Atmospheric propagation

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Outline

- Part I. Ionospheric effects on microwave signals[(1)]
- Part II. Path delays in the neutral atmosphere[(2)]

following

(1) Alizadeh, M., et al. (2013) Ionospheric effects on microwave signals

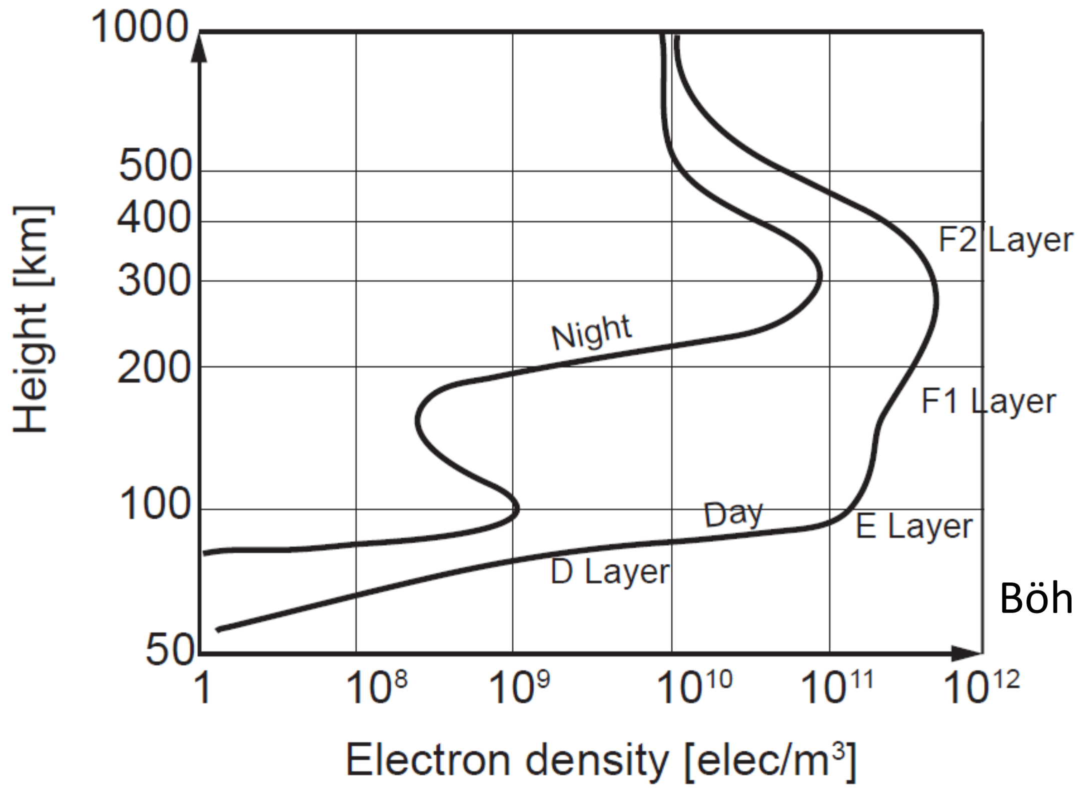
(2) Nilsson, T., et al. (2013) Path delays in the neutral atmosphere

both in Böhm, J., and Schuh, H. (eds.) Atmospheric Effects in Space Geodesy, Springer, in press 2013.

I. Ionospheric effects on microwave signals

Ionosphere

- Chapman electron density profile and the ionospheric layers D, E, and F



Böhm et al. 2013

Outline

- Group and phase velocity
- Ionosphere refractive index
- Ionospheric delay
- How to deal with ionospheric delays in geodetic VLBI

1 Group and phase velocity

- Dispersive medium (Ionosphere):
  - Propagation velocity of an electromagnetic wave is dependent on its frequency
  - Phase velocities v_{ph} and group velocities v_{gr} are different

$$v_{ph} = \lambda f \qquad v_{gr} = v_{ph} - \lambda \frac{dv_{ph}}{d\lambda}$$

- Non-dispersive medium (Neutral atmosphere):
  - Phase and group velocities are the same and are equal or lower than the speed of light c

2 Ionosphere refractive index

- Phase and group refractive index

$$v_{ph} = \frac{c}{n_{ph}}$$ >=c

not in contradiction with Theory of Relativity

$$v_{gr} = \frac{c}{n_{gr}}$$ <=c

$$n_{gr} = n_{ph} - \lambda \frac{dn_{ph}}{d\lambda}$$

2 Ionosphere refractive index

- Appleton-Hartree formula for phase refractive index

$$n_{ph}^2 = 1 - \frac{X}{1 - \frac{\frac{1}{2}Y^2\sin^2\theta}{1-X} \pm \frac{1}{1-X}\left(\frac{1}{4}Y^4\sin^4\theta + Y^2\cos^2\theta(1-X)^2\right)^{1/2}}\,, \qquad (16)$$

where

$$X = \frac{\omega_0^2}{\omega^2}\,, \qquad Y = \frac{\omega_H}{\omega}\,,$$

$$\omega_0 = 2\pi f_0 = \sqrt{\frac{N_e e^2}{\varepsilon_0 m_e}}\,, \qquad \omega_H = 2\pi f_H = \frac{B_0|e|}{m_e}\,,$$

n	complex refractive index	ν	electron collision frequency
ω	$= 2\pi f$ (radial frequency)	f	wave frequency
ω_0	electron plasma frequency	ω_H	electron gyro frequency
ε_0	permittivity of free space	B_0	magnitude of the magnetic field vector $\mathbf{B}_0$
θ	angle between the ambient magnetic field vector and the wave vector	e	electron charge
		m_e	electron mass

2 Ionosphere refractive index

- Higher order terms may be neglected (Hawarey et al. 2005)
- Phase refractive index

$$n_{ph}^{ion} = 1 - C_2 \frac{N_e}{f^2} = 1 - 40.31 \frac{N_e}{f^2}$$

- Group refractive index

$$n_{gr}^{ion} = 1 + C_2 \frac{N_e}{f^2} = 1 + 40.31 \frac{N_e}{f^2}$$

3 Ionospheric delay

- Group delay or phase advance of signals

$$\Delta\rho^{ion} = \int n ds - \int ds_0$$

- First order approximation

$$\Delta\rho_{ph}^{ion1} = -\frac{C_2}{f^2}\int N_e \, ds_0$$

- Phase advance and group delay

$$\Delta\rho_{ph}^{ion1} = -\frac{40.31}{f^2}\int N_e \, ds_0 \qquad \Delta\rho_{gr}^{ion1} = \frac{40.31}{f^2}\int N_e \, ds_0$$

3 Ionospheric delay

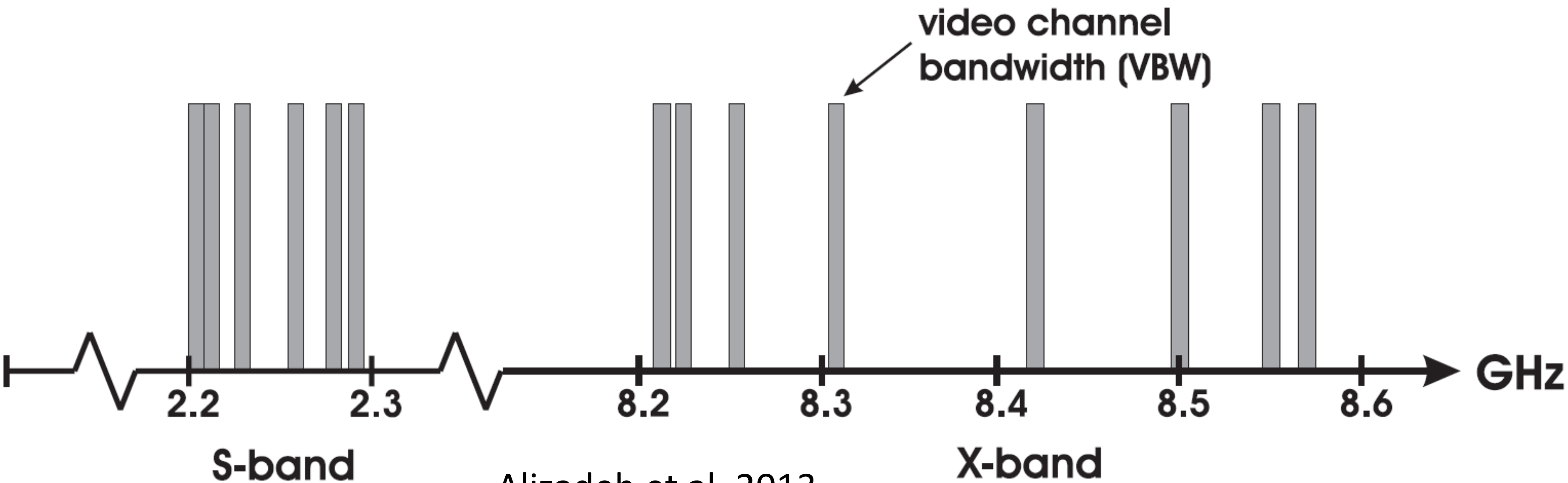
- Integrated electron density: Total Electron Content
  - TEC: Total amount of free electrons in a cylinder with a cross section of 1 m^2
- 1 TECU = 10^{16} electrons per m^2

$$\Delta\rho_{gr}^{ion} = \frac{40.31}{f^2} STEC \qquad [\mathrm{m}]$$

- Ionospheric delays in VLBI due to 1 TECU
  - 7.6 cm at S-Band (2.3 GHz)
  - 0.6 cm at X-Band (8.4 GHz)

4.2.4 VLBI and the ionosphere

- Typical channel distribution of a geodetic VLBI experiment



Alizadeh et al. 2013

4.2.4 VLBI and the ionosphere

- Group delay is determined as the slope of the fringe phases across the band
- VLBI group delays are not assigned to a reference frequency that is actually observed (unlike GNSS)
- "Effective frequency" is used to calculate the delays in the usual way

4.2.4 VLBI and the ionosphere

- Effective frequency

$$f_{gr} = \sqrt{\frac{\sum_{i=1}^{N}\rho_i \cdot \sum_{i=1}^{N}\rho_i(f_i - f_0)^2 - \left(\sum_{i=1}^{N}\rho_i(f_i - f_0)\right)^2}{\sum_{i=1}^{N}\rho_i(f_i - f_0) \cdot \sum_{i=1}^{N}\frac{\rho_i}{f_i} - \sum_{i=1}^{N}\rho_i \cdot \sum_{i=1}^{N}\rho_i\frac{f_i - f_0}{f_i}}}$$

  - f_0 reference sky frequency
  - f_i channel frequency
  - ρ_i correlation amplitude at channel i

4.2.4 VLBI and the ionosphere

- Ionospheric delay per baseline observation per band

$$\tau_{gr} = \tau_{if} + \frac{\alpha}{f_{gr}^2}$$

$$\alpha = \frac{40.31}{c}\left(\int N_e ds_1 - \int N_e ds_2\right) = \frac{40.31}{c}(STEC_1 - STEC_2)$$

  - τ_{gr} observed group delays
  - τ_{if} ionosphere free group delays

4.2.4 VLBI and the ionosphere

- Elimination with ionosphere free linear combination

$$\tau_{gx} = \tau_{if} + \frac{\alpha}{f_{gx}^2} \qquad \tau_{gs} = \tau_{if} + \frac{\alpha}{f_{gs}^2}$$

$$\tau_{if} = \frac{f_{gx}^2}{f_{gx}^2 - f_{gs}^2}\tau_{gx} - \frac{f_{gs}^2}{f_{gx}^2 - f_{gs}^2}\tau_{gs}$$

- Ionospheric contribution in X-Band

$$\tau_{igx} = \frac{\alpha}{f_{gx}^2} = -\frac{f_{gs}^2}{f_{gx}^2 - f_{gs}^2}(\tau_{gx} - \tau_{gs})$$

4.2.4 VLBI and the ionosphere

- Ambiguity resolution and ionosphere delays
  - have to be calculated together in an iterative approach

4.2.4 VLBI and the ionosphere

- Instrumental biases
  - Observations contain extra delay term caused by instrumental effects

$$\tau'_{gx} = \tau_{if} + \frac{\alpha}{f_{gx}^2} + \tau_{inst,x} \qquad \tau'_{gs} = \tau_{if} + \frac{\alpha}{f_{gs}^2} + \tau_{inst,s}$$

$$\begin{aligned} \tau'_{if} &= \frac{f_{gx}^2}{f_{gx}^2 - f_{gs}^2}\tau'_{gx} - \frac{f_{gs}^2}{f_{gx}^2 - f_{gs}^2}\tau'_{gs} \\ &= \tau_{if} + \underbrace{\frac{f_{gx}^2}{f_{gx}^2 - f_{gs}^2}\tau_{inst,x} - \frac{f_{gs}^2}{f_{gx}^2 - f_{gs}^2}\tau_{inst,s}}_{\hat{\tau}} \end{aligned}$$

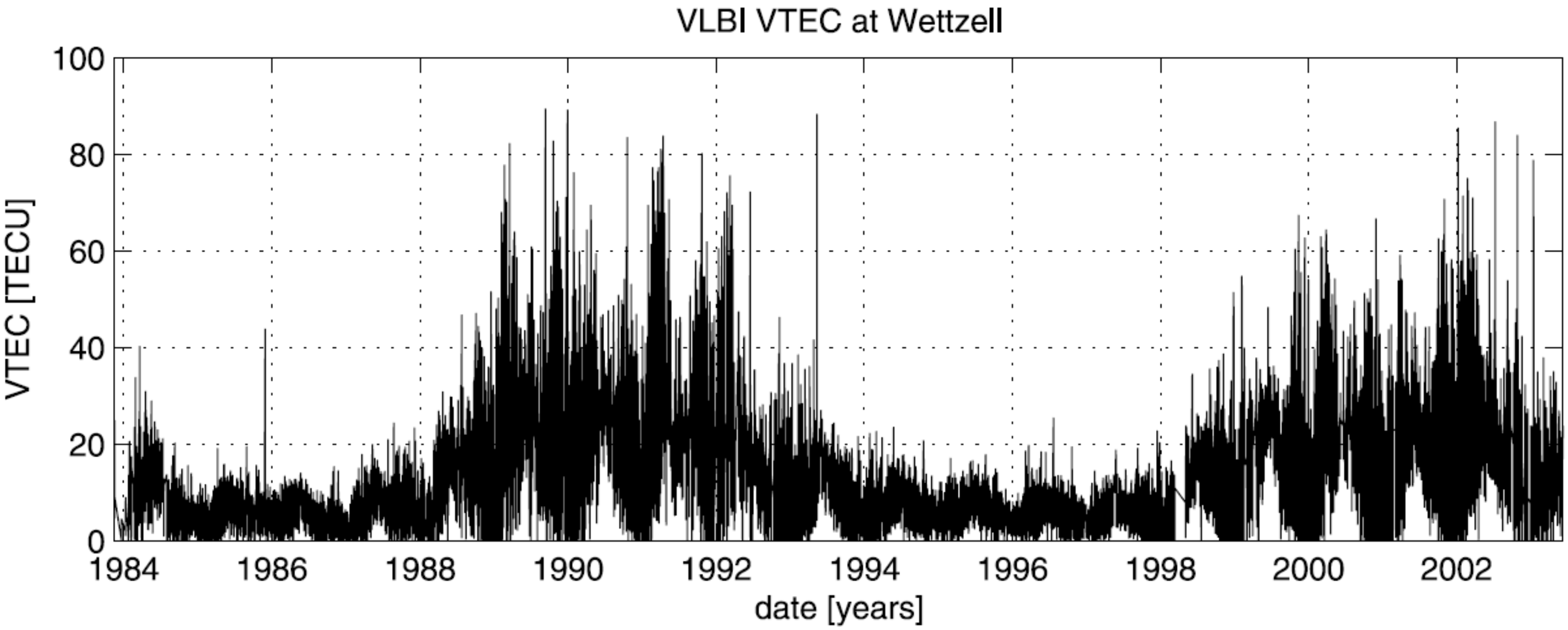
4.2.4 VLBI and the ionosphere

- Instrumental effects absorbed in clock estimates
- Ionosphere delays contain instrumental effects

$$\tau'_{igx} = -\frac{f_{gs}^2}{f_{gx}^2 - f_{gs}^2}(\tau'_{gx} - \tau'_{gs})$$
$$= \frac{\alpha}{f_{gx}^2} + \underbrace{\frac{f_{gs}^2}{f_{gx}^2 - f_{gs}^2}(\tau_{inst,s} - \tau_{inst,x})}_{\tau_{inst}}$$

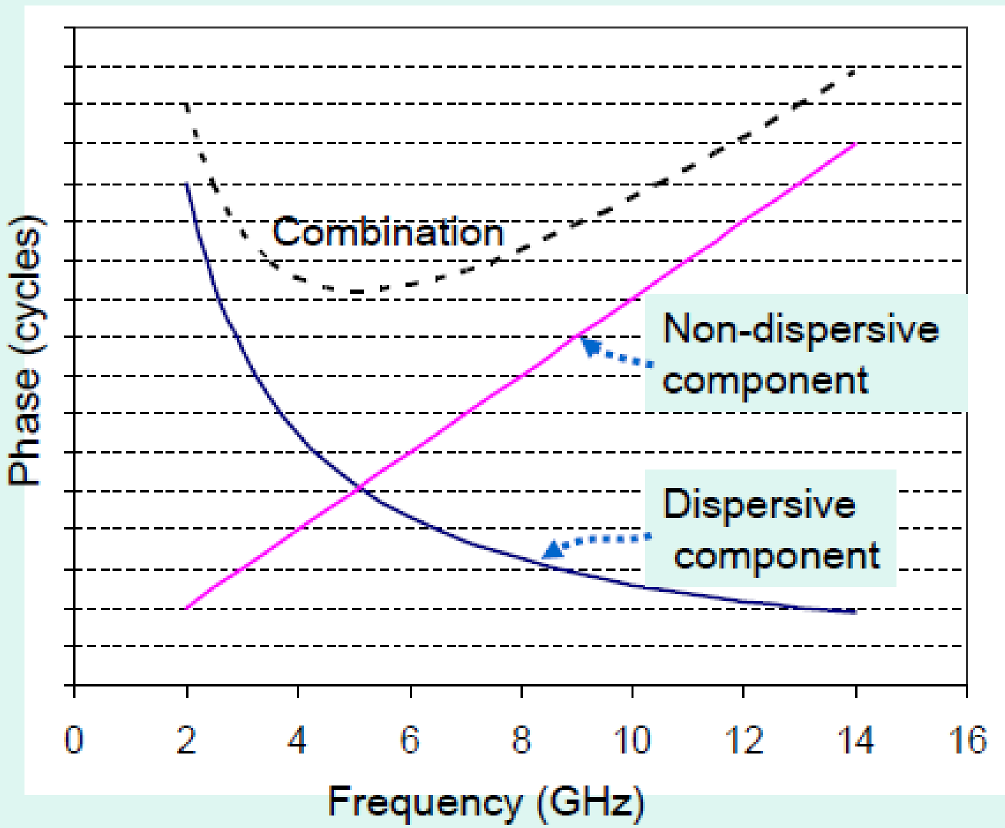
4.2.4 VLBI and the ionosphere

- VLBI is only sensitive to differences in ionospheric conditions, however it is possible to derive TEC values
  - Hobiger et al. (2006)



4.2.4 VLBI and the ionosphere

- VLBI2010: Separation of dispersive and non-dispersive delays during fringe detection



Non-dispersive delay. Delay is independent of frequency (phase is linear wrt frequency)

$$\phi = f \cdot \tau$$

$$\phi = f \cdot \left(\tau_g + \tau_{clk} + \tau_{atm} + \ldots\right)$$

Dispersive delay. Delay varies with frequency. Variation is due to the Ionosphere.

$$\phi_{Ion} = \frac{K}{f} \qquad \tau_{Ion} = -\frac{K}{f^2}$$

Petrachenko, 2013

II. Path delays in the neutral atmosphere

Outline

- Basics
- Definition of the path delay in the neutral atmosphere
- Modelling delays in the neutral atmosphere
- Atmospheric turbulence
- Application of space geodetic techniques for atmospheric studies

1 Introduction

- Neutral atmosphere vs. troposphere
  - We need to consider layers of the atmosphere up to about 100 km (stratosphere)
  - "Tropospheric delays"
- There is no frequency-dependency for VLBI observations in the neutral atmosphere (unlike the ionosphere)

2 Basics

- In general, the propagation of electromagnetic waves is described by Maxwell's equations
- Refractive index n versus refractivity N

$$N = (n - 1) \cdot 10^6$$

  - $n \approx 1.0003$; $N \approx 300$
- N is complex number

$$N = N_0 + N'(\nu) - iN''(\nu)$$

  - ν is frequency

2 Basics

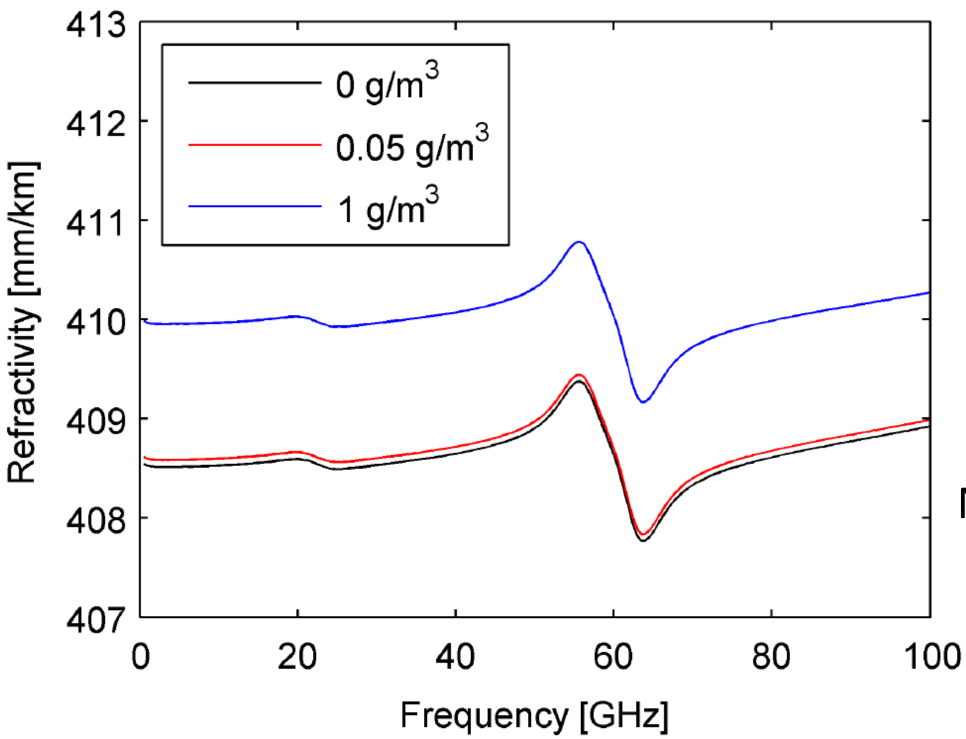
- Imaginary part causes absorption (used for WVR)
  - of [no] importance for delays
- Real part causes refraction and propagation delay

$$N = \sum_i \left(A_i(\nu)\rho_i + B_i(\nu)\frac{\rho_i}{T} \right)$$

  - Debye (1929)
  - B_i term for permanent dipole moment of molecules (water vapour)

2.1 Microwaves

- p = 1013 hPa, T = 300 K, rh = 100%, different concentrations of liquid water (e.g., fog or clouds)



Nilsson et al. 2013

2.1 Microwaves

- Dry and wet refractivity

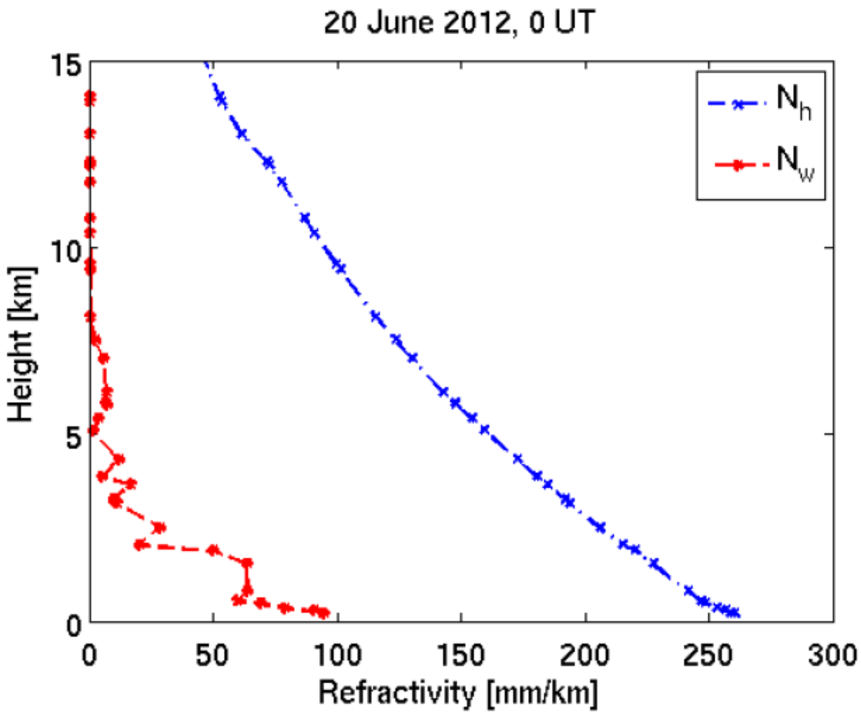
$$N = k_1 \frac{p_d}{T} Z_d^{-1} + k_2 \frac{p_w}{T} Z_w^{-1} + k_3 \frac{p_w}{T^2} Z_w^{-1}$$

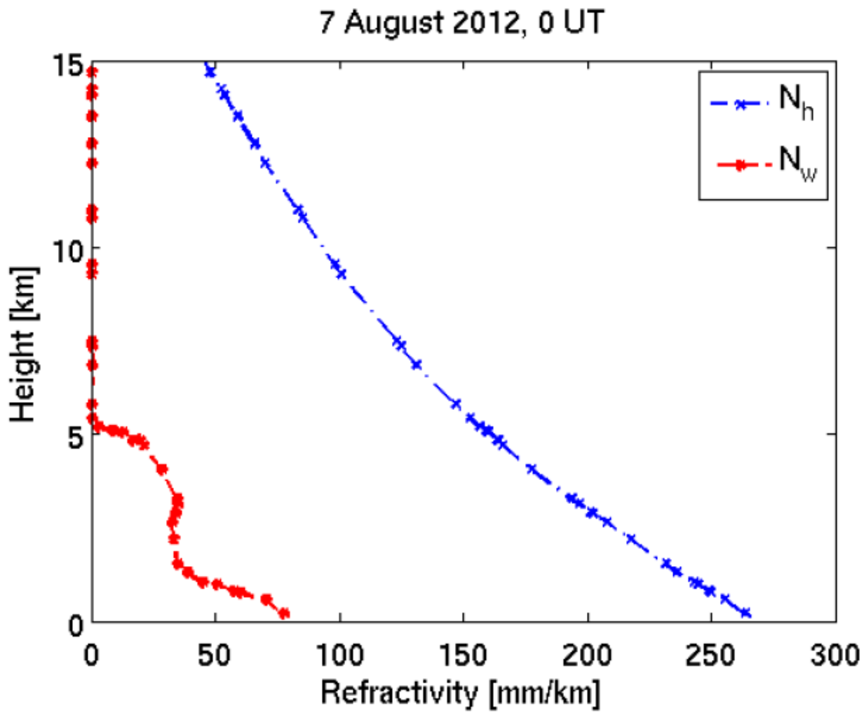
- Hydrostatic and non-hydrostatic ("wet")

$$N = k_1 \frac{R}{M_d} \rho + k_2' \frac{p_w}{T} Z_w^{-1} + k_3 \frac{p_w}{T^2} Z_w^{-1} = N_h + N_w$$

2.1 Microwaves

- Radiosonde profiles Vienna





Nilsson et al. 2013

3 Definition of path delay in the neutral atmosphere

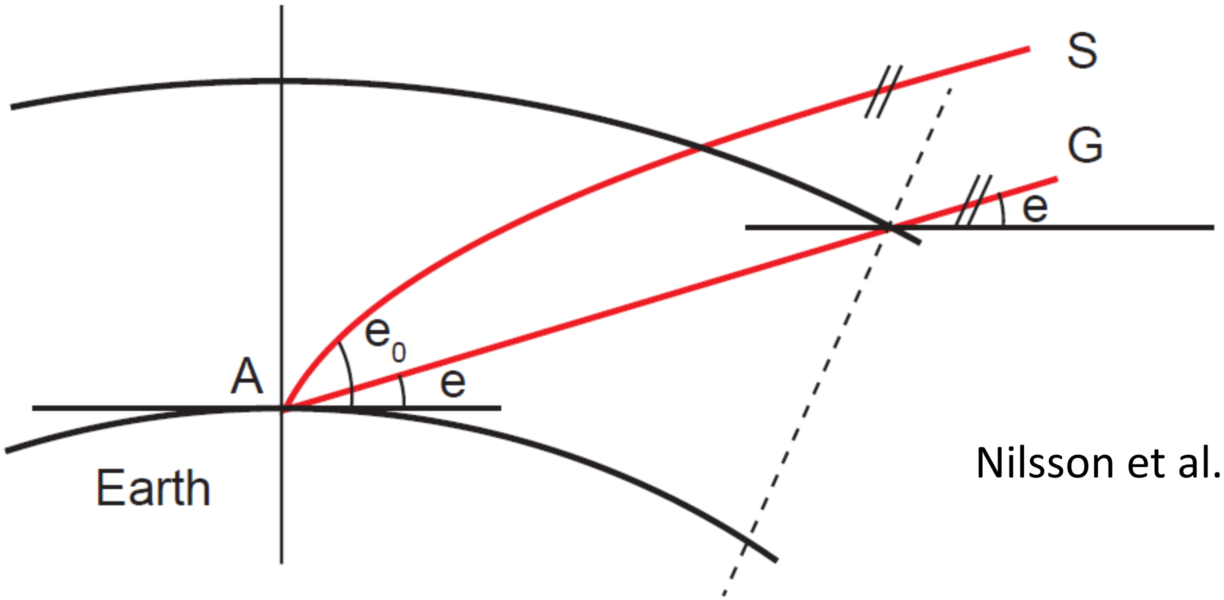
- In VLBI, the difference in travel time to a quasar from two telescopes is measured
- Propagation speed of the signal is lower than speed of light in vacuum
- Phase and group delays are equal in the neutral atmosphere
- If variation in refractivity over the distance of one wavelength is negligible, we can describe the propagation as a ray and apply geometrical optics

3 Definition of path delay in the neutral atmosphere

- Electric path length L along the path S

$$L = \int_S n(s)\,\mathrm{d}s$$

- Principle of Fermat: L is minimized



Nilsson et al. 2013

3 Definition of path delay in the neutral atmosphere

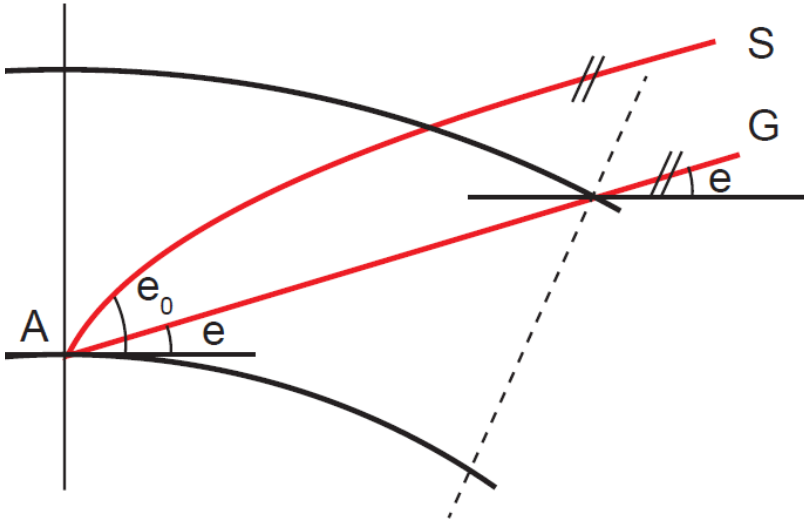
- The atmospheric delay ΔL is defined as the excess electric path length

$$\Delta L = L - G = \int_S n(s)\,\mathrm{d}s - G =$$

Nilsson et al. 2013

$$= 10^{-6}\int_S N(s)\,\mathrm{d}s + S - G$$

$$= \Delta L_h + \Delta L_w + S - G$$



3 Definition of path delay in the neutral atmosphere

- Bending effect S–G considered in hydrostatic mapping function (about 2 dm at 5 degrees)
- Zenith hydrostatic and wet delay

$$\Delta L_h^z = 10^{-6} \int_{h_0}^{\infty} N_h(z)\,\mathrm{d}z$$

$$\Delta L_w^z = 10^{-6} \int_{h_0}^{\infty} N_w(z)\,\mathrm{d}z$$

3.1 Hydrostatic delay

- Hydrostatic equation

$$\frac{\mathrm{d}p}{\mathrm{d}z} = -\rho(z)\,g(z)$$

- The pressure tells us how much mass is above the site but not its vertical distribution
  - That is enough information about the zenith hydrostatic delay, if we have a rough estimate of the height of the atmospheric centre of mass above the site

3.1 Hydrostatic delay

- Zenith hydrostatic delay with equation by Saastamoinen (1972) as refined by Davis et al. (1985)

$$\Delta L_h^z = 10^{-6} k_1 \frac{R\, p_0}{M_d\, g_{eff}}$$

$$\Delta L_h^z = 0.0022768 \frac{p_0}{f(\theta, h_0)}$$

- Thus, we need the pressure at the site
- 1000 hPa $\rightarrow$ 2.227 m zenith hydrostatic delay

3.1 Hydrostatic delay

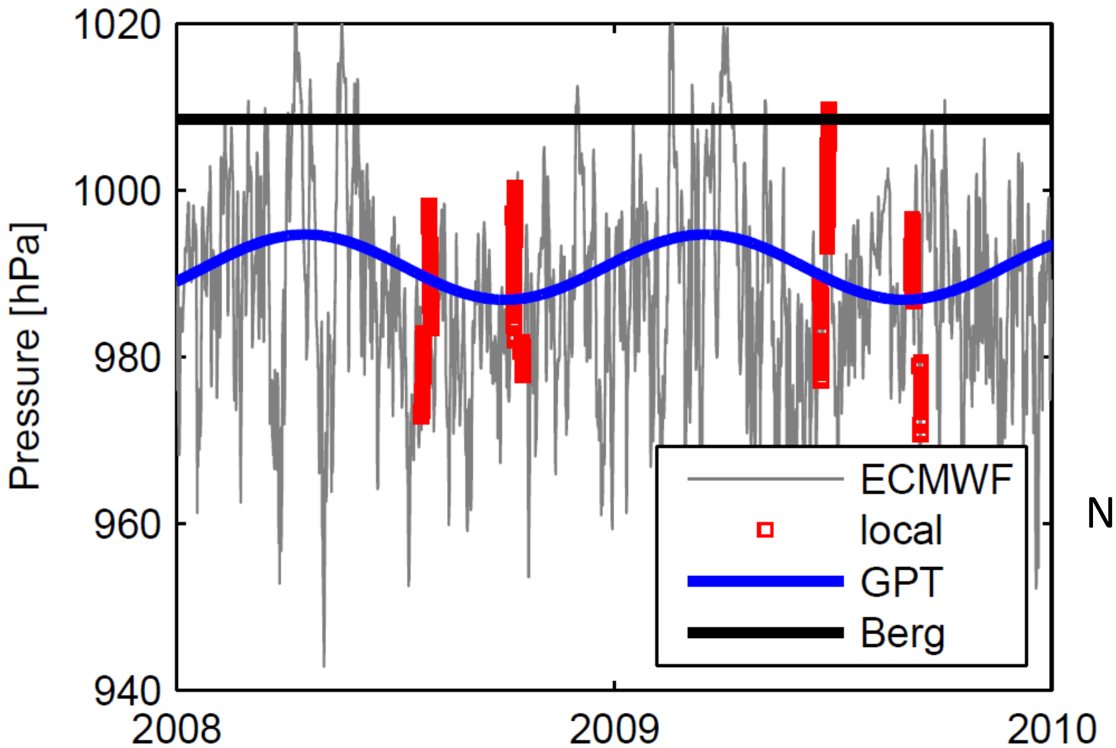
- Empirical models for the pressure like Berg (1948) or Hopfield (1969)

pressure	local recordings	grid values	GPT
availability	at sites	all (by interpolation)	all
time span	per observation	since 1994	unlimited
spatial resolution	per site	2.0 x 2.5°	spherical harmonics (9/9)
time resolution	per observation	6 hours	annual

- Local recordings recommended, but be careful with breaks
- GPT2!

3.1 Hydrostatic delay

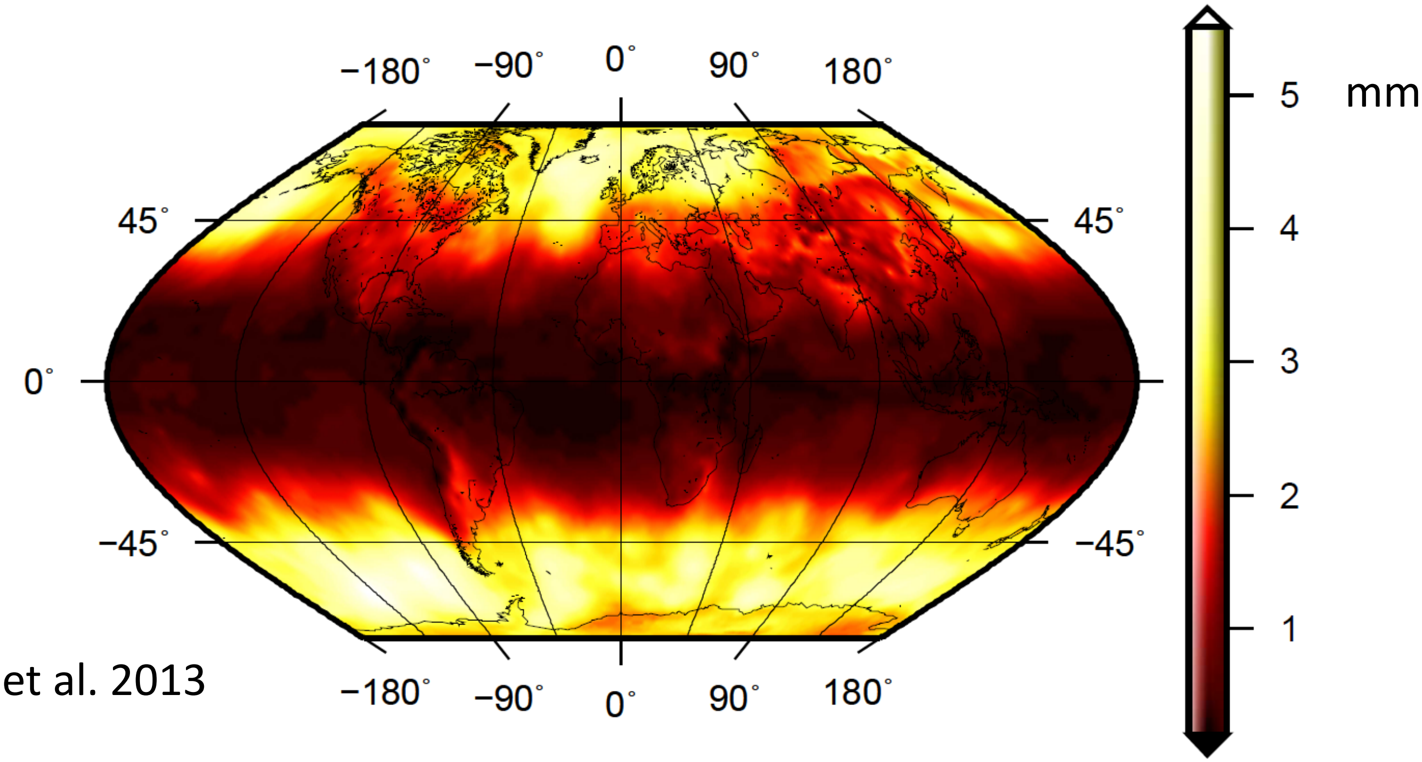
- Pressure values at O'Higgins (Antarctica)



Nilsson et al. 2013

3.1 Hydrostatic delay

- 3 hPa → 1 mm height
- Height standard deviation between GPT and ECMWF



Nilsson et al. 2013

3.1 Hydrostatic delay

- Be aware of destructive effects between atmospheric loading and empirical pressure values for the determination of zenith hydrostatic delays
  - When you apply empirical pressure values like those from GPT or GPT2 for the determination of the a priori zenith hydrostatic delay, you already do a bit of atmosphere loading correction

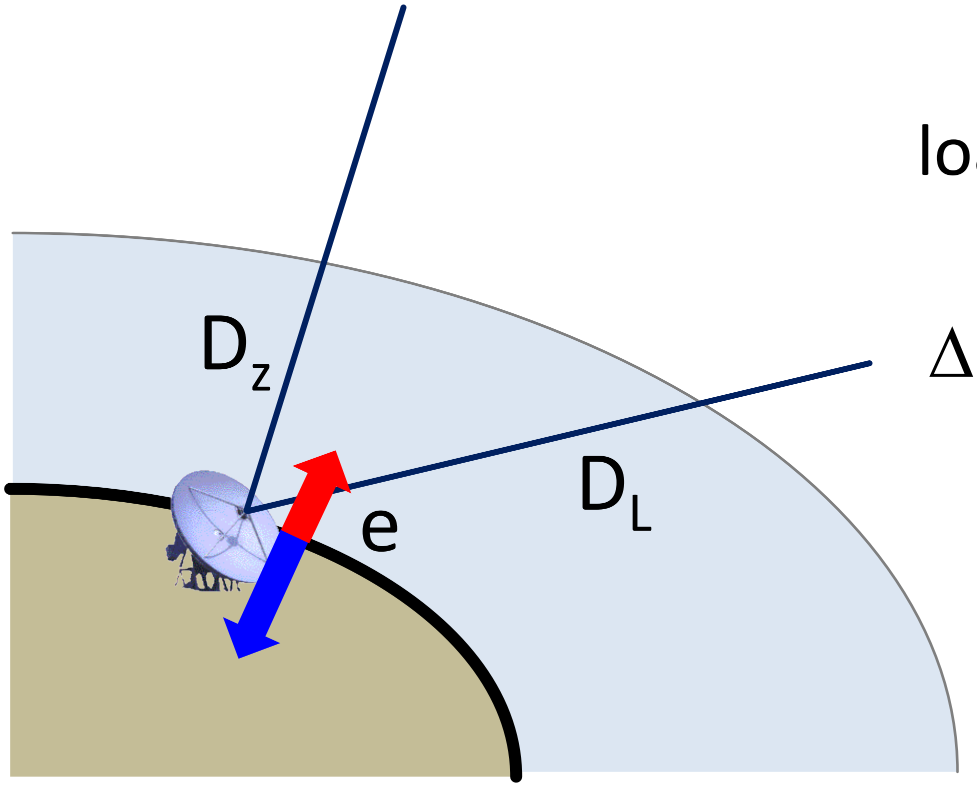
3.1 Hydrostatic delay

true pressure: 1020 hPa

mean pressure (GPT): 1000 hPa

loading: 8 mm

Δ height: 7 mm



3.2 Wet delay

- Varies between 0 cm (e.g., poles) and 40 cm

$$\Delta L_w^z = 10^{-6}\left[\int_{h_0}^{\infty}(k_2' \frac{p_w}{T} Z_w^{-1})\mathrm{d}z + \int_{h_0}^{\infty}(k_3 \frac{p_w}{T^2} Z_w^{-1})\mathrm{d}z\right]$$

$$\Delta L_w^z\,[\mathrm{cm}] \approx p_{w0}\,[\mathrm{hPa}]$$

3.2 Wet delay

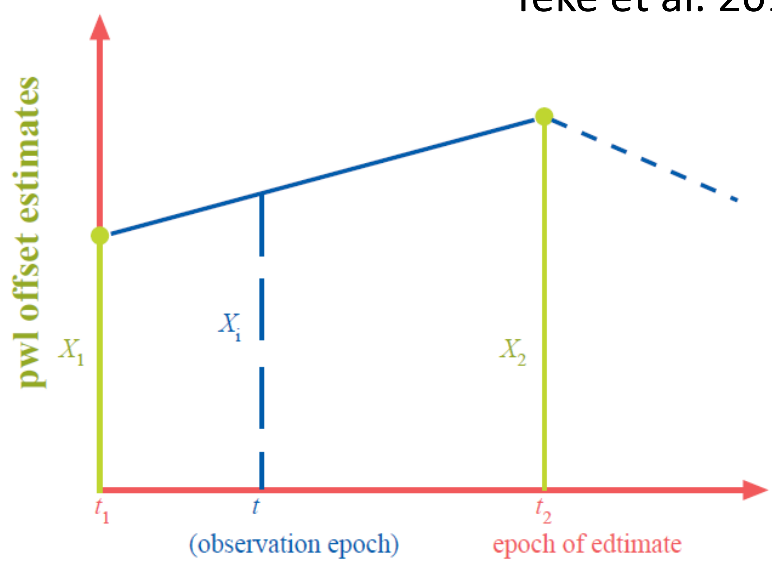
- Zenith wet delays must be estimated in VLBI analysis
- For example, as piecewise linear offsets with constraints (quasi observation equations)

$$\Delta L_w(t) = mf_w(t) \cdot x_1 + mf_w(t) \cdot \frac{t - t_1}{t_2 - t_1} \cdot (x_2 - x_1)$$

$$\frac{d\Delta L_w}{dx_1} = mf_w(t) - mf_w(t) \cdot \frac{t - t_1}{t_2 - t_1}$$

$$\frac{d\Delta L_w}{dx_2} = mf_w(t) \cdot \frac{t - t_1}{t_2 - t_1}$$

Teke et al. 2012



3.2 Wet delay

- Conversion of zenith wet delay to Integrated Water Vapour (IWV) and Precipitable Water (PW)

$$IWV = \Pi \, \Delta L_w^z$$

$$PW = \frac{IWV}{\rho_{w,fl}}$$

$$\Pi = \frac{10^6 M_w}{\left[k_2' + \frac{k_3}{T_m}\right] R}$$

$$PW = \kappa \Delta L_w^z$$

4 Modelling delays in the neutral atmosphere

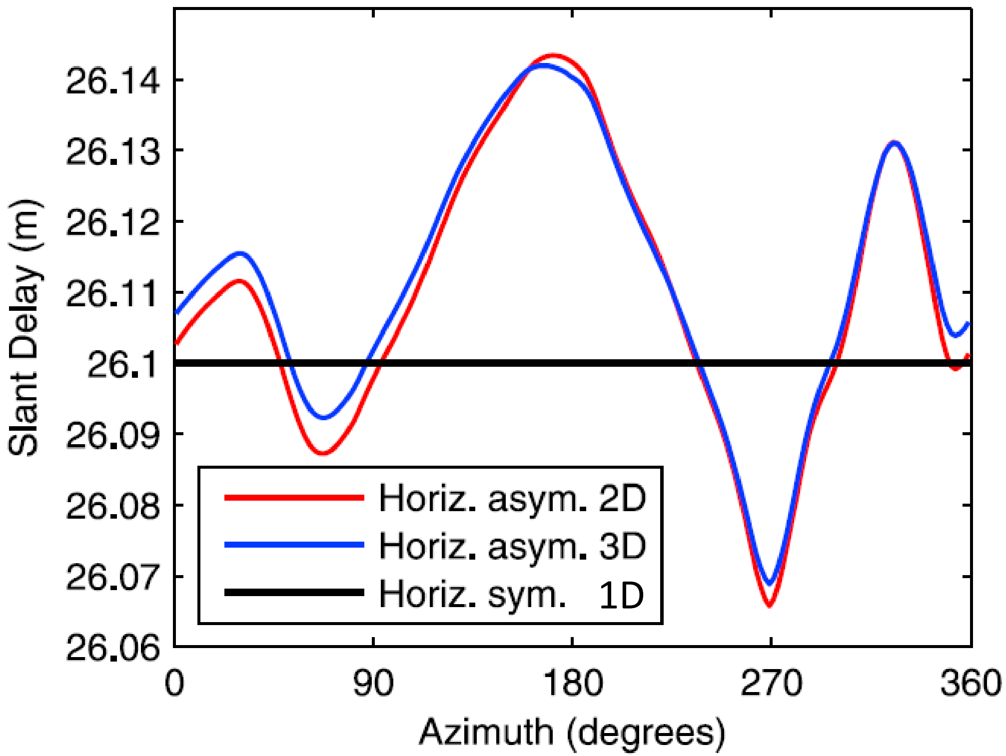
- Ray-tracing
- Mapping functions and gradients
- Water vapour radiometry

4.1 Ray-tracing

- To find the ray-path from the source to the telescope (has to be done iteratively, "shooting")
- Coupled differential equations need to be solved
- Easier in 2D case (6 equations), because not out-of-plane components
- "1D" for Vienna Mapping Function 1

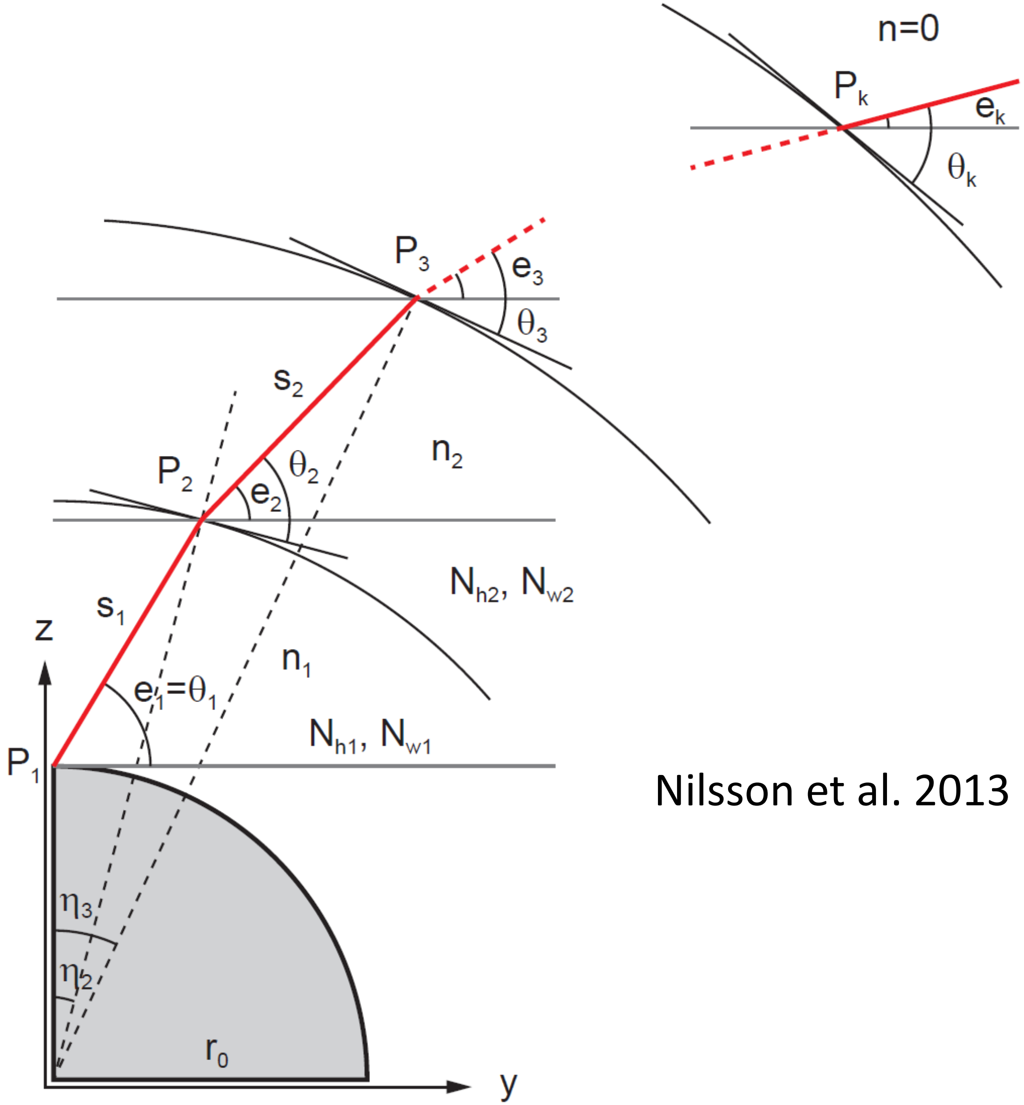
4.1 Ray-tracing

- Total delays at 5° outgoing elevation angle at Tsukuba on 12 August 2008 (2D not always shorter!)



Nafisi et al., 2012

4.1 Ray-tracing



Nilsson et al. 2013

4.1 Ray-tracing

- Many groups working on ray-tracing
- Improved Length-of-Day values from Intensives with ray-traced delays
- Regular 24 hour sessions not that sensitive to asymmetric delays

4.2 Mapping functions

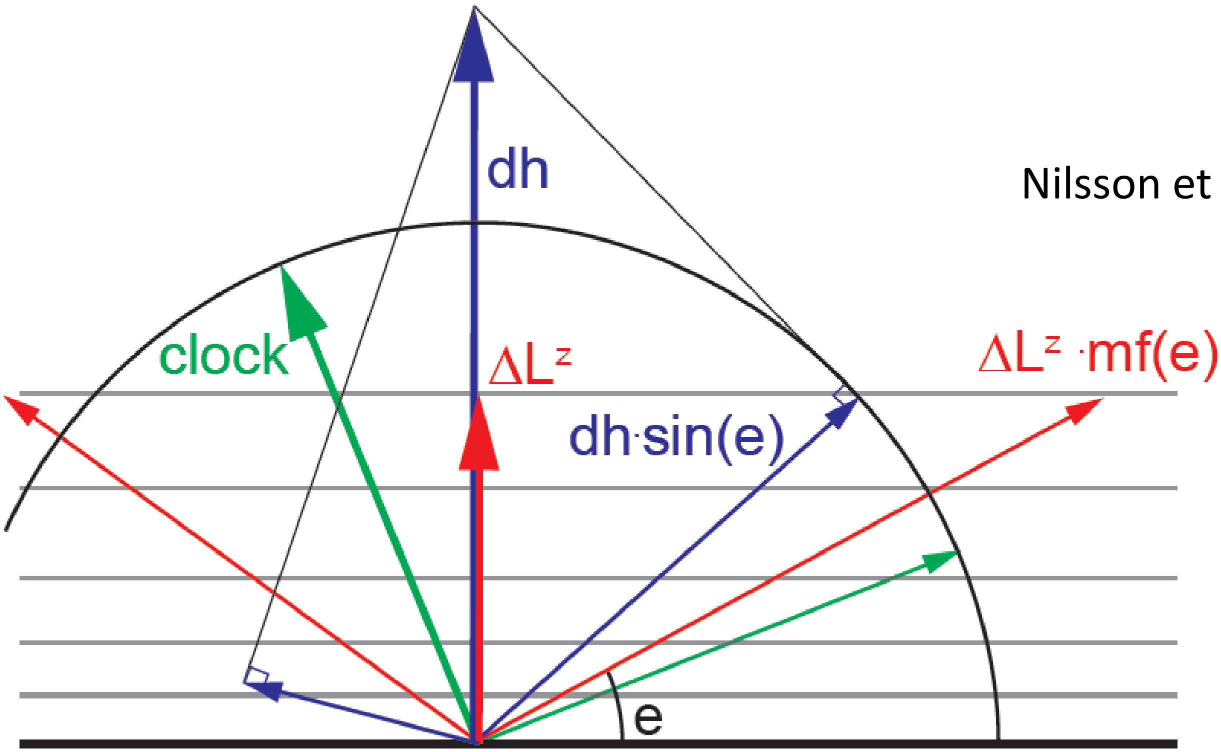
- Slant delay = zenith delay times mapping function

$$\Delta L(e) = \Delta L^z \cdot mf(e)$$

- Mapping functions to map down a priori zenith delays and to estimate residual delays
- Estimation every 20 to 60 min; this allows a least-squares adjustment

4.2 Mapping functions

- Different elevation dependencies for zenith delays (mf), clocks (1), and station heights (sin e)



Nilsson et al. 2013

4.2 Mapping functions

- Mapping function not perfectly known
- Errors via correlations also in station heights (and clocks)
- Low elevations necessary to de-correlate heights, clocks, and zenith delays
- Trade-off $\rightarrow$ about 7 degrees cut off elevation angle (sometimes with down-weighting)

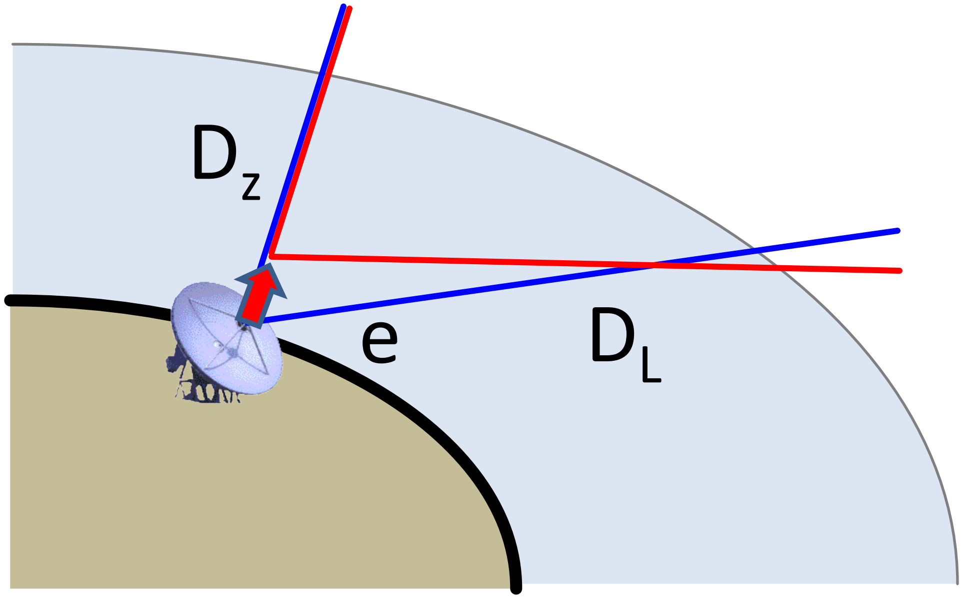
4.2 Mapping functions

- Mapping function too large $\rightarrow$ zenith delay too small $\rightarrow$ station height goes up
- Rule of thumb: "Station height error is about one fifth of the delay error at 5 degrees"

4.2 Mapping functions

$D_L(e) = D_z \cdot m(e)$

$D_L(e) = D_z' \cdot m(e)'$



4.2 Mapping functions

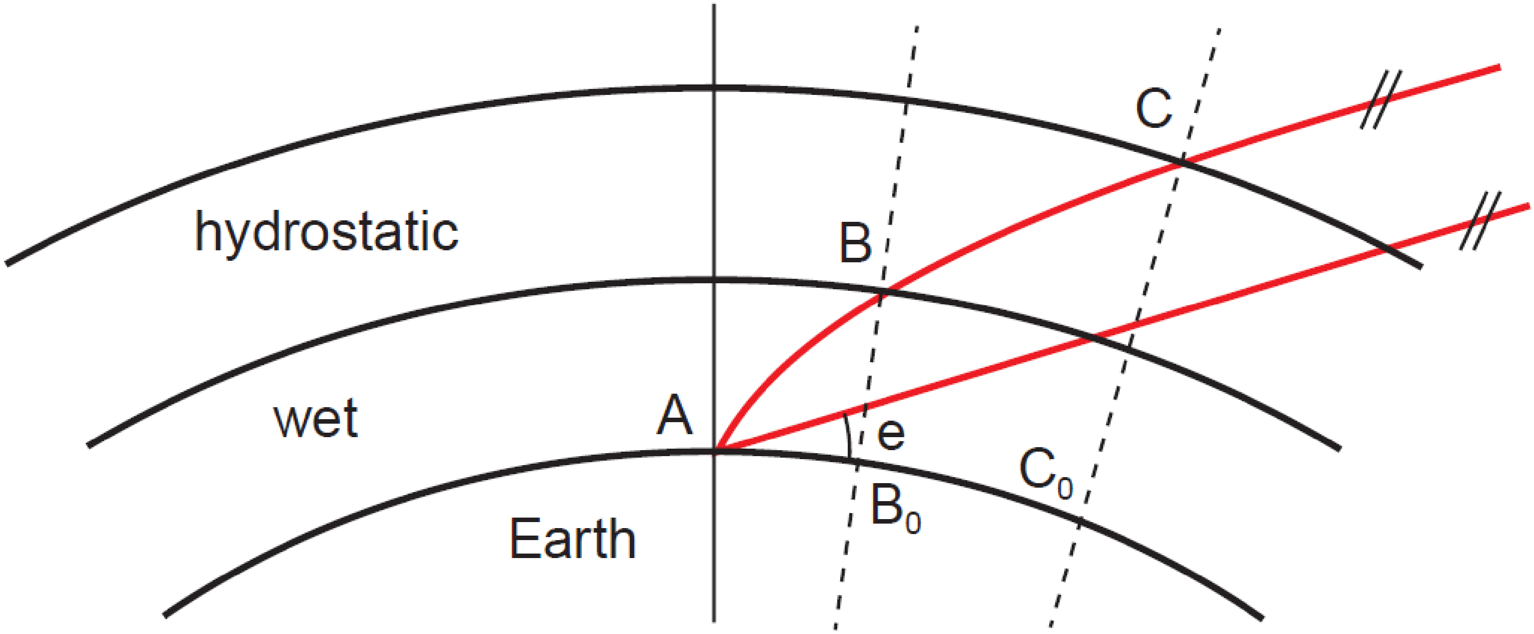
- Hydrostatic and wet mapping functions

$$\Delta L(e) = \Delta L_h^z \cdot mf_h(e) + \Delta L_w^z \cdot mf_w(e)$$

- Example:
  - Zenith hydrostatic and wet delays shall be 2000 mm and 200 mm, respectively;
  - Hydrostatic mapping function at 5° too large by 0.01 (10.16 instead of 10.15);
  - Slant delay at 5° too large by 20 mm
  - Station height too large by 4 mm (one fifth)

4.2 Mapping functions

- Wet mapping function larger than hydrostatic mf
- Mapping functions are a measure for the thickness of the atmosphere (1/sin e means flat)



Nilsson et al. 2013

4.2 Mapping functions

- Modern mapping functions use continued fractions form as specified by Herring (1992)

$$mf(e) = \frac{1 + \cfrac{a}{1 + \cfrac{b}{1 + c}}}{\sin(e) + \cfrac{a}{\sin(e) + \cfrac{b}{\sin(e) + c}}}$$

4.2 Mapping functions

- Saastamoinen (1972), Chao (1974), CfA2.2 (Davis et al., 1985), MTT (Herring, 1992), ...
- New Mapping Functions (Niell, 1996)
- Isobaric Mapping Functions (Niell, 2001)
- Vienna Mapping Functions 1 (Böhm et al., 2006)
- Global Mapping Functions (Böhm et al., 2006)
- Global Pressure and Temperature 2 (Lagler et al., 2013)

4.2 Mapping functions

- Vienna Mapping Functions 1
  - empirical functions for b and c coefficients
  - coefficients a by ray-tracing at initial elevation angle 3.3°
  - "1D ray-trace"
  - available for all VLBI sites (resolution 0.25°) and on global grid

http://ggosatm.hg.tuwien.ac.at/

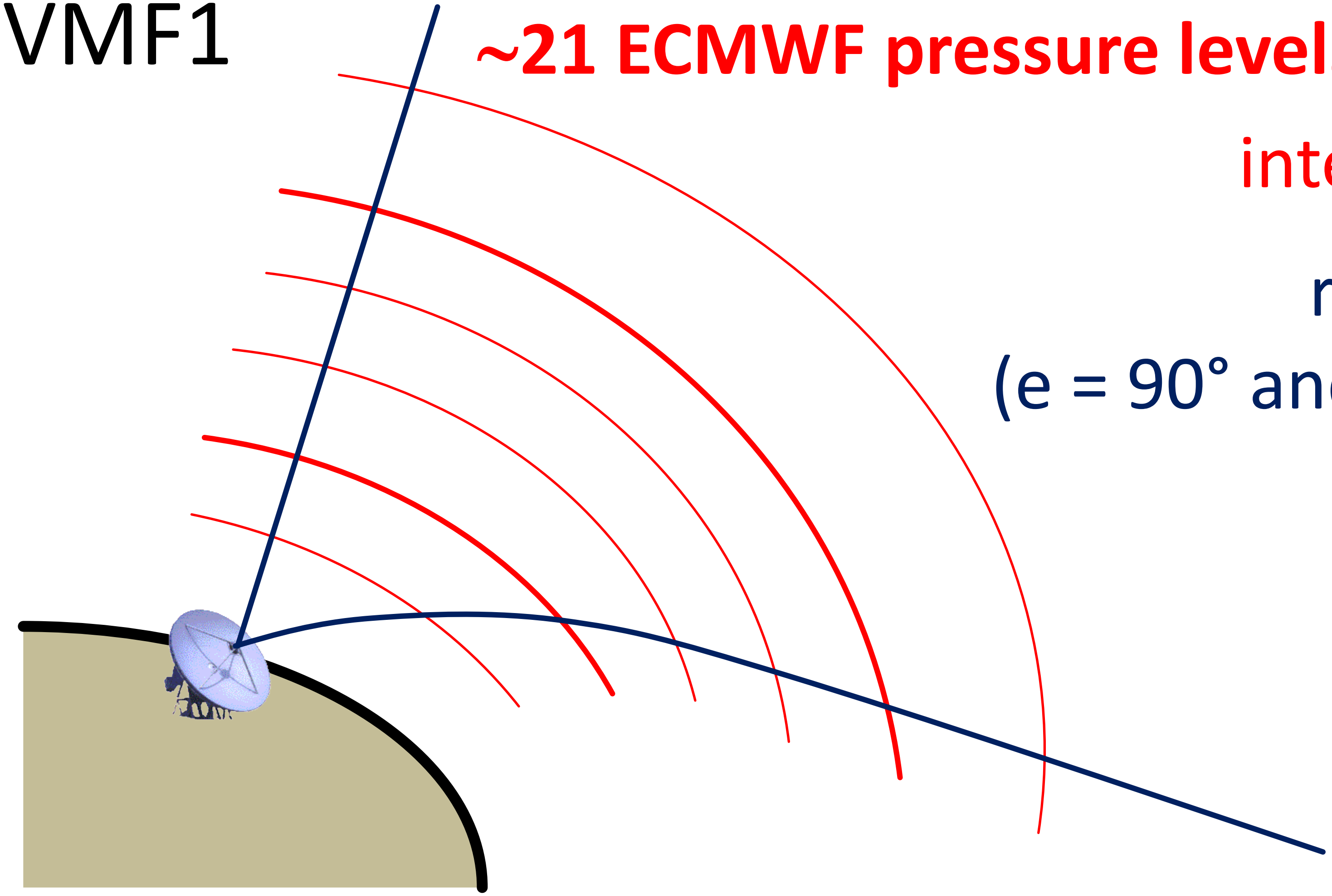
4.2 Mapping functions

VMF1

~21 ECMWF pressure levels: T, p_{wv}, h

interpolation

ray-tracing
(e = 90° and e = 3.3°)



4.2 Mapping functions

VMF1

$$m(e) = \frac{1 + \frac{a}{1 + \frac{b}{1 + c}}}{\sin(e) + \frac{a}{\sin(e) + \frac{b}{\sin(e) + c}}}$$

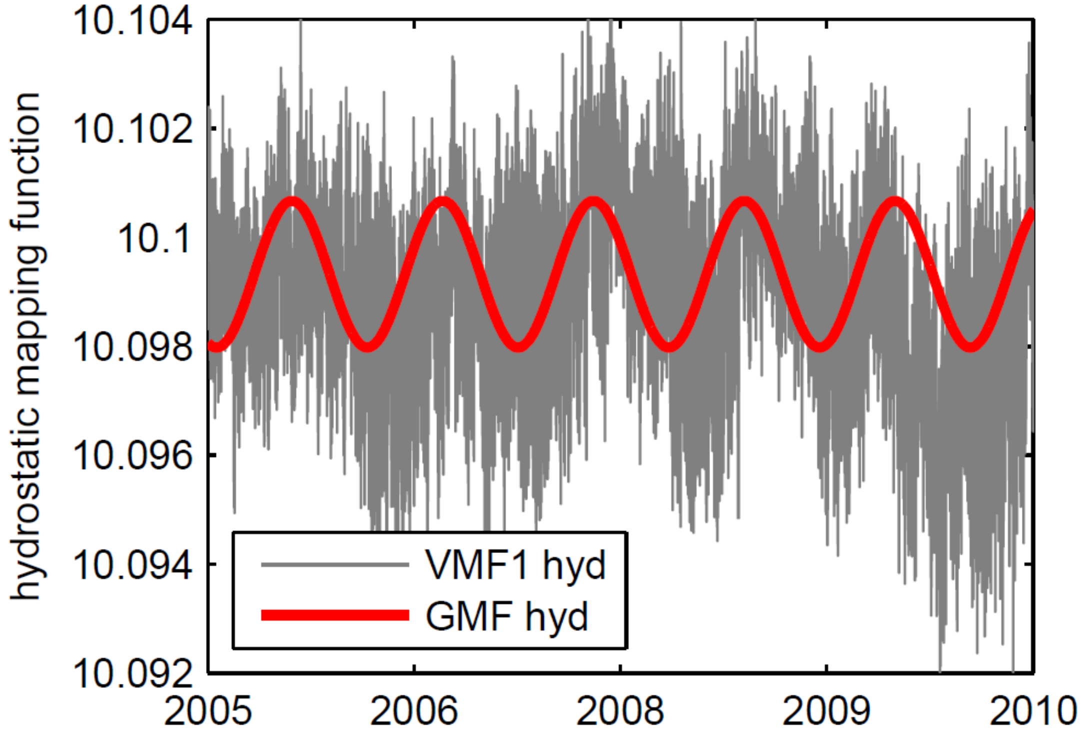
variable in time and space

ray-tracing

analytical functions

4.2 Mapping functions

- Hydrostatic VMF 1 versus GMF at 5° at Fortaleza



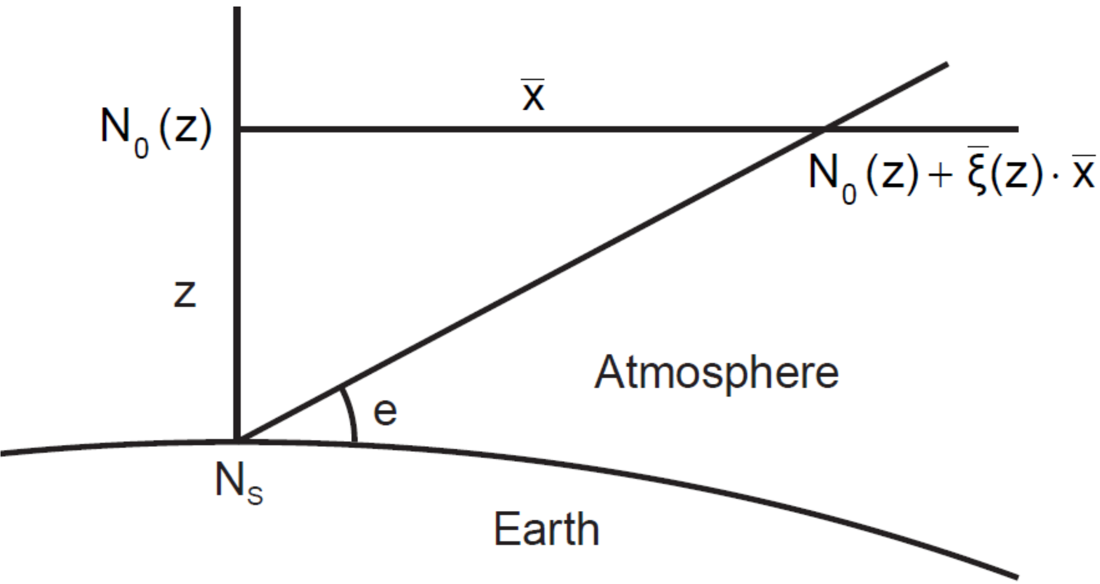
Nilsson et al. 2013

4.2 Gradients

- Modelling azimuthal asymmetries
  - to account for higher atmosphere above the equator
  - systematic effects, e.g. at coasts
  - local weather phenomena
- Gradients typically estimated every 6 hours
- Order of magnitude
  - 1 mm gradient $\rightarrow$ 100 mm delay at 5° elevation

4.2 Gradients

- "Linear horizontal gradients" of refractivity



Nilsson et al. 2013

$$N(\mathbf{x}, z) = N_0(z) + \boldsymbol{\xi}(z) \cdot \mathbf{x}$$

$$\xi_i(z) = \left. \frac{\partial N(\mathbf{x}, z)}{\partial x_i} \right|_{\mathbf{x}=0}$$

4.2 Gradients

- MacMillan (1995)

$$\Delta L(a,e) = \Delta L_0(e) + mf_h(e)\cot(e)(G_n\cos(a) + G_e\sin(a))$$

- Chen and Herring (1997)

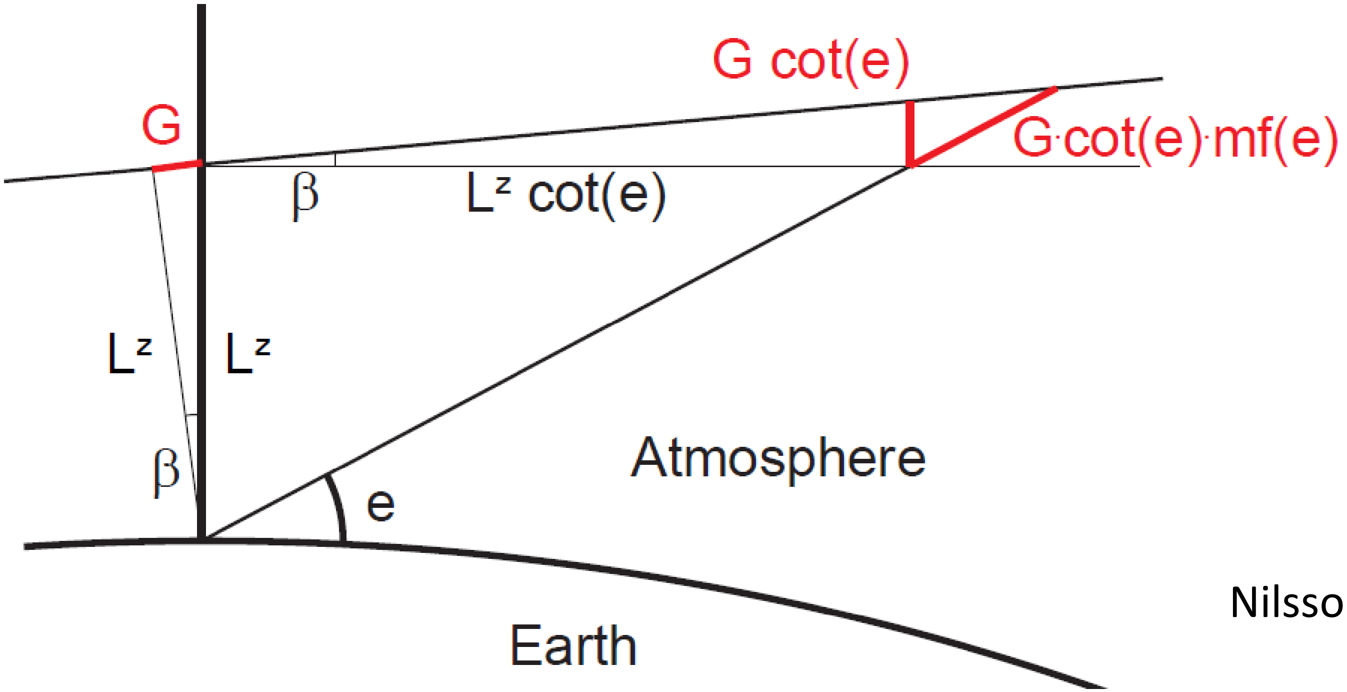
$$\Delta L(a,e) = \Delta L_0(e) + mf_g(e)(G_n\cos(a) + G_e\sin(a))$$

$$mf_g(e) = \frac{1}{\sin(e)\tan(e) + C}$$

e.g., C = 0.0032

4.2 Gradients

- Tilting of the mapping function



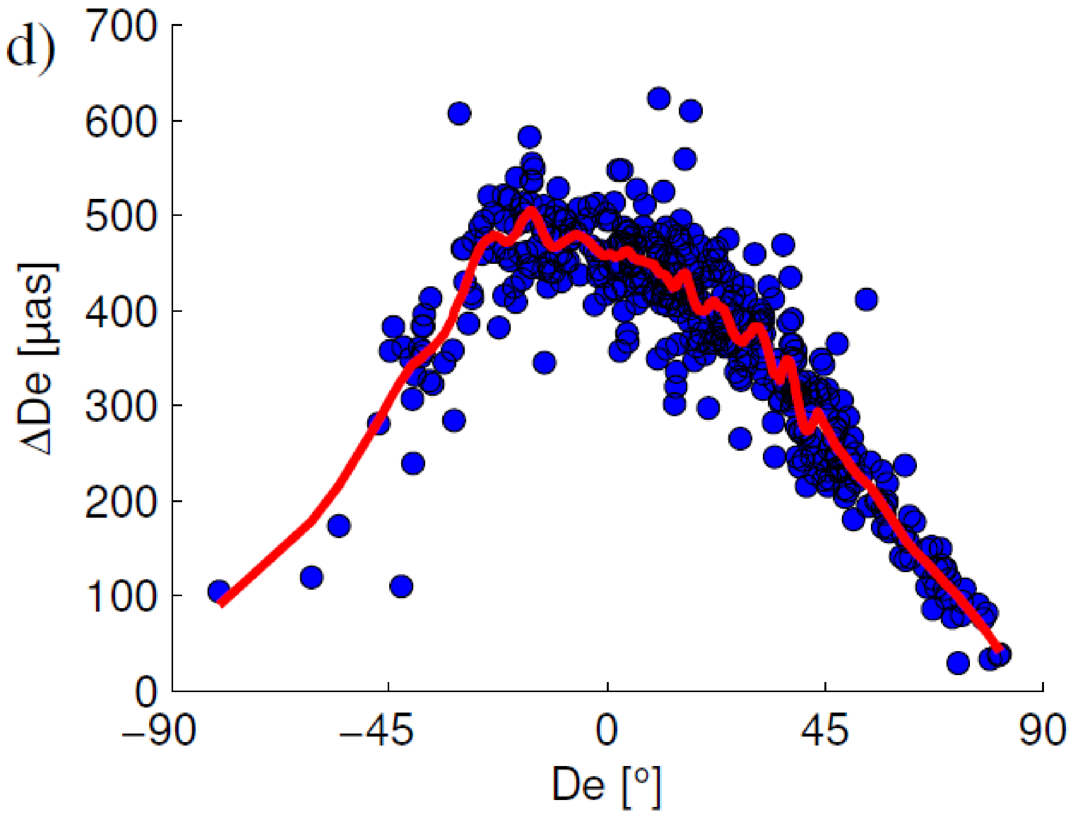
Nilsson et al. 2013

4.2 Gradients

- In the early years of VLBI (before 1990) gradient estimates need to be constrained because of poor observation geometry
- If possible, estimates should be constrained to a priori values (different from zero, accounting for the atmospheric bulge above the equator and local effects)

4.2 Gradients

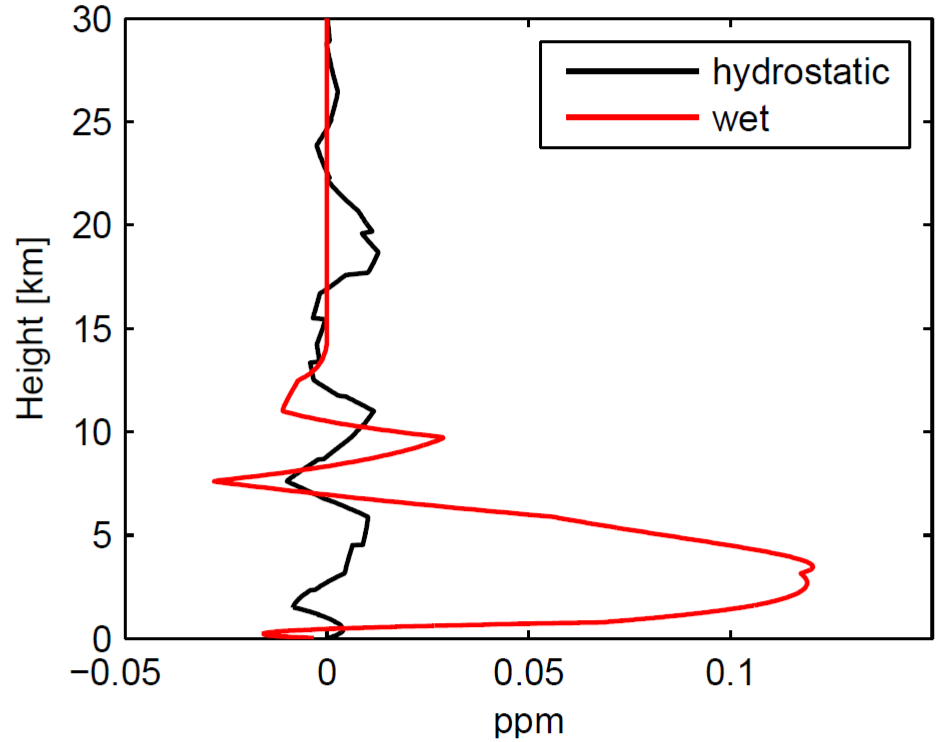
- Source declination differences between estimating and not estimating gradients



Hofmeister, 2013

4.2 Gradients

- Goddard provides static gradients
- Vienna provides 6 hourly gradients from the ECMWF
  - Weighted (with height) refractivity gradients toward east at Fortaleza



Nilsson et al. 2013

4.4 Water vapour radiometry

- WVR measure the thermal radiation from the sky at microwave frequencies where the atmospheric attenuation due to water vapour is relatively high

5 Atmospheric turbulence

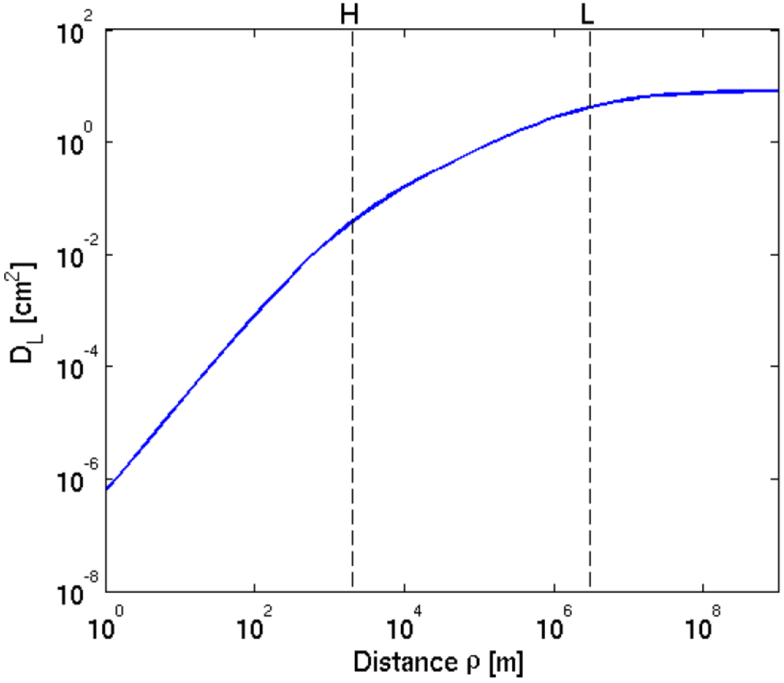
- Random fluctuations in refractivity distribution
- Structure function as modified by Treuhaft and Lanyi (1997)

$$D_n(\mathbf{R}) = \left\langle [n(\mathbf{r}) - n(\mathbf{r}+\mathbf{R})]^2 \right\rangle = C_n^2 \frac{\|\mathbf{R}\|^{2/3}}{1 + \left[\frac{\|\mathbf{R}\|}{L}\right]^{2/3}}$$

- C_n^2 is the refractive index structure constant
- L is the saturation length scale

5 Atmospheric turbulence

- Spatial structure function for the zenith wet delay



Nilsson et al. 2013

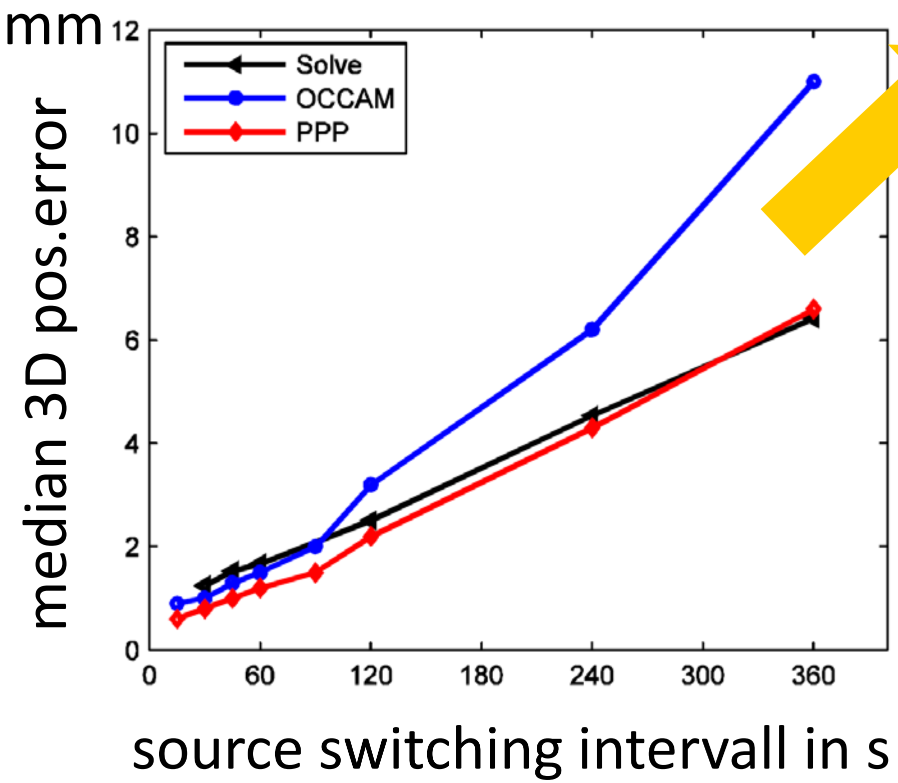
- Frozen flow theory

5 Atmospheric turbulence

- Turbulence simulator (Nilsson et al., 2007) very useful for (VLBI2010) simulations

5 Atmospheric turbulence

- Simulations



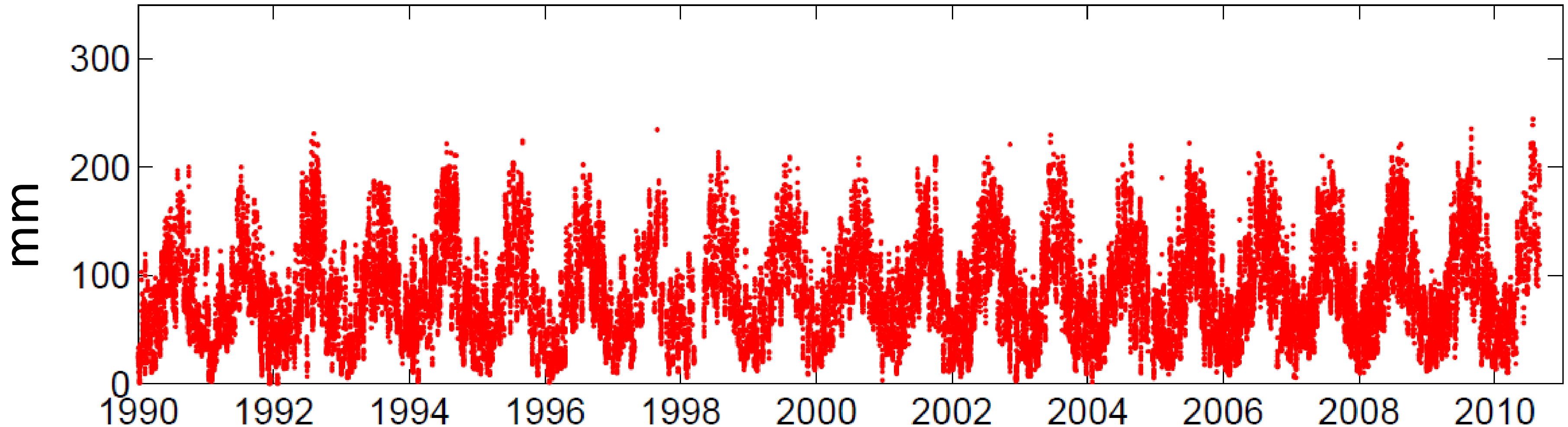
- fast 12m telescopes
- twin telescopes

Troposphere is limiting factor!

[Petrachenko et al., 2009]

Application for atmospheric studies

- Zenith wet delays at Wettzell (Nilsson, 2011)



THANKS FOR YOUR ATTENTION