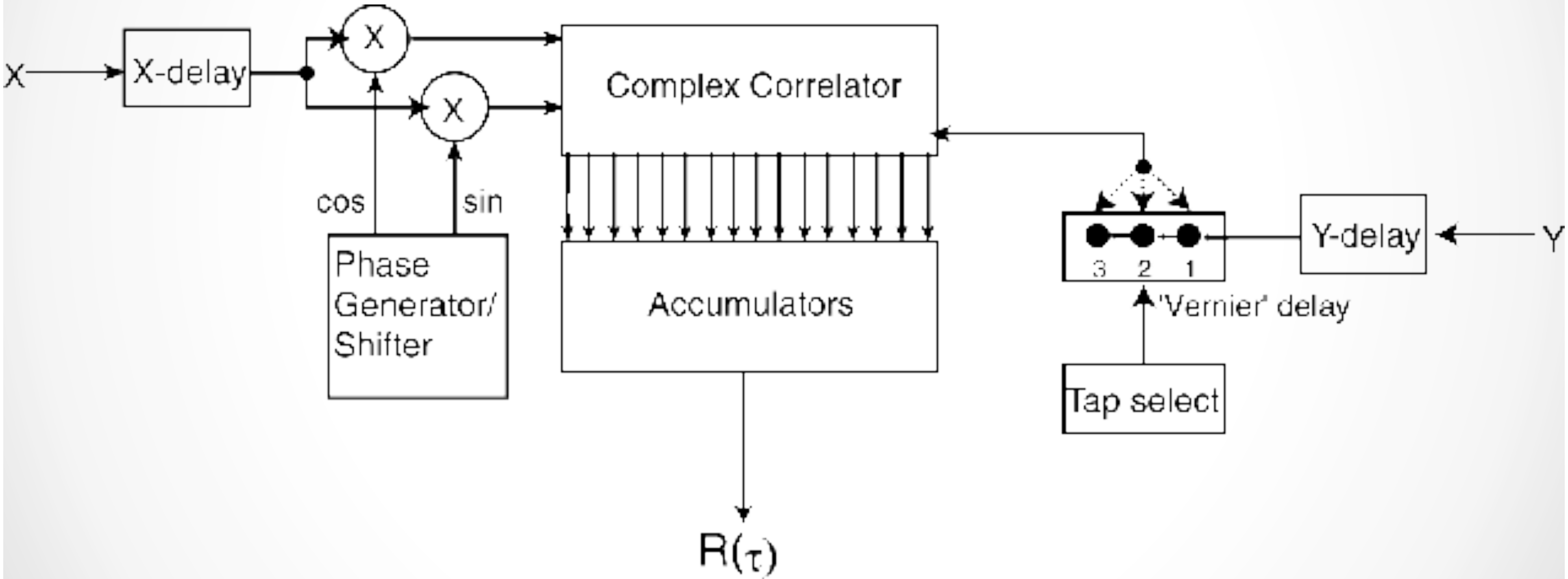
FX, software

Mark IV @ Haystack

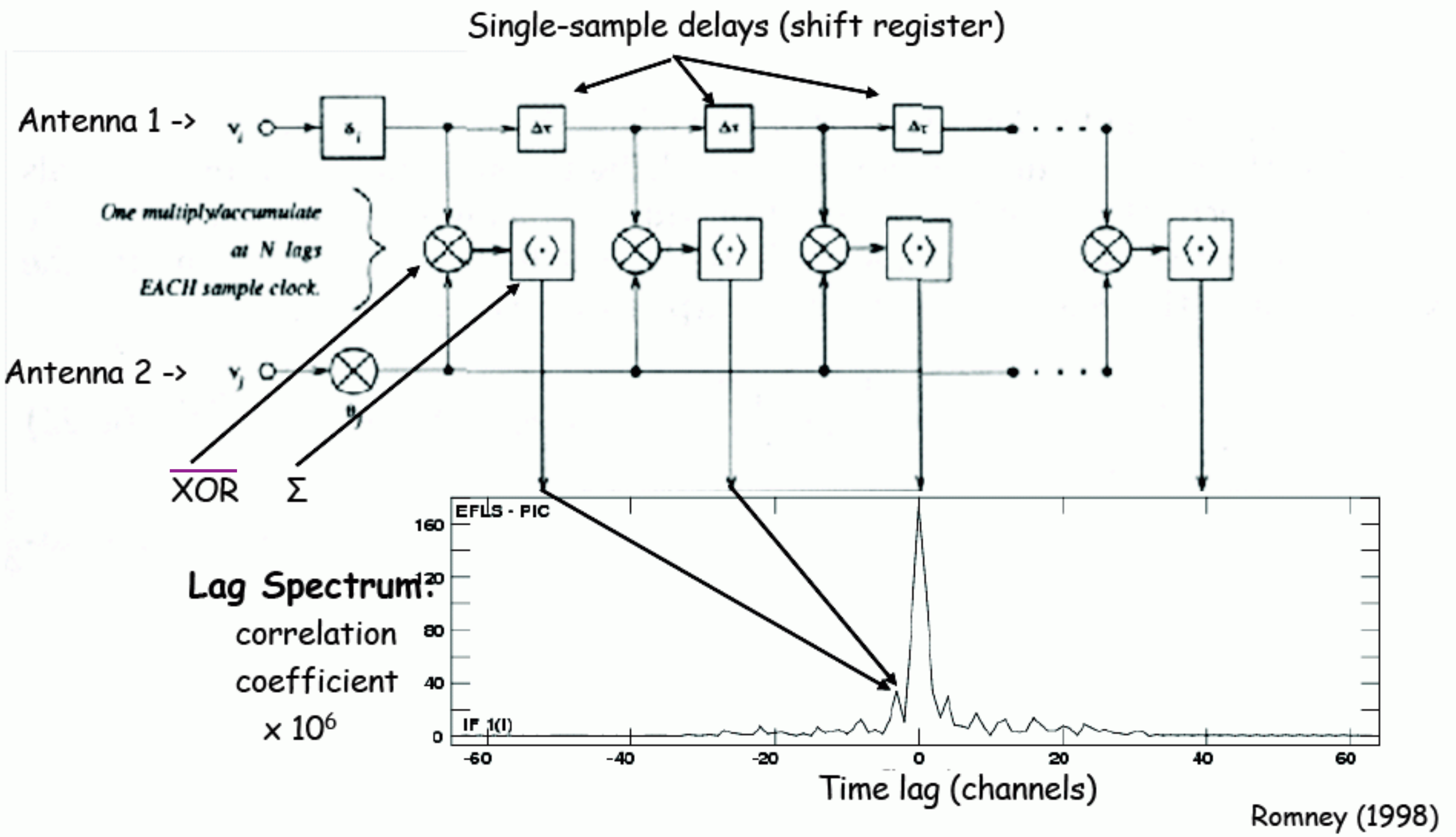
XF, hardware

Hardware Correlation

Mark 4 correlation cell



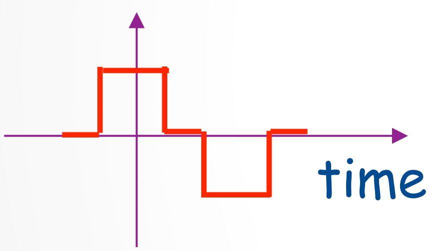
Lag Correlators: Block Schematic

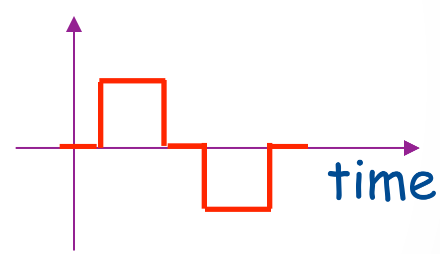


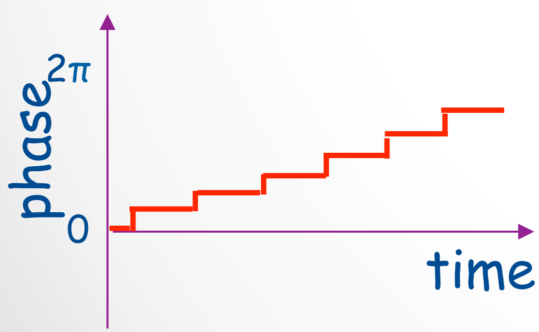
Fringe Rotation

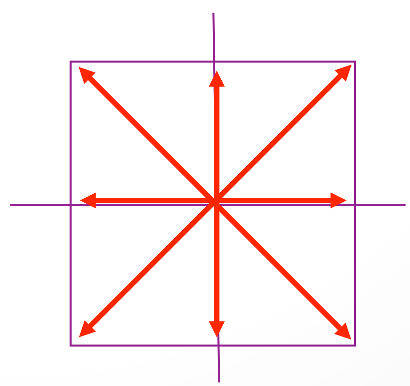
Multiplication the data by a 3-level approx. of sine wave:

$\cos(-2\pi\omega_{LO}\tau)$ **and** $\sin(-2\pi\omega_{LO}\tau)$





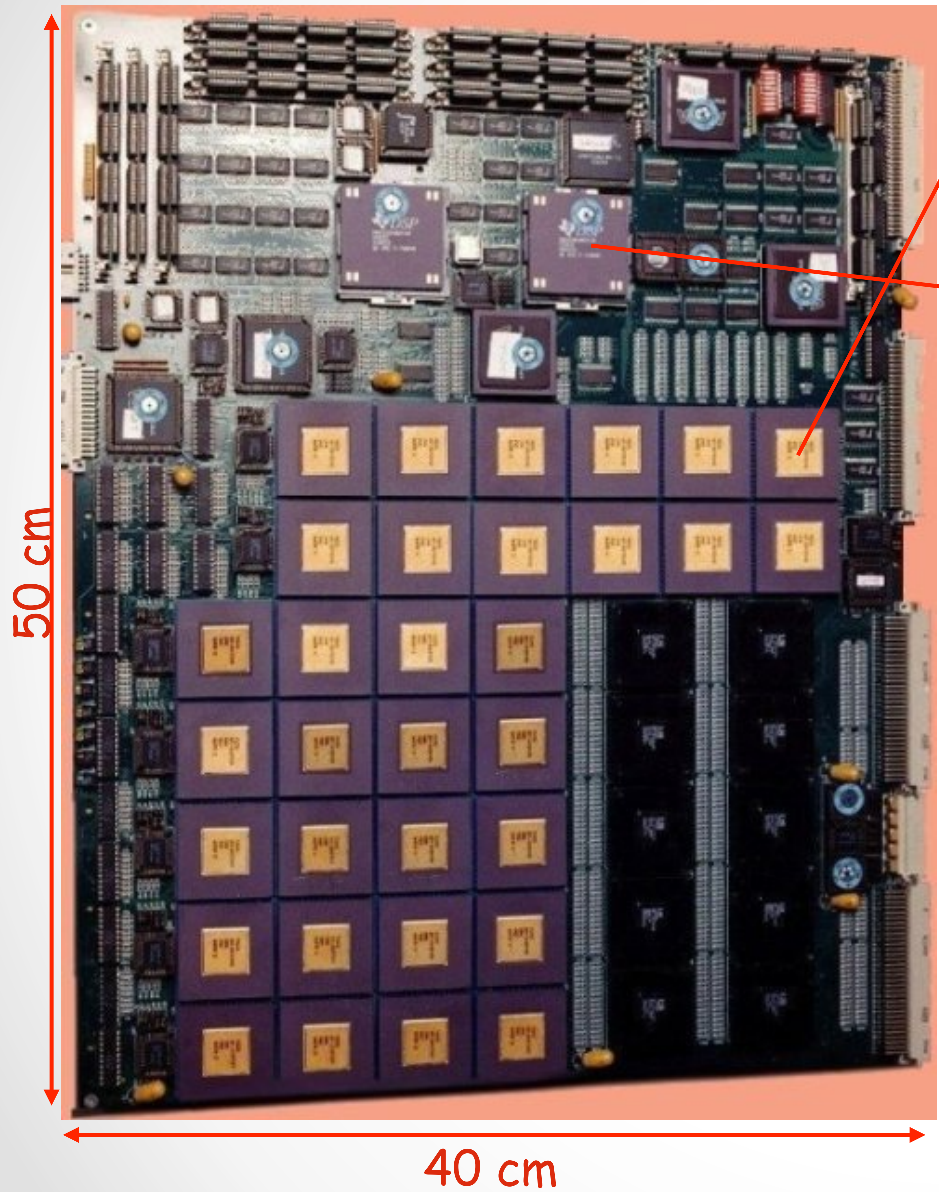




3-level phasor

Plots adapted from Thomson Moran and Swenson

Mark IV Correlator Board



32 correlator chips

8 cross-bar chips

2 digital signal processors (DSP) chips (read CF header)

1 crate:

8 correlator boards

2 input boards

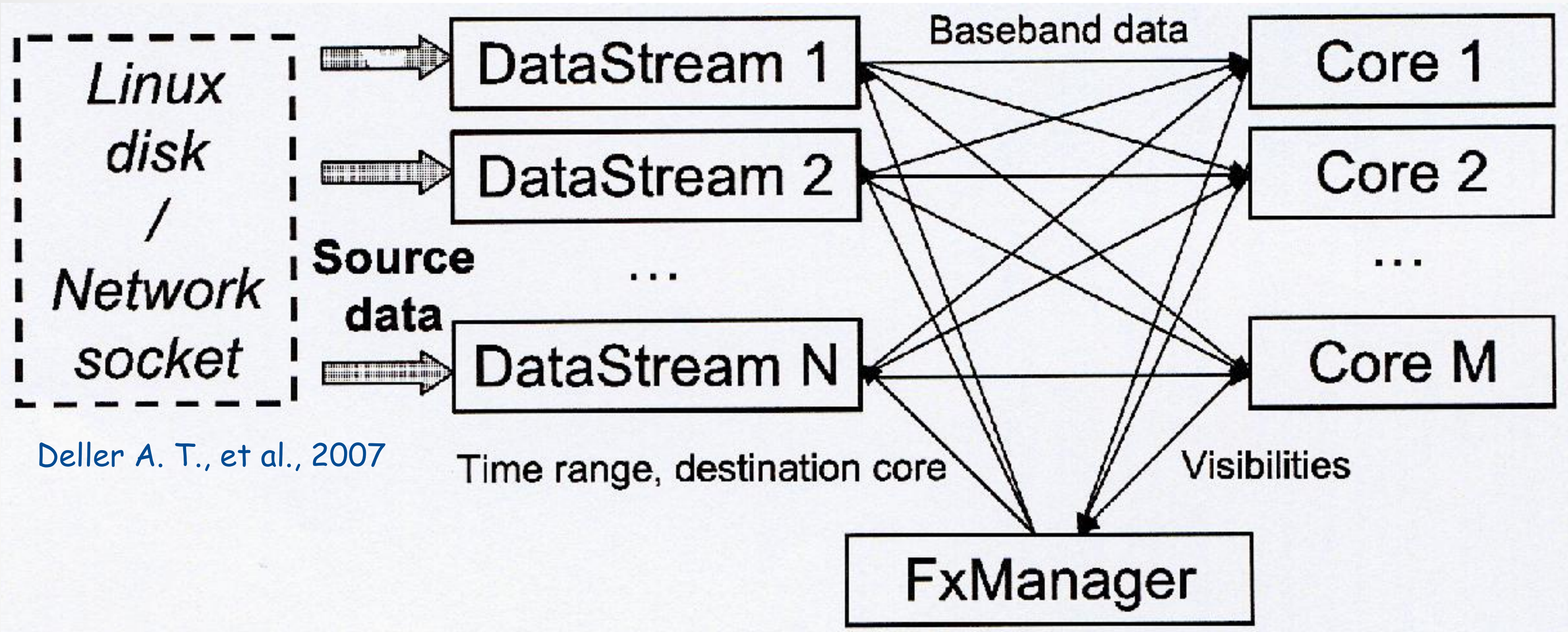
1 control board

Mark IV has 2 crates =>
Total power ~ 1000
Pentiums of the 90's

DiFX Software Correlator

- *FX correlator, originally written by Adam Deller to facilitate his PhD research in 2006*
- *Now improved & maintained by an international team of ~12 developers*
- *Widely used (VLBA, LBA, MPIfR, …)*
- *Executes on a cluster of high-performance servers, using MPI and Intel's IPP library*
- *Extremely flexible (e.g.):*
  - *Multiple phase centers within FOV*
  - *Pulsar binning*
  - *eVLBI*
  - *RFI excision*
  - *Phase cal and Tsys extraction*
  - *Input data formats: LBA, Mk4, VLBA, VDIF*
  - *Output datasets to FITS-IDI or Mk4/fourfit*

DiFX Architecture

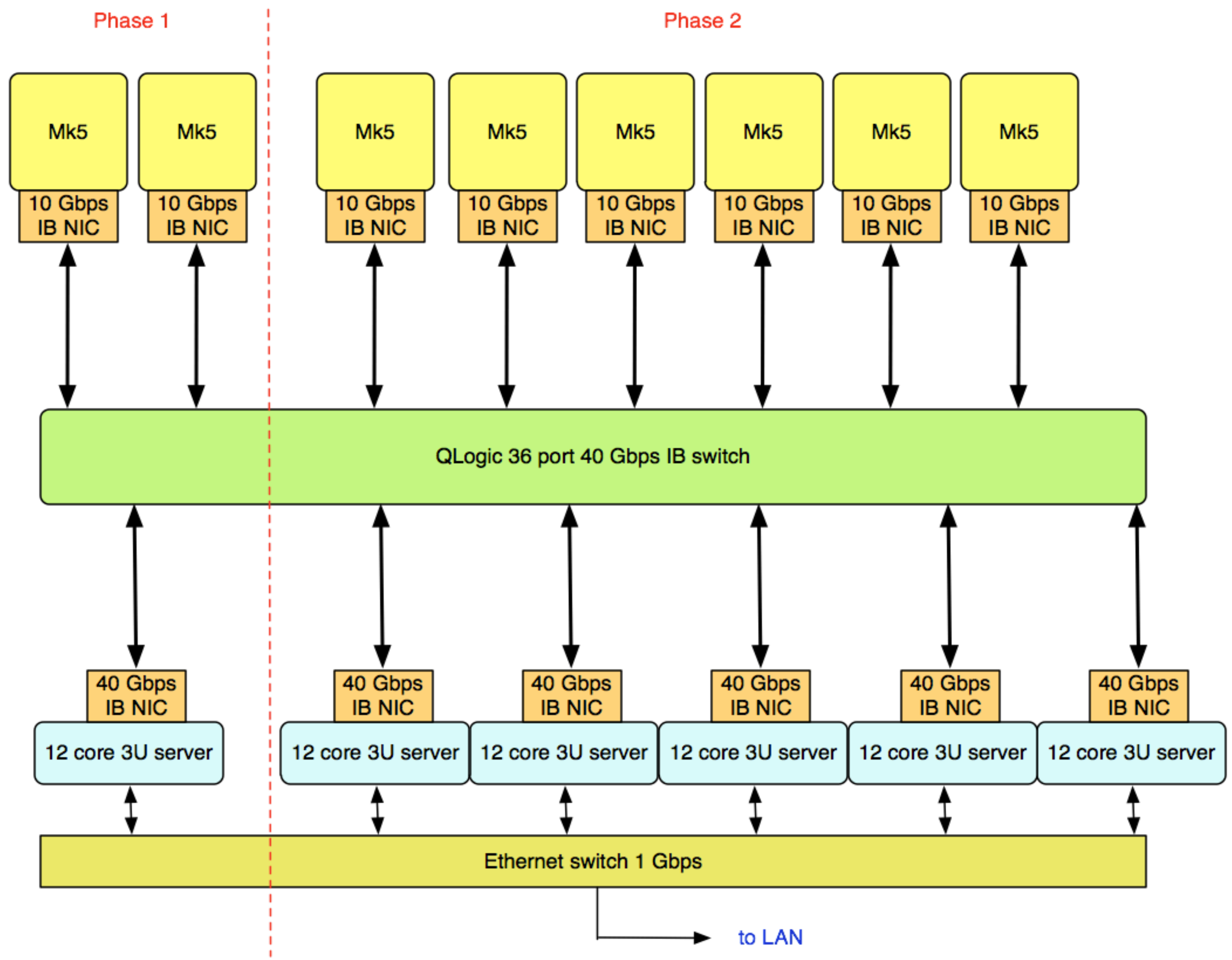


- datastream nodes deformat and delay
- time slice for all antennas sent to same core node
- fringe rotation, FFT, complex mult., sum done in core
- FX manager orchestrates all, and writes results out

Software Correlation Hardware

- high speed signal transport & routing infrastructure
  - InfiniBand
  - 10 gigE
- server-class motherboards
  - possibly multiple CPU chips
  - multiple (4-8) cores per chip
- modest amounts of RAM for buffering (~8 GB)
- Intel architecture w/ Intel Performance Primitives package (no longer essential)

can be run on a desktop machine!


Phase 1
Phase 2
Mk5
10 Gbps IB NIC
QLogic 36 port 40 Gbps IB switch
40 Gbps IB NIC
12 core 3U server
Ethernet switch 1 Gbps
to LAN

Haystack DiFX Cluster

- Six Supermicro 3U server chassis, each having:
  - X8DAH+-F motherboard
  - 2 hexacore Intel Xeon X5650 CPU's
  - 24 GB RAM
  - 1 TB hard disk
  - 40 Gb/s IB/HCA
- QLogic Infiniband switch
  - 36 ports
  - 40 Gb/s
- Eight 10 Gb/s Infiniband adapter cards for the Mk5's
- Total cost ~$34K

MPIfR DiFX Cluster

RAIDs

RAIDs + fxmanager

nodes + frontend and frontend2

nodes

- 60 compute nodes
- 8 cores per node
- 20 GBps InfinBand
- Network cards
- 10 RAIDs (220 TB)
- 1 service node (with keybord & monitor)
- 2 user interaction nodes (frontend & frontend2)

Correction Factors

	amplitude mk4	amplitude difx	snr mk4	snr difx
Van Vleck - 2 level	0.64	0.64	0.64	0.64
Van Vleck - 4 level	0.87*	0.88	0.87	0.88
rotator – 3 level	0.85	-	0.96	-
fractional bit	0.966	1.0	0.966	1.0

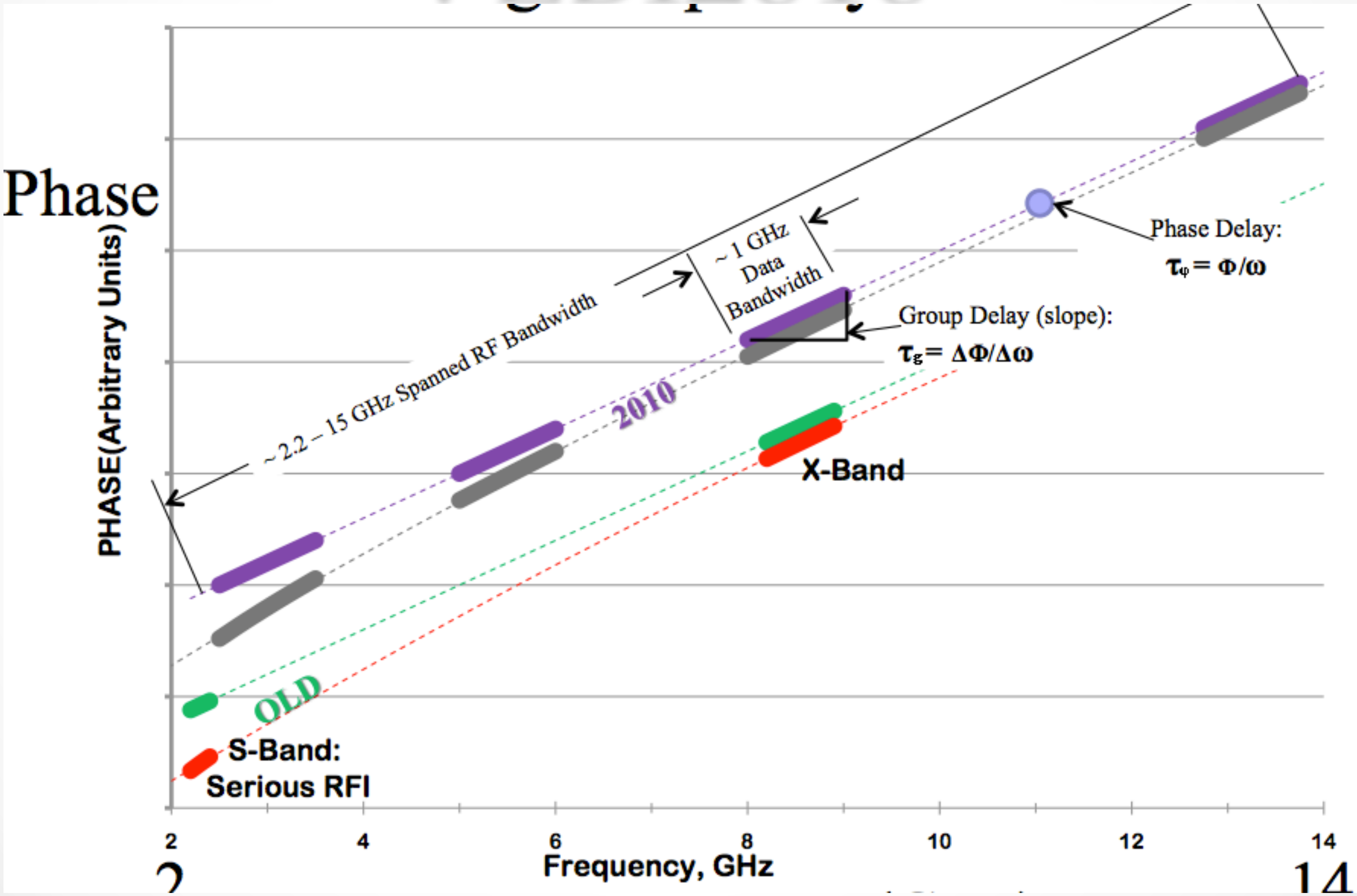
- arithmetic
- sampler state statistics
- bandpass

* assuming inner products discarded

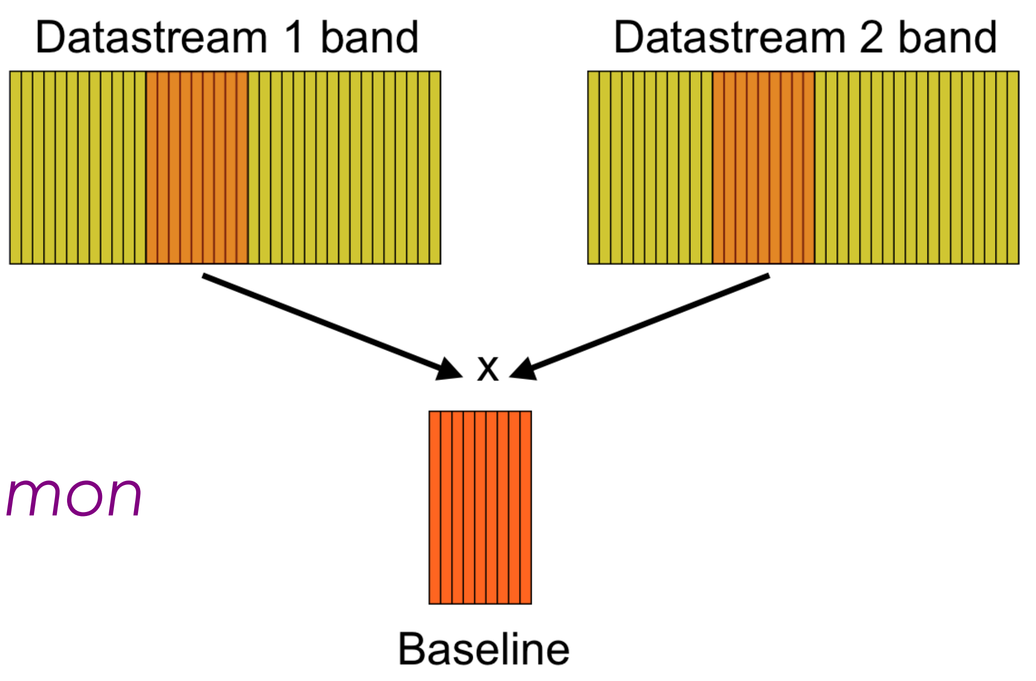
Summary of Software Correlator Advantages

- hardware design enormously leveraged
- faster development cycle
- economy of scale in PC marketplace
- floating point precision
- characteristics good match for FX architecture
- features easily added
- most new VLBI correlators are software – possible exception of SKA

VLBI2010


Phase
PHASE(Arbitrary Units)
~2.2 – 15 GHz Spanned RF Bandwidth
~ 1 GHz Data Bandwidth
Phase Delay:
$\tau_\varphi = \Phi/\omega$
Group Delay (slope):
$\tau_g = \Delta\Phi/\Delta\omega$
2010
X-Band
OLD
S-Band:
Serious RFI
2
4
6
8
10
12
14
Frequency, GHz

Example of DiFX Flexibility: correlating 32 MHz x 8 MHz ch

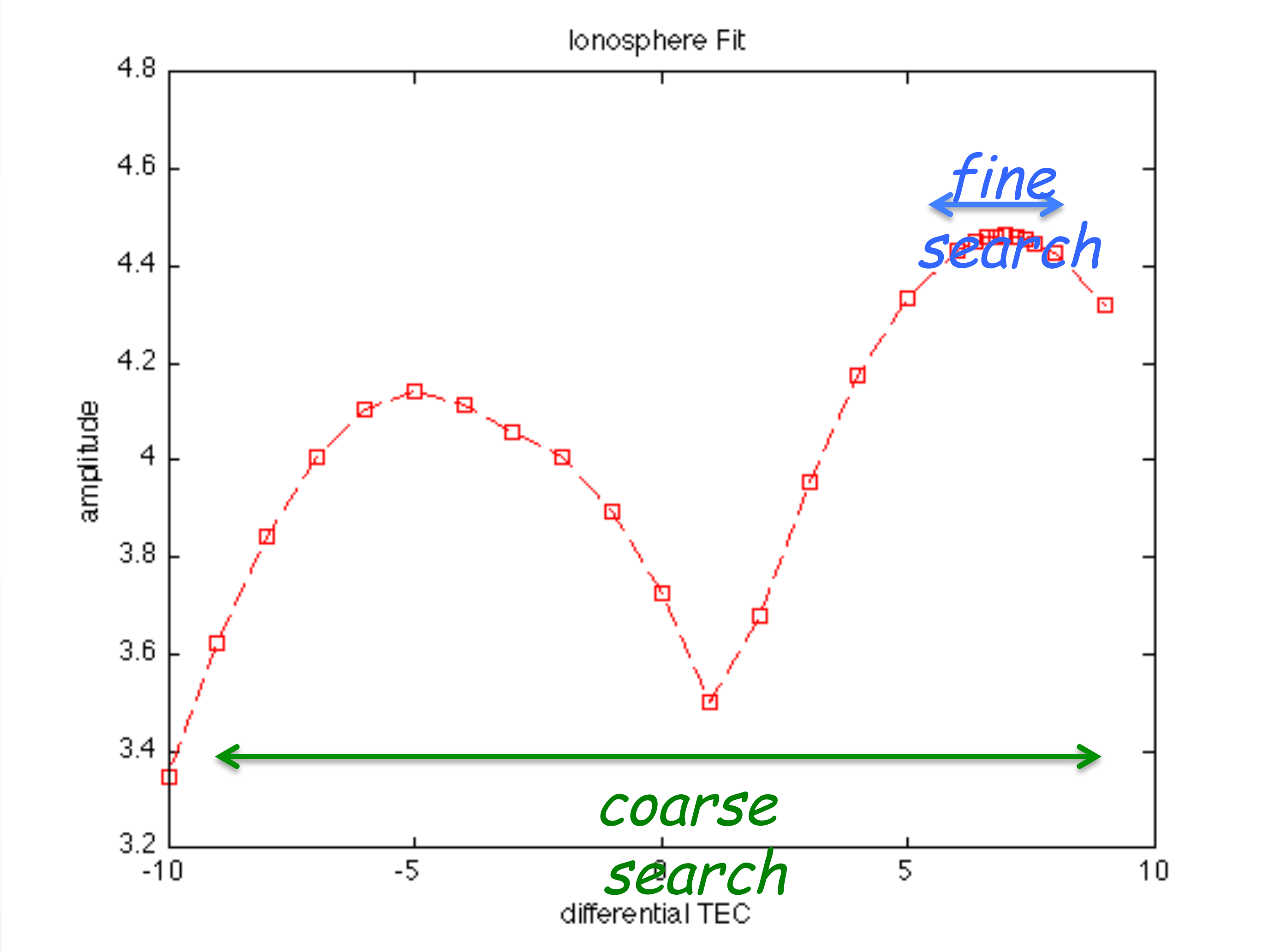


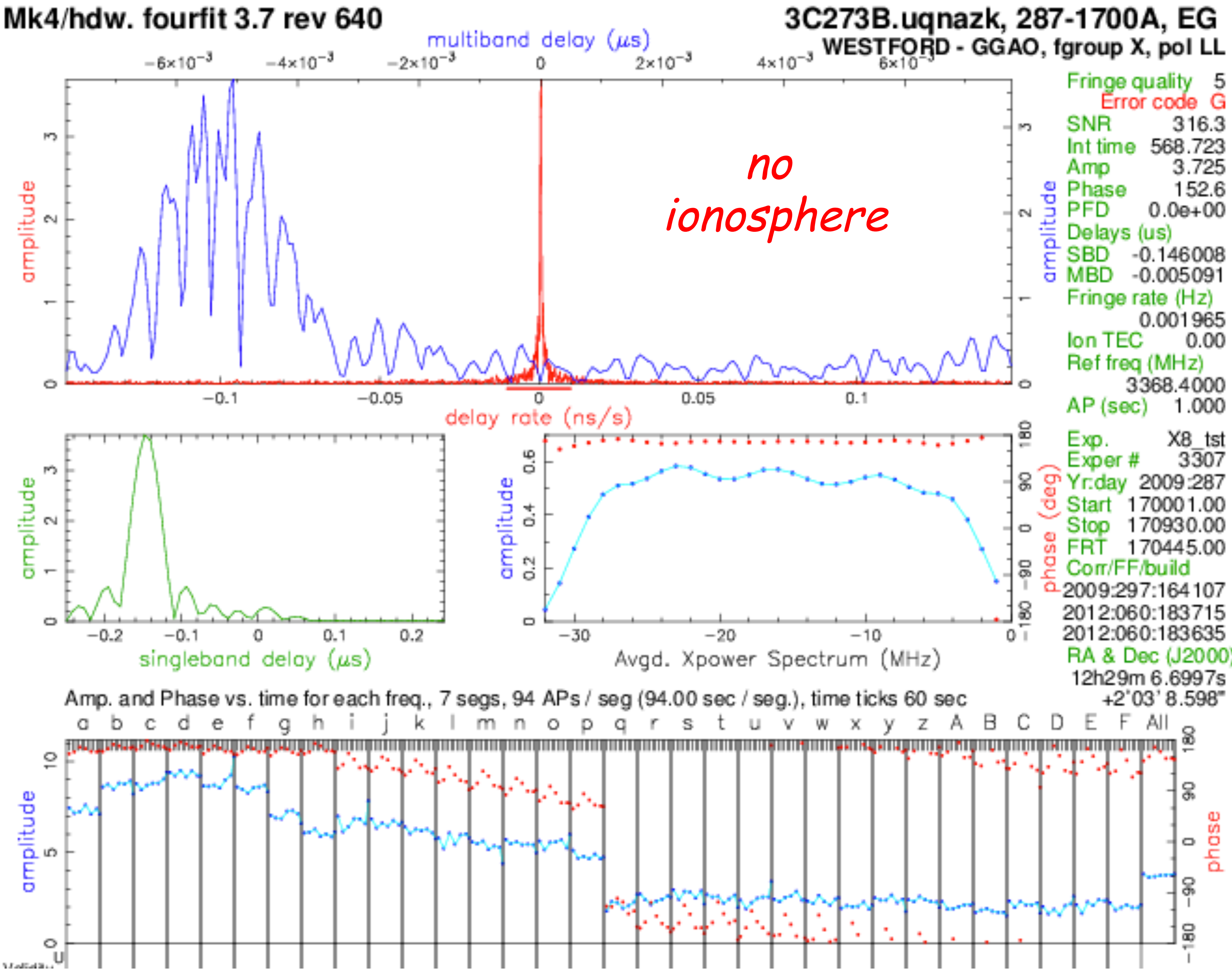
- *use zoom mode*
- *channelize to a common divisor frequency*
- *e.g. 0.5 MHz*
- *match corresponding channels prior to XY* multiply*

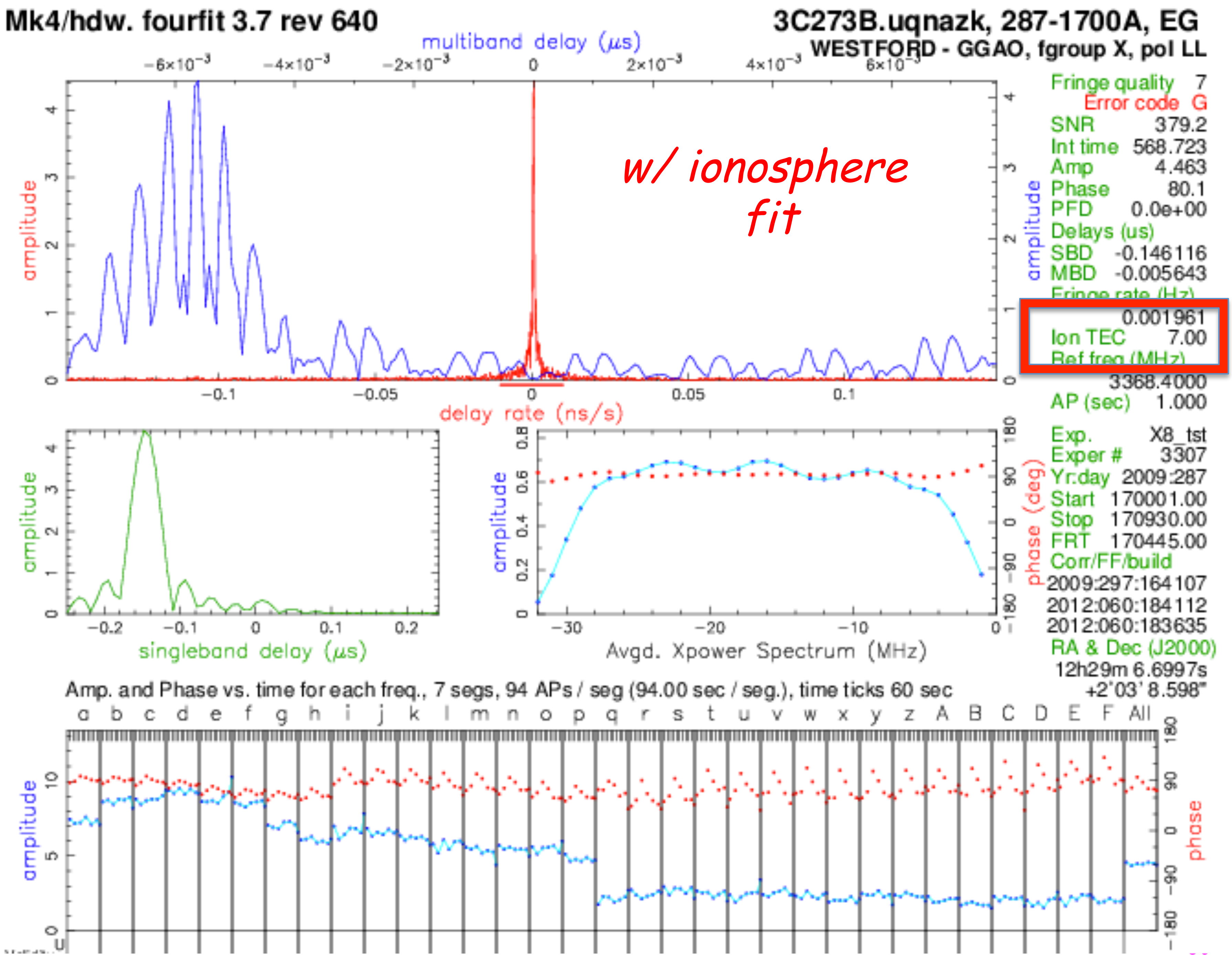
Ionosphere

- phase of each freq channel affected by differential path integral of charges (Total Electron Content)
- 1 TEC unit = 10^{16} electrons / m^2
- $\Delta \phi = c \times \Delta TEC / f$
- differential TEC can be fit and/or specified *a priori*
  - all-sky models from GPS available, but not yet used
  - fit made difficult by nonlinearity
  - search for peak of coherent sum of all bands

Fourfit Ionosphere Fit




Mk4/hdw. fourfit 3.7 rev 640
3C273B.uqnazk, 287-1700A, EG
WESTFORD - GGAO, fgroup X, pol LL
multiband delay (μs)
no ionosphere
amplitude
delay rate (ns/s)
Fringe quality 5
Error code G
SNR 316.3
Int time 568.723
Amp 3.725
Phase 152.6
PFD 0.0e+00
Delays (us)
SBD -0.146008
MBD -0.005091
Fringe rate (Hz)
0.001965
Ion TEC 0.00
Ref freq (MHz)
3368.4000
AP (sec) 1.000
Exp. X8_tst
Exper # 3307
Yr:day 2009:287
Start 170001.00
Stop 170930.00
FRT 170445.00
Corr/FF/build
2009:297:164107
2012:060:183715
2012:060:183635
RA & Dec (J2000)
12h29m 6.6997s
+2°03' 8.598"
singleband delay (μs)
Avgd. Xpower Spectrum (MHz)
phase (deg)
Amp. and Phase vs. time for each freq., 7 segs, 94 APs / seg (94.00 sec / seg.), time ticks 60 sec
a b c d e f g h i j k l m n o p q r s t u v w x y z A B C D E F All
phase


Mk4/hdw. fourfit 3.7 rev 640
3C273B.uqnazk, 287-1700A, EG
WESTFORD - GGAO, fgroup X, pol LL
multiband delay (μs)
delay rate (ns/s)
amplitude
w/ ionosphere fit
Fringe quality 7
Error code G
SNR 379.2
Int time 568.723
Amp 4.463
Phase 80.1
PFD 0.0e+00
Delays (us)
SBD -0.146116
MBD -0.005643
Fringe rate (Hz)
0.001961
Ion TEC 7.00
Ref freq (MHz)
3368.4000
AP (sec) 1.000
Exp. X8_tst
Exper # 3307
Yr:day 2009:287
Start 170001.00
Stop 170930.00
FRT 170445.00
Corr/FF/build
2009:297:164107
2012:060:184112
2012:060:183635
RA & Dec (J2000)
12h29m 6.6997s
+2°03' 8.598"
singleband delay (μs)
Avgd. Xpower Spectrum (MHz)
phase (deg)
Amp. and Phase vs. time for each freq., 7 segs, 94 APs / seg (94.00 sec / seg.), time ticks 60 sec
a b c d e f g h i j k l m n o p q r s t u v w x y z A B C D E F All
phase

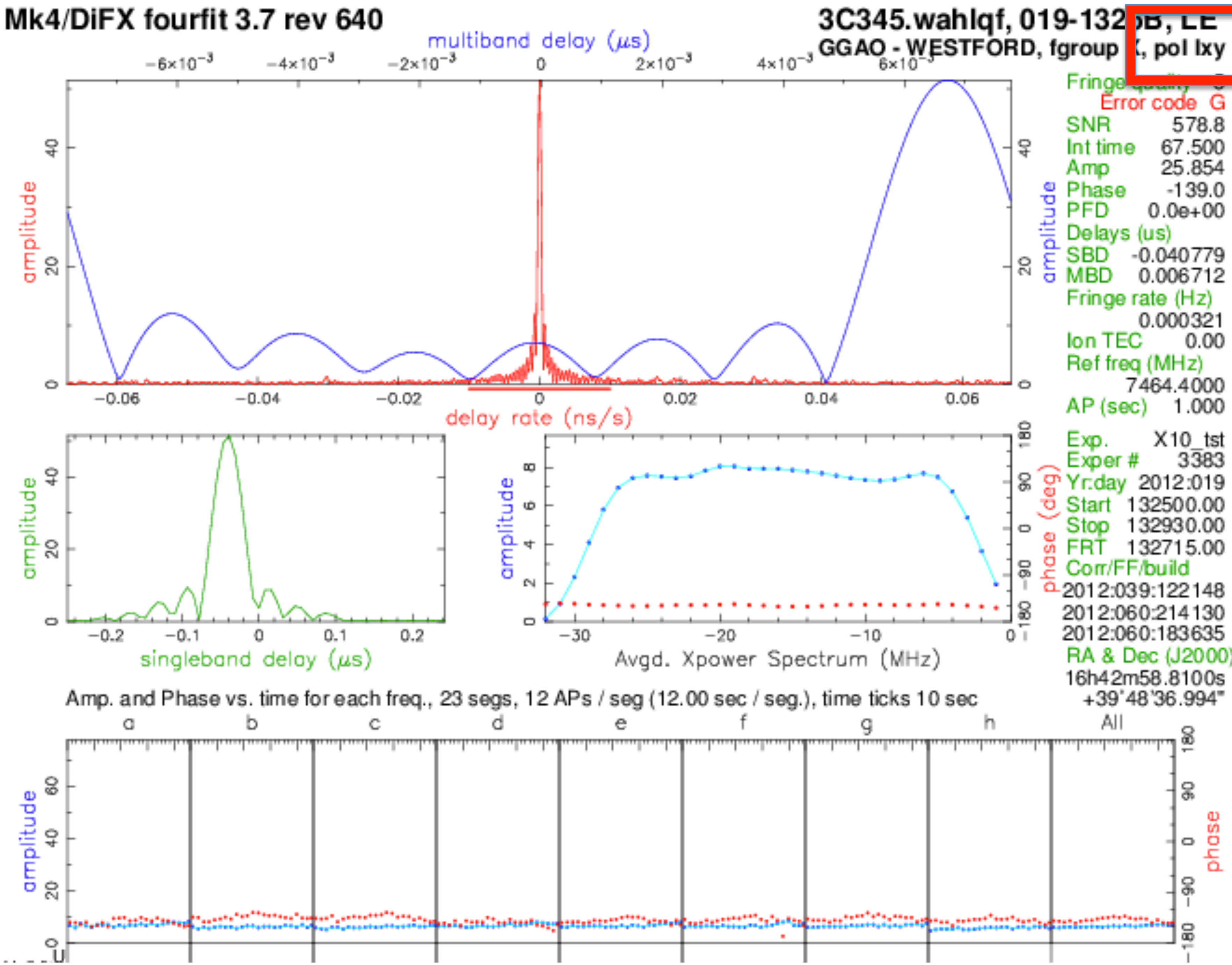
Polarization in HOPS

- Maximize sensitivity in τ_g by combining all 4 Stokes polarization products
- Form an approximation to Stokes I:
  - from the 4 correlation products form

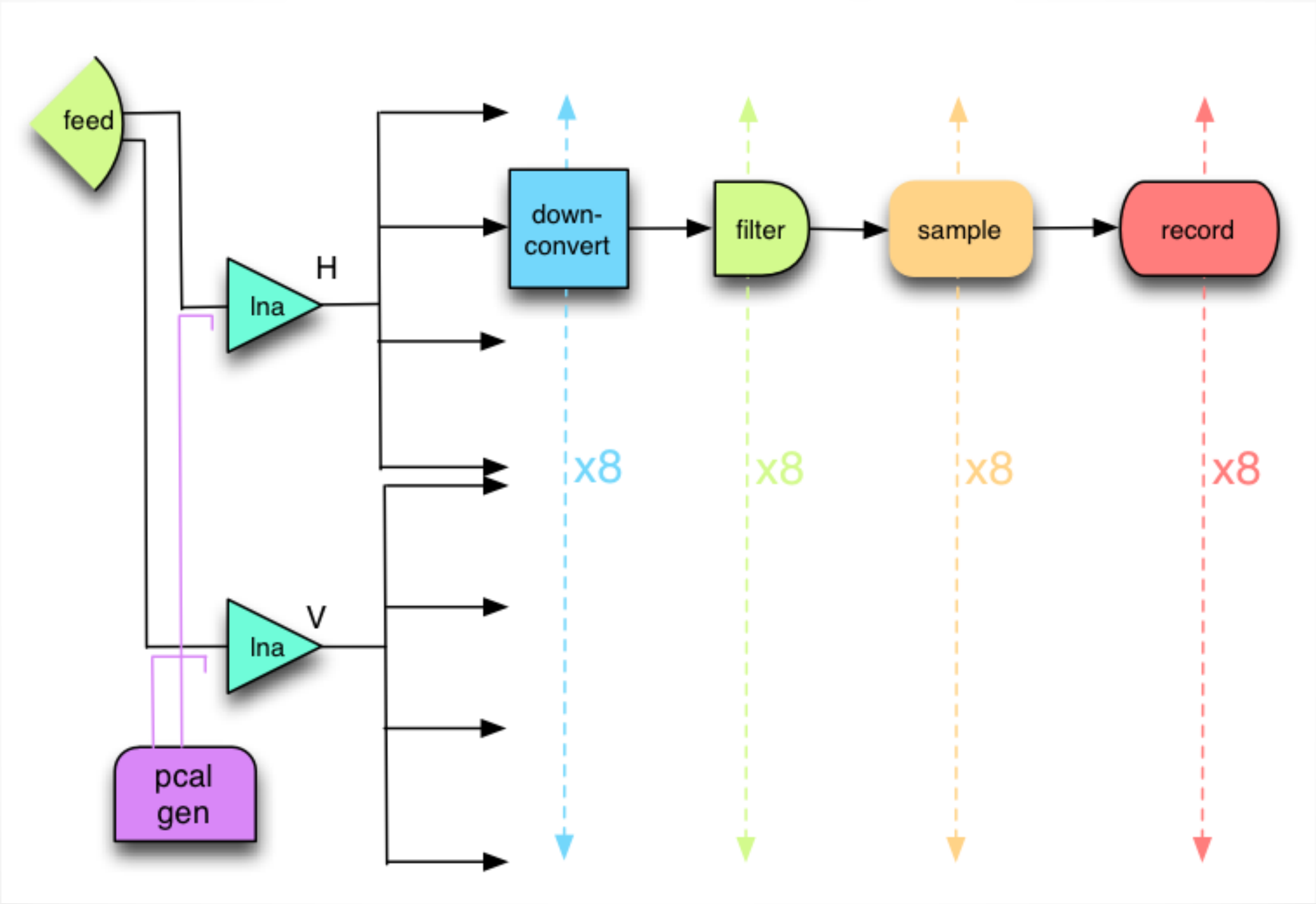
    $$I \cong (HxH + VxV) \cos\Delta + (HxV - VxH) \sin\Delta$$

    Δ = differential parallactic angle
  - correct to first order in the D terms
- Also have mixed combinations to legacy stations

  e.g. {RxV, RxH, LxV, LxH}


Mk4/DiFX fourfit 3.7 rev 640
3C345.wahlqf, 019-1325B, LE
GGAO - WESTFORD, fgroup X, pol lxy
multiband delay (μs)
delay rate (ns/s)
amplitude
Error code G
SNR 578.8
Int time 67.500
Amp 25.854
Phase -139.0
PFD 0.0e+00
Delays (us)
SBD -0.040779
MBD 0.006712
Fringe rate (Hz)
0.000321
Ion TEC 0.00
Ref freq (MHz)
7464.4000
AP (sec) 1.000
Exp. X10_tst
Exper # 3383
Yr:day 2012:019
Start 132500.00
Stop 132930.00
FRT 132715.00
Corr/FF/build
2012:039:122148
2012:060:214130
2012:060:183635
RA & Dec (J2000)
16h42m58.8100s
+39°48'36.994"
singleband delay (μs)
Avgd. Xpower Spectrum (MHz)
phase (deg)
Amp. and Phase vs. time for each freq., 23 segs, 12 APs / seg (12.00 sec / seg.), time ticks 10 sec
a
b
c
d
e
f
g
h
All
phase

VLBI2010 Signal Path



Increased Postprocessing Setup Complexity

- Now have 4 frequency bands and 4 polarization products
- if 4 passes, need to be merged (*fourmer*)
- Need to correct for separate delays and phases in each signal path
- pcal has a delay ambiguity of 200 ns – need to ensure that right ambiguity is used to stitch together the broad bands

fourfit features for VLBI2010

- If necessary, explicit delay and phase offsets per
  - polarization
  - channel
  - station

- Multitone phasecal extraction uses all (desired) tones in each band to derive instrumental delay for groups of chan/pols sharing a sampler

```
if station A
  pc_mode multitone
  pc_period 30
  pc_tonemask abcdefgh 0 0 8 0 4 0 5 0
  pc_phases_l abcdefgh 12 13 11 12 24 -6 38 110
  pc_phases_r abcdefgh 11 29 14 11 64 -2 44 132
  samplers 2 abcd efgh
  delay_offs bdh 2.7 -3.65 4.778
```