VLBI modeling and data analysis

Thomas Artz

Institut für Geodäsie und Geoinformation

Rheinische Friedrich-Wilhelms-Universität Bonn

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Educational Objectives

you should understand

1. different steps of VLBI analysis
2. geometry of VLBI observations
3. set-up criteria
   - estimated params
   - datum definition
4. how to constrain params
5. individual and global solutions

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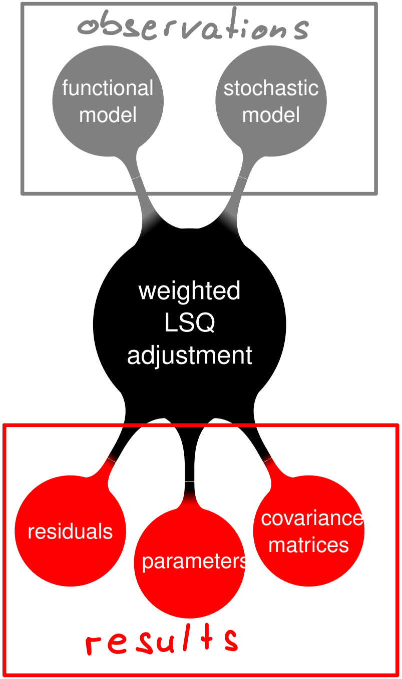
Modeling and analysis → adjustment theory



http://www.blogohblog.com

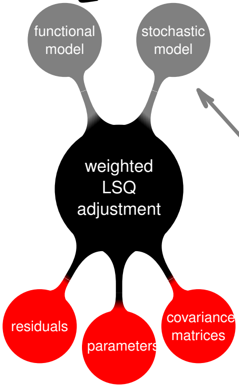
modeling

understanding physical and stochastical properties of observations
→ mathematical models

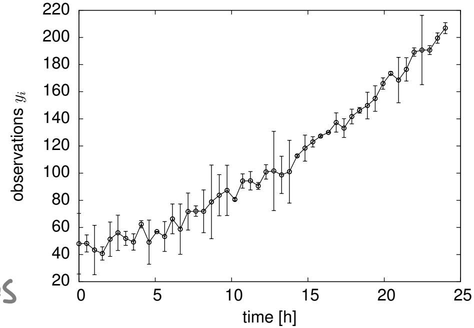
analysis

investigate estimated parameters and their stochastical properties

Modeling



- obs. y_i (vector: $\underline{y}$)
- params x_i
- functions $\underline{y} = f(\underline{x})$
  $= \underline{A} \cdot \underline{x}$

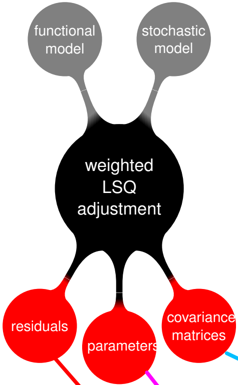


- standard deviations σ_i
- correlations $\rho_{ij} = \frac{\sigma_{ij}}{\sigma_i \sigma_j}$ ← covariance

=> covariance matrix

$$\Sigma_{yy} = \begin{pmatrix} \sigma_1^2 & \cdots & \sigma_{1n} \\ \vdots & \ddots & \vdots \\ \sigma_{n1} & \cdots & \sigma_n^2 \end{pmatrix}$$

Adjustment



given: $y_i, \sigma_i, \rho_{ij} = 0$

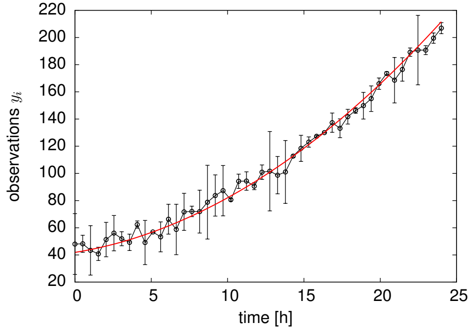
2nd deg. Polynomial

$\Rightarrow y_i = a + b t_i + c t_i^2$

$\underline{x} = (a \quad b \quad c)^T$

$\underline{A} = (\underline{1} \quad \underline{t} \quad \underline{t}^2)$

$\Sigma_{yy} = diag(\underline{\sigma}^2)$

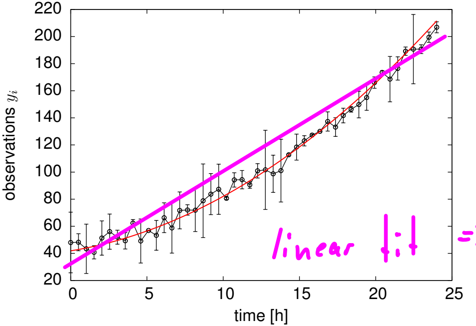


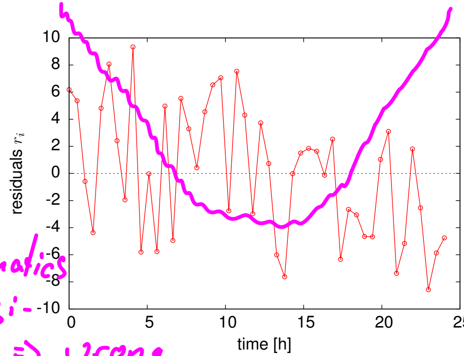
LSQ

$$\mathbf{r} = \mathbf{y} - \mathbf{A}\mathbf{x} \qquad \text{idea: } \sum r_i^2 \rightarrow min$$

$$\Rightarrow \underbrace{(\mathbf{A}^T \Sigma_{yy}^{-1} \mathbf{A})}_{\underline{N}} \mathbf{x} = \underbrace{\mathbf{A}^T \Sigma_{yy}^{-1} \mathbf{y}}_{\underline{n}}; \qquad \Sigma_{xx} = (\mathbf{A}^T \Sigma_{yy}^{-1} \mathbf{A})^{-1} = \underline{N}^{-1}$$

Analysis





linear fit => systematics in residuals => wrong funct. model

$$
\left.
\begin{aligned}
a &= 41.84; & \sigma_a &= 1.07\\
b &= 1.84; & \sigma_b &= 0.25\\
c &= 0.21; & \sigma_c &= 0.01
\end{aligned}
\right\}
$$

params significant

$$\tilde{\sigma}_0 = \sqrt{\frac{\mathbf{r}^T \boldsymbol{\Sigma}_{yy}^{-1} \mathbf{r}}{n-u}} = 1.1 \sim 1$$

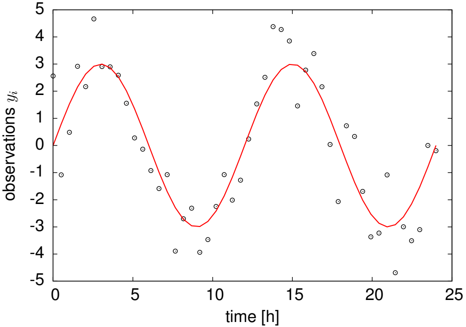
=> global test $\tilde{\sigma}_0^2 \sim 1$

=> functional & stochastic model o.k.

$$P(c - 3\sigma_c \le \mu_c \le c + 3\sigma_c) = 99{,}7\%$$

expectation value

Non-linear models



$$\mathbf{y}_i = \varphi(\mathbf{x}) = a \cdot \sin(b \cdot t_i)$$

$$\Rightarrow \mathbf{x} = \begin{pmatrix} a \\ b \end{pmatrix}$$

non-linear model $\Rightarrow$ linearization

=> partial derivatives

$$\mathbf{A} = \frac{\partial \varphi(\mathbf{x})}{\partial \mathbf{x}}$$ Jacobian matrix

$$\Rightarrow \mathbf{A}^T = \begin{pmatrix} \sin(b_0 \cdot \mathbf{t}) \\ \underline{-a_0} \cdot \cos(\underline{b_0} \cdot \mathbf{t}) \cdot \mathbf{t} \end{pmatrix}$$

aprioris

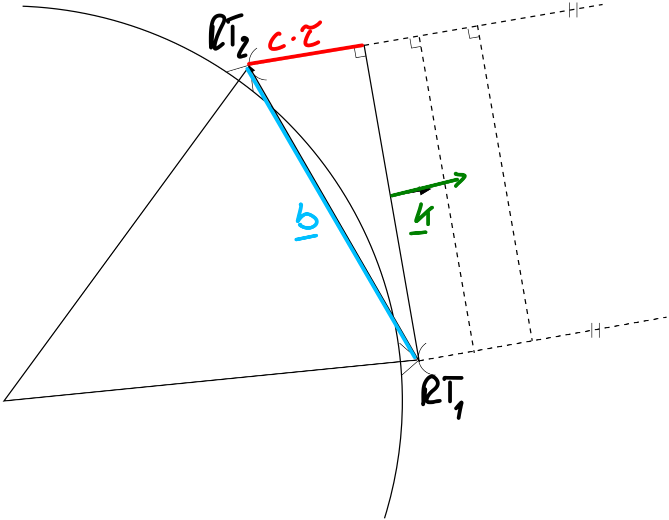
$$\Rightarrow \Delta\mathbf{y} = \mathbf{y} - \varphi(\mathbf{x}_0) = o - c$$

$$\Rightarrow \Delta\mathbf{x} = (\mathbf{A}^T \mathbf{\Sigma}_{yy}^{-1} \mathbf{A})^{-1} \mathbf{A}^T \mathbf{\Sigma}_{yy}^{-1} \Delta\mathbf{y}$$

modeling

determination of good apriori values $\Rightarrow$ convergence of LSQ to global minimum

VLBI functional model



$$\tau = -\frac{1}{c}\,\underline{b}\,\underline{k}$$

b and k are not in the same system!

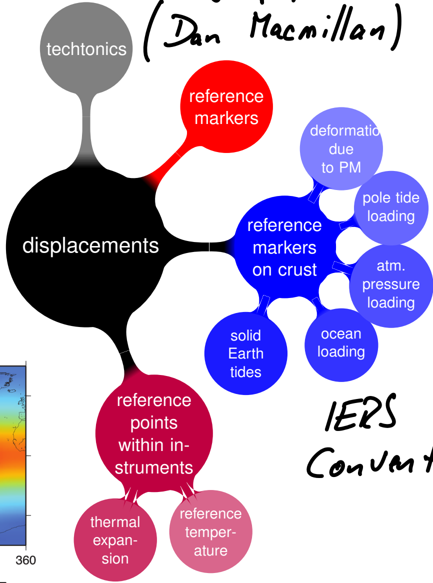
formulation of delay in frame where radio sources have no apparent motions w.r.t. telescopes

Station positions at reception epoch

station positions

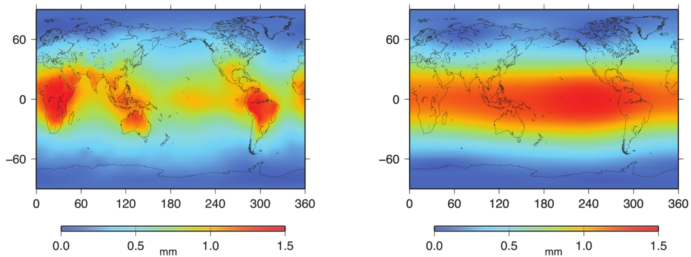
- are given for reference epoch
- various well known geophysical effects
- used as a priori information

see also "geophys. modeling" (Dan Macmillan)



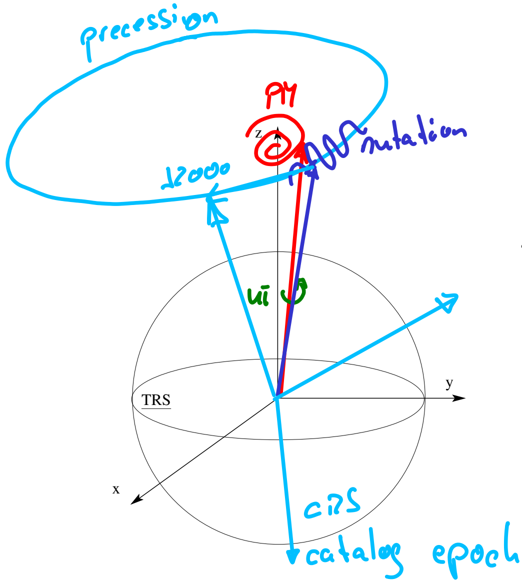
IERS Conventions

e.g., S1/S2 atm. loading



Transformations → CRF

sources and (corrected) station positions not in same reference system ⇒ transformations necessary (Earth orientation)



rotation matrix

$$\underline{R} = \underline{W}\,\underline{S}\,\underbrace{\underline{N}\,\underline{P}}_{\underline{PN}}$$

$$\Rightarrow \tau = -\frac{1}{c}\,\underline{b}\,\underline{R}\,\underline{k}$$

$$\text{e.g. } \underline{W}^T = \underline{R}_2(x)\cdot\underline{R}_1(y)$$

$$\underline{W}^T = \underline{R}_2(x) \cdot \underline{R}_1(y) = \begin{pmatrix} \cos x & 0 & -\sin x \\ 0 & 1 & 0 \\ \sin x & 0 & \cos x \end{pmatrix} \begin{pmatrix} 1 & 0 & 0 \\ 0 & \cos y & \sin y \\ 0 & -\sin y & \cos y \end{pmatrix}$$

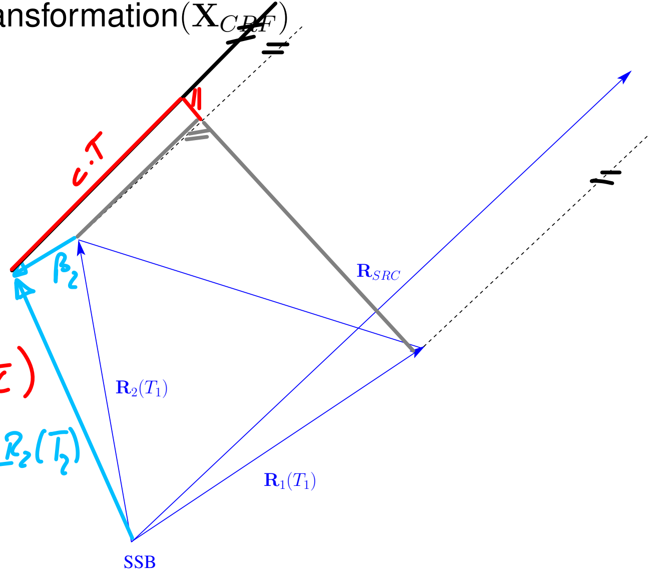
x & y small $\Rightarrow$ $\sin x = x$, $\cos x = 1$

$$\Rightarrow \underline{W}^T = \begin{pmatrix} 1 & 0 & -x \\ 0 & 1 & 0 \\ x & 0 & 1 \end{pmatrix} \begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & y \\ 0 & -y & 1 \end{pmatrix} = \begin{pmatrix} 1 & xy & -x \\ 0 & 1 & y \\ x & -y & 1 \end{pmatrix}$$

Solar System Barycenter (SSB) frame

most natural frame for calculation of theoretical delay

$$\mathbf{X}_{SSB} \leftarrow \text{Lorentz transformation}(\mathbf{X}_{CRF})$$



$\tau \leftarrow$ Lorentz transformation (τ)

see also "geophysical models" D. Macmillan

Observation equations

functional model

$$y_i = \tau = -\frac{1}{c}\left(\mathbf{x}_a(t_i) - \mathbf{x}_b(t_i)\right) \cdot \mathbf{R} \cdot \mathbf{k} \cdot \left(1 - F(\mathbf{v}, \mathbf{v}^b)\right) + \tau_{corr}$$

$$\begin{aligned}
\mathbf{R} =& \mathbf{W}(x_p, y_p) \cdot \mathbf{S}(UT1) \cdot \mathbf{PN}(X, Y) \\
F(\mathbf{v}, \mathbf{v}^b) =& \frac{(\mathbf{v} + \mathbf{v}^b) \cdot \mathbf{k}}{c} - \frac{(\mathbf{v} \cdot \mathbf{k})^2 - 2(\mathbf{v} \cdot \mathbf{k})(\mathbf{v}^b \cdot \mathbf{k})}{c^2} - \frac{(\mathbf{b} \cdot \mathbf{v})(\mathbf{v}^b \cdot \mathbf{k})}{c^3} \\
& - \frac{(\mathbf{b} \cdot \mathbf{v})(\mathbf{v} \cdot \mathbf{k})}{2c^3}
\end{aligned}$$

$\mathbf{W}$	polar motion matrix	t_i	epoch of reception at a
$\mathbf{S}$	rotational motion matrix	$\mathbf{b}$	baseline vector
$\mathbf{N}$	nutation matrix	$\mathbf{k}$	source unit vector in CRF
$\mathbf{P}$	precession matrix	F	abberation
$\mathbf{x}_{a/b}(t_i)$	stations positions of a and b	$\mathbf{v}$	velocioty of geocenter in SSB frame
	geophysical modeling applied	$\mathbf{v}^b$	velocity of station b w.r.t. geocenter

Corrections

$$y_i = \tau = -\frac{1}{c}\mathbf{b}(t_i) \cdot \mathbf{R} \cdot \mathbf{k} \cdot \big(1 - F(\mathbf{v}, \mathbf{v}^b)\big) + \tau_{corr}$$

τ_{corr}

signal variations which are not assigned to station or source positions or Earth orientation

- clock synchronization
- ionosphere
- troposphere
- gravitational bending
- (thermal expansion)
- ...

Theoretical delay

$$y_i = \tau = -\frac{1}{c}\mathbf{b}(t_i) \cdot \mathbf{R} \cdot \mathbf{k} \cdot \left(1 - F(\mathbf{v}, \mathbf{v}^b)\right) + \tau_{corr}$$

inserting a priori knowledge about the individual components into functional model $\Rightarrow$ <u>theoretical delay τ_{comp}</u>

a priori information

is not sufficient to describe observations $\Rightarrow$ systematics in

$$\Delta\mathbf{y} = \boldsymbol{\tau}_{comp} - \boldsymbol{\tau}_{obs} = \mathbf{o} - \mathbf{c}$$

$\Rightarrow$ estimate parameters

Theoretical delay in a nutshell

1. get TRF station positions at reference epoch $(\mathbf{x}_0 + \dot{\mathbf{x}} \cdot (t_i - t_0))$
2. apply displacements
3. build rotation matrix due to a priori EOPs
4. calculate geometric delay and account for special relativistics
5. calculate and apply correction terms if possible

Clocks

$$y_i = \tau = -\frac{1}{c}\,\mathbf{b}(t_i)\cdot\mathbf{R}\cdot\mathbf{k}\cdot\big(1 - F(\mathbf{v},\mathbf{v}^b)\big) + \tau_{corr}$$

station clocks are not perfectly synchronized ⇒ offsets and drifts and ...
between time tags which directly appear as polynomial systematics in residuals

$$\tau_{cl} = \tau_{cl}^{b} - \tau_{cl}^{a}$$

$$\tau_{cl}^{a/b} = cl_0^{a/b} + cl_1^{a/b}\cdot(t_i - t_0) + cl_2^{a/b}\cdot(t_i - t_0)^2$$

$$\frac{\partial \tau}{\partial cl_0^a} = \frac{\partial \tau_{cl}}{\partial cl_0^a} = -1 \;; \quad \frac{\partial \tau}{\partial cl_1^a} = -\Delta t$$

$$\Rightarrow \underline{A} = \Big(\cancel{-\underline{1} \quad \underline{\Delta t} \quad \underline{\Delta t}^2} \quad 1 \quad \underline{\Delta t} \quad \underline{\Delta t}^2 \; \ldots\Big)$$

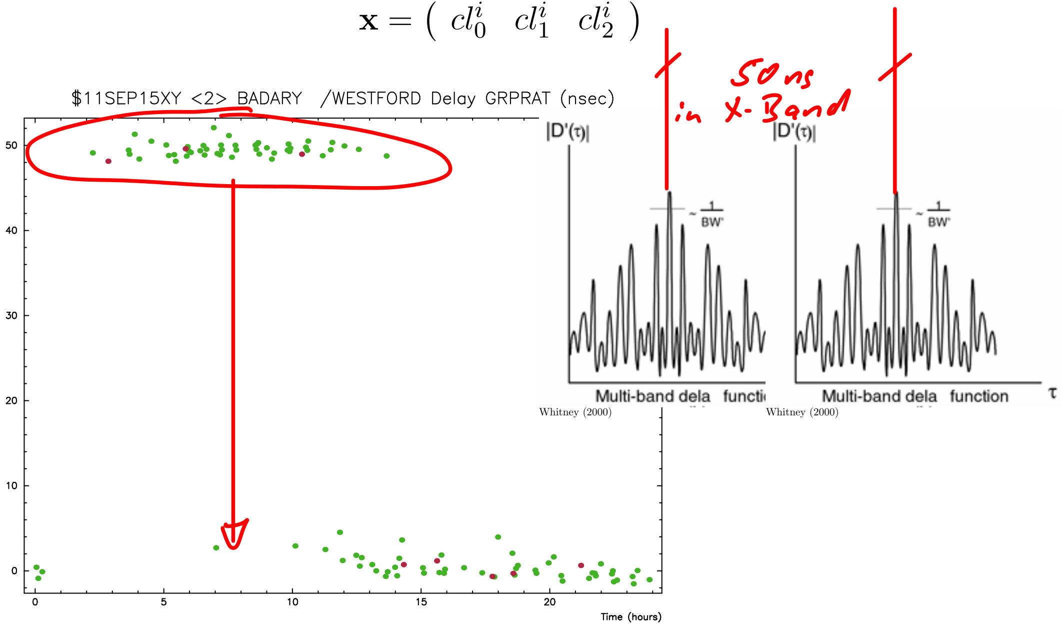
reference clock

rank deficiency

delays provide only relative information on clock behavior

Ambiguity resolution

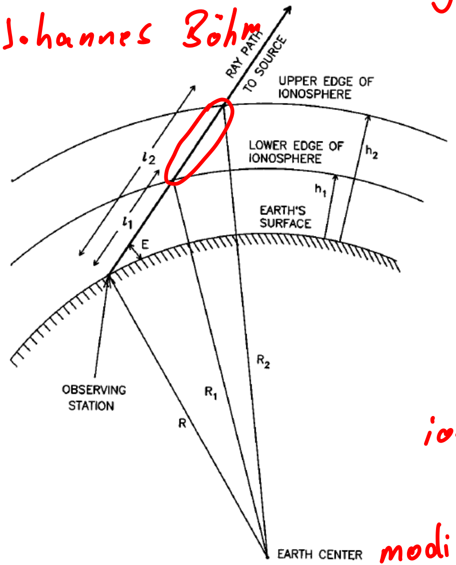
$$\mathbf{x} = \begin{pmatrix} cl_0^i & cl_1^i & cl_2^i \end{pmatrix}$$



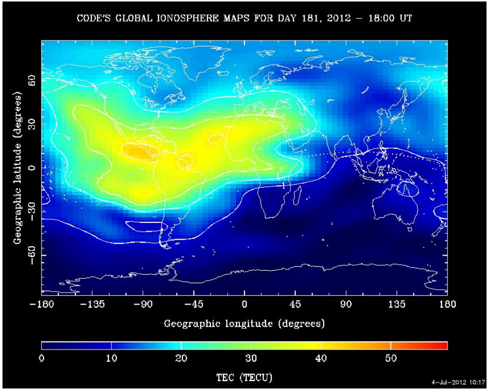
Ambiguity spacing can be determined from channels in bandwidth synthesis

Ionosphere

see "atmospheric propagation"
Johannes Böhm



Sovers et al. (1998)



http://aiuws.unibe.ch

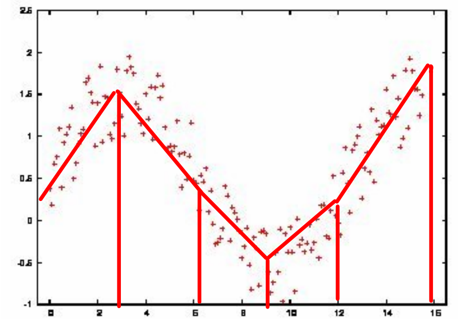
ion. corr.:

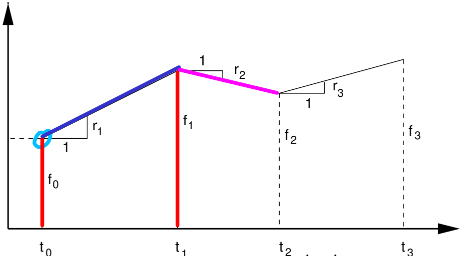
$$\tau_{ion,X} = (\tau_x - \tau_s)\frac{f_S^2}{f_X^2 - f_S^2}$$

modification of stochastic model

$$\sigma_{\tau_{ion,X}} = \sqrt{\sigma_{\tau_X}^2 + \sigma_{\tau_S}^2}\,\frac{f_S^2}{f_X^2 - f_S^2}$$

Clock refinement





$$\frac{\partial f}{\partial f_{i-1}} = \begin{cases} 1 - \frac{t-t_{i-1}}{t_i-t_{i-1}} \\ 0 \end{cases}$$

CPWLF:

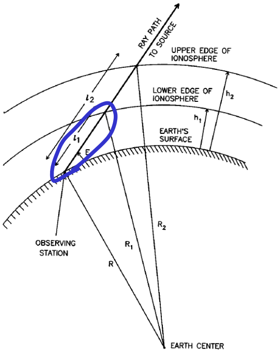
$$f(t) = f(t_0) + r_1(t_1 - t_0) + r_2(t_2 - t_1) + \ldots + r_n(t - t_{n-1})$$

$$r_i = \frac{f(t_i) - f(t_{i-1})}{t_i - t_{i-1}}$$

$$f(t) = f(t_0) + \frac{f(t_1) - f(t_0)}{t_1 - t_0}(t_1 - t_0) + \frac{f(t_2) - f(t_1)}{t_2 - t_1}(t_2 - t_1) + \ldots$$

$$\frac{\partial f}{\partial f_i} = \begin{cases} \frac{t-t_{i-1}}{t_i-t_{i-1}} & \text{for } t_i < t < t_{i+1} \\ 0 & \text{all other cases} \end{cases}$$

Troposphere 1



Sovers et al. (1998)

see "atmospheric propagation" J. Böhm

$$\tau_{trop} = 10^{-6} \int_S N ds$$
$$= 10^{-6} \int_S (N_h(T,p) + N_w(T,p,e)) ds$$
$$\tau_{trop}(\epsilon) = \underbrace{\tau_{at,h} \cdot m_h(\epsilon)}_{\text{apriori}} + \underbrace{\tau_{at,w}}_{\text{estimated}} \cdot m_w(\epsilon)$$

hydrostatic part

90 % of the tropospheric delay, modeled, e.g, Saastamoinen (1973)

$$\tau_{at,h}(\epsilon) = m_h(\epsilon) \frac{0.0022768 \cdot p}{1 - 0.00266 \cdot \cos(2\phi) - 0.28 \cdot 10^{-6} h}$$

mapping function m_h: hydrostatic NMF (Niell, 1996) or VMF (Boehm et al., 2006)

Troposphere 2

wet part

estimated ⇒ partial derivatives

$$\tau_{at,w} = at \cdot m_w(\epsilon)$$
$$\frac{\partial \tau}{\partial at} = \frac{\partial \tau_{at,w}}{\partial at} = m_w(\epsilon)$$

typically CPWLF with 1 h and below

troposphere gradients

azimuthal asymmetries ⇒ estimation of troposphere gradients in north-south and east-west direction (e.g., MacMillan 1995 or Chen and Herring 1997)

Initial solution in a nutshell

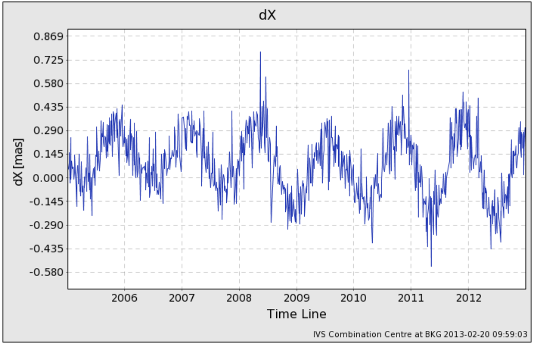
1. estimate clock polynomial
2. resolve ambiguities
3. calculate ionosphere correction
4. estimate clocks and ZWDs: CPWLF with 300 min resolution
5. find and remove outliers
6. estimate clocks and ZWDs: CPWLF with 60 min resolution and 24 h troposphere gradients
7. find and remove outliers or possibly restore earmarked observations
8. export V4 DB

see also:
`http://lacerta.gsfc.nasa.gov/mk5/help/solve_guide_01.html`

Independent solution

aim: individual solution for each experiment to generate time series of

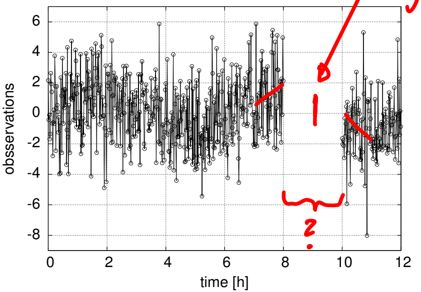
- station positions
- EOPs
- source positions
- nuisance parameters



http://ccivs.bkg.bund.de

Constraints on CPWLF params

singularity



$$\underline{y} = \begin{pmatrix} \underline{y} \\ 0 \end{pmatrix}$$

no pseudo-observation: no rate between sub-sequent params

$$cl^a_{i+1} - cl^a_i = 0$$

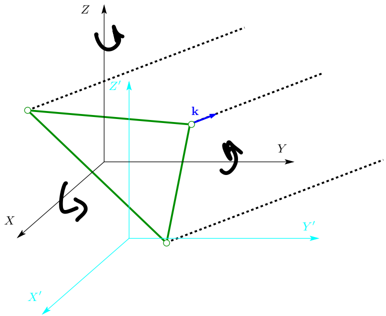
$$\underset{n+1}{\mathbf{A}} = \begin{pmatrix} \cdots & \frac{\partial \tau}{\partial cl^a_i} & \frac{\partial \tau}{\partial cl^a_{i+1}} & \cdots \\ \underline{0}^T & -1 & 1 & \underline{0}^T \end{pmatrix}$$

$$\mathbf{\Sigma}_{yy} = \begin{pmatrix} \begin{pmatrix} \sigma^2_{\tau_1} & & 0 \\ & \ddots & \\ 0 & & \sigma^2_{\tau_n} \end{pmatrix} & \boxed{\underline{0}} \\ \boxed{\underline{0}^T} & \boxed{\sigma} \end{pmatrix}$$

typical constraint σ

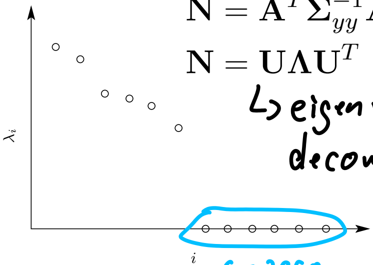
- clocks: 10^{-14}
- ZWD: 20 ps/h
- gradients: 2 mm/d & 1 mm

Datum definition



STA -> 3
EOP -> 3

6 rank deficiency



6 = zero nullspace

$$\mathbf{N} = \mathbf{A}^T \boldsymbol{\Sigma}_{yy}^{-1} \mathbf{A}$$

$$\mathbf{N} = \mathbf{U} \boldsymbol{\Lambda} \mathbf{U}^T$$

↳ eigenvalue decomposition

condition: optimal station estimates ⇒ minimizing $trace(\boldsymbol{\Sigma}_{xx})$

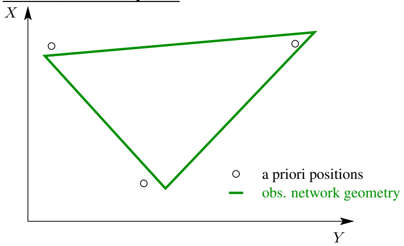
$$\boldsymbol{\Sigma}_{xx} = \mathbf{N}^{-1}$$ singularity

$$\boldsymbol{\Sigma}_{xx} = \sum_{i=1}^{m} \frac{1}{\lambda_i} \mathbf{u}_i \cdot \mathbf{u}_i^T, \quad m : \#\lambda \neq 0$$

↳ pseudo inverse

Datum: geometrical interpretation

2D example



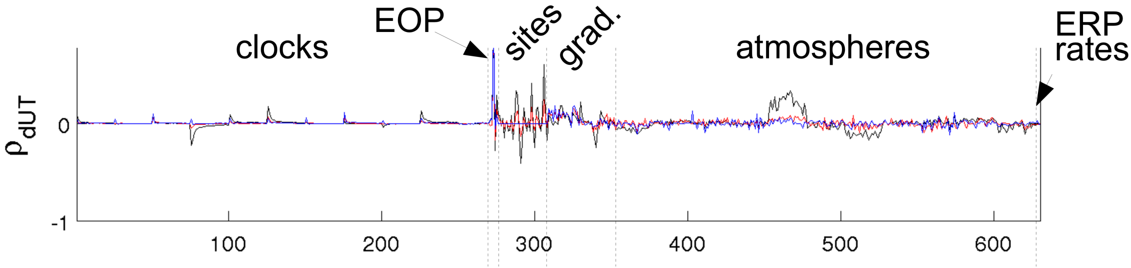
NNR/NNT condition: helmert parameter = 0

$$\mathbf{B}_i = \begin{pmatrix} 1 & 0 & 0 & 0 & -Z_i & Y_i \\ 0 & 1 & 0 & Z_i & 0 & -X_i \\ 0 & 0 & 1 & -Y_i & X_i & 0 \end{pmatrix}$$

$$\mathbf{A} = \left(\cdots \quad \underbrace{\frac{\partial \tau}{\partial X_1} \quad \frac{\partial \tau}{\partial Y_1} \quad \frac{\partial \tau}{\partial Z_1}}_{\boxed{B_1^T}} \quad \cdots \quad \underbrace{\frac{\partial \tau}{\partial X_2} \quad \frac{\partial \tau}{\partial Y_2} \quad \frac{\partial \tau}{\partial Z_2}}_{\boxed{B_2^T}} \quad \cdots \right)$$

EOP determination

significant correlations between EOPs and other parameter when NNR/NNT conditions are applied



reliable VLBI EOPs

can only be determined when station positions are fixed

Stacking sessions

identical param in 2 sessions

$$
\mathrm{N}: \begin{pmatrix}
N_{11}^{(1)} & \cdots & N_{1n}^{(1)} & & & \\
\vdots & \ddots & \vdots & & \mathbf{0} & \\
N_{n1}^{(1)} & \cdots & N_{nn}^{(1)} & & & \\
 & & & N_{11}^{(2)} & \cdots & N_{1n}^{(2)} \\
 & \mathbf{0} & & \vdots & \ddots & \vdots \\
 & & & N_{n1}^{(2)} & \cdots & N_{nn}^{(2)}
\end{pmatrix}
$$

$$
\rightarrow \begin{pmatrix}
N_{11}^{(1)} & \cdots & N_{1n}^{(1)} & & \mathbf{0} & \\
\vdots & \ddots & \vdots & & & \\
N_{n1}^{(1)} & \cdots & N_{nn}^{(1)} + N_{11}^{(2)} & N_{12}^{(2)} & \cdots & N_{1n}^{(2)} \\
 & & N_{21}^{(2)} & N_{22}^{(2)} & \cdots & N_{1n}^{(2)} \\
 & \mathbf{0} & \vdots & \vdots & \ddots & \vdots \\
 & & N_{n2}^{(2)} & N_{n2}^{(2)} & \cdots & N_{nn}^{(2)}
\end{pmatrix}
$$

CRF/TRF solution

- clocks, tropospheric parameters, EOPs and other nuisance params stay session parameters
- station, sources and axis offsets positions are stacked
- some stations and sources stay session parameters
- station velocities are set-up

$\Rightarrow$ consistent TRF, CRF, and EOPs

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