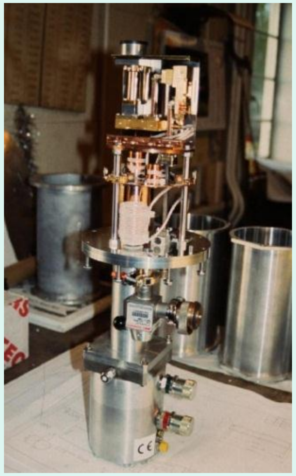


Technical Equipment at Stations

Bill Petrachenko, NRCan

2nd IVS Training School on

VLBI for Geodesy and Astrometry

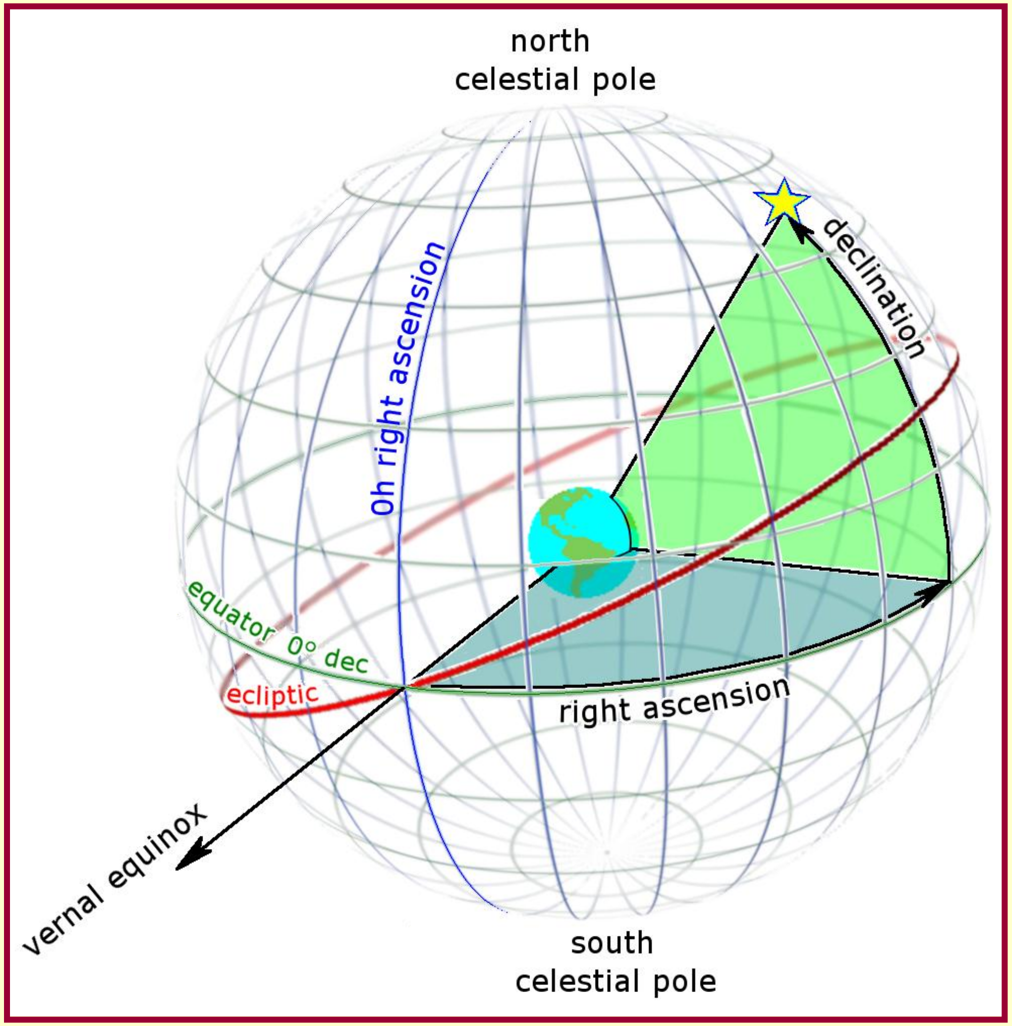
March 9-12, 2016

Hartebeesthoek Radio Observatory, South Africa

Radiation Basics – Celestial Coordinates

- The location of any point in the sky (celestial sphere) is defined by two coordinate angles:
  - Right Ascension (α)
  - Declination (δ)

- Area on the celestial sphere is called solid angle (Ω)
  - Units of solid angle are steradians (sr).



Radiation Basics – Surface Brightness

- Radiation is received from all points of the celestial sphere.
- The distribution of radiation is referred to as the ***Surface Brightness or Intensity***, $I_f(\alpha, \delta)$, with units ($Wm^{-2}Hz^{-1}sr^{-1}$).
- ***Surface Brightness*** is a power density with respect to:
  - Solid angle (Ω) of the source
  - Bandwidth of the signal, Δf
  - Area through which the radiation passes

$$\therefore P = \iiint I_f(\alpha, \delta)\, d\Omega\, df\, dA$$

Since Surface Brightness is continuously variable with position on the sky, it is the parameter used by astronomers to image a source.

Radiation Basics – Power Flux Density

- The ***power flux density***, S_f(Wm^{-2}Hz^{-1}), is the integral of brightness distribution over the solid angle of a source, i.e.

$$S_f = \int I_f(\alpha, \delta)\, d\Omega$$

- S_f is the most commonly used parameter to characterize the strength of source.
- It is often referred to simply as ***Flux Density*** or even ***Flux***.

- Because the flux of a typical radio source is very small, a unit of flux, the ***Jansky***, was defined for radio astronomy,

i.e. 1 Jy = 10^{-26} Wm^{-2}Hz^{-1}

- The power from a 1 Jy source collected in 1 GHz bandwidth by a 12 m antenna would take about 300 years to lift a 1 gm feather by 1 mm.

Radiation Basics – Brightness Temperature

- For a *Black Body* in *thermal equilibrium* and in the non-quantum Rayleigh-Jeans limit of the Planck Equation (i.e. $\hbar\nu \ll kT$), which is good for all radio frequencies,

$$I_f(\alpha, \delta) = \frac{2kT_B(\alpha, \delta)}{\lambda^2}, \qquad \lambda = \frac{c}{f}$$

where k=1.38e10-23 (m^2 Kg s^{-2} K^{-1}) is the Boltzmann Constant.

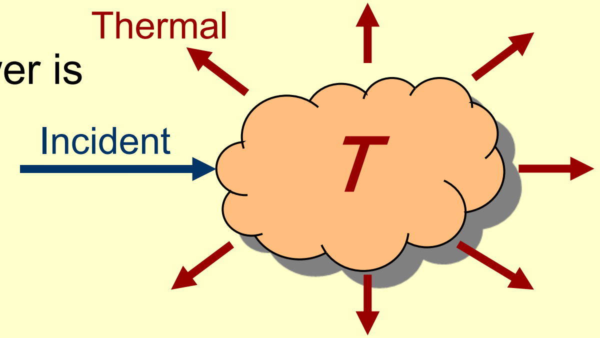
- *Brightness Temperature*, $T_B(\alpha, \delta)$, is often used as a proxy for $I_f(\alpha, \delta)$ regardless of whether of not the radiation mechanism is that of a thermal Black Body (although it is not an strictly proportional since the λ^2 still needs to be taken into account).

- In a similar way ***power in a resistor*** at temperature, T, can be written

$$P = kT$$

Radiation Basics – Radiative Transfer

For a ***Black Body***, i.e. a perfect absorber, the radiated power is



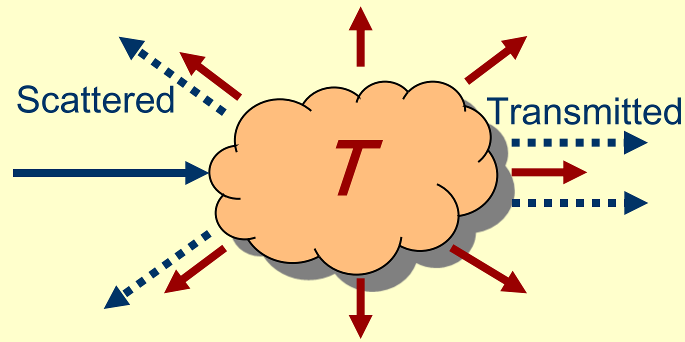
$$I_f = \frac{2kT}{\lambda^2}$$

(see previous page)

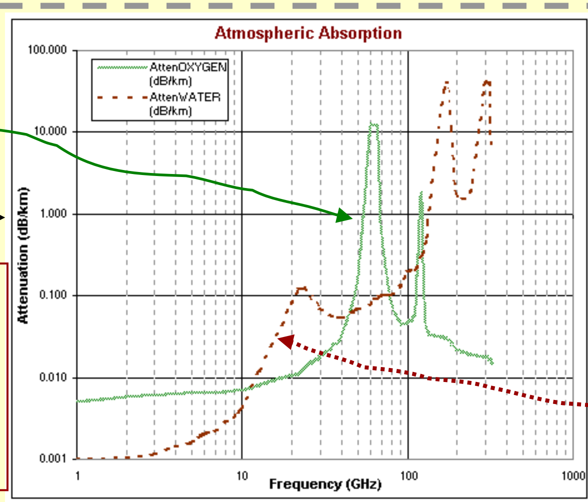
For an ***imperfect absorber***, the radiated power is

$$I_f = \frac{2kT_B}{\lambda^2} < \frac{2kT}{\lambda^2}$$

where T_B is the brightness temperature,
T is the physical temperature, and
$T_B = T\left(1 - e^{-\kappa \Delta s}\right)$.
κ is the absorption coefficient and
Δs is the length of the absorption path



For the ***atmosphere*** →



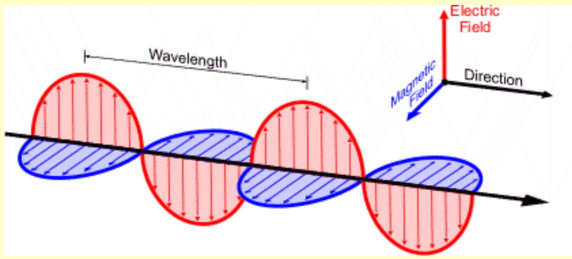
Imperfect absorption is why zenith atmosphere at x-band is 3°K and not 300°K

Absorption is a Lose-lose effect:
- the desired signal is attenuated
- thermal noise is added to the absorber

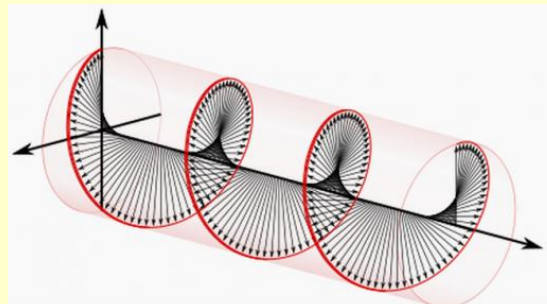
Radiation Basics - Polarization

The Polarization vector is in the instantaneous direction of the E-field vector

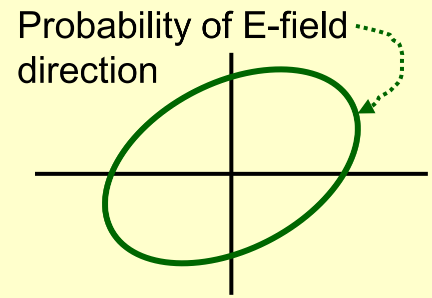
Linear Polarization



Circular Polarization



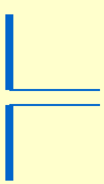
Random Polarization



Most geodetic VLBI sources have nearly circular distributions, i.e. are nearly unpolarized.

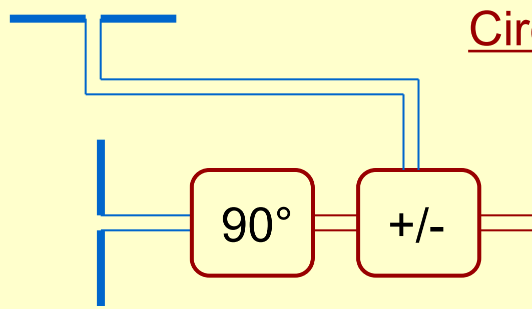
Regardless of the input signal, all of the radiated power can be detected with two orthogonal detectors, either Horizontal and Vertical linear polarization or Left and Right circular polarization. With random polarization this is the only option for detecting all the power.

Linear Detector



e.g. dipole

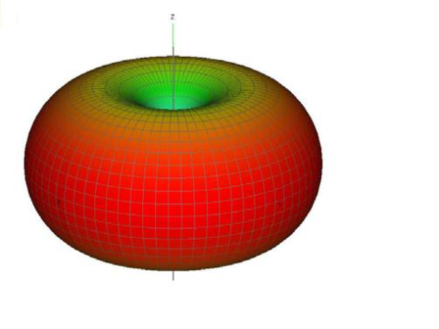
Circular Detector



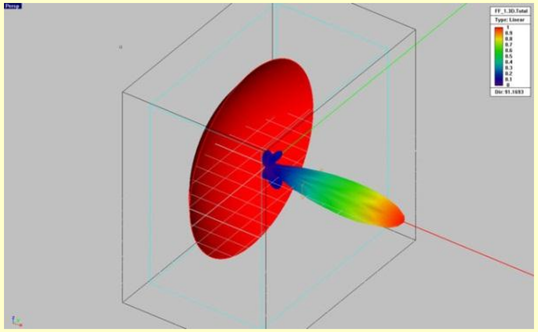
e.g. quadrature combination of dipole outputs

Antenna Basics

- A *Radio antenna* is a device for converting electromagnetic radiation in free space to electric current in conductors
- An *Antenna Pattern* is the variation of power gain (or receiving efficiency) with direction.



Antenna pattern: Dipole antenna



Antenna pattern: Parabolic antenna

- *Reciprocity* is the principle that an antenna pattern is the same whether the antenna is transmitting or receiving.
  - *Transmitting* antennas are generally characterized by ***gain***
  - *Receiving* antennas are generally characterized by ***effective area***

Antenna Gain – Characterizes a *Transmitting* Antenna

- *Antenna gain* is defined as $G(\theta, \varphi) = \frac{P_f(\theta,\varphi)}{P_{iso}}$, where

  $P_f(\theta, \varphi)$ ~ power per unit solid angle transmitted in direction (θ, φ)

  P_{iso} ~ power per unit solid angle transmitted by an isotropic antenna (i.e. a ***hypothetical*** lossless antenna that transmits equal power in all directions).

- Functionally

$$P_{out}(\theta, \varphi) = \mathrm{G}(\theta, \varphi) P_{in}$$

- For a *lossless antenna*, $P_{out} = P_{in}$, and

$$\langle G \rangle = G_{iso} = 1$$

Effective Area – Characterizes a *Receiving* Antenna

Effective area is defined as $A_e(\theta, \varphi) = \frac{2P_f(\theta,\varphi)}{I_f(\theta,\varphi)}$, where

$P_f(\theta, \varphi)$ ~ power received from direction (θ, φ).
$I_f(\theta, \varphi)$ ~ surface brightness received from direction (θ, φ).

Note: I_f includes all radiated flux. For an unpolarized source, only half of the flux is received per polarization detector. Hence we need to use $\frac{I_f(\theta,\varphi)}{2}$.

The power received from a source in direction (θ, φ) can be written

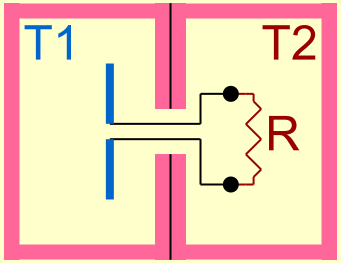
$$P_f(\theta, \varphi) = \int_{\Omega\text{-source}} A_e(\theta, \varphi) \frac{I_f(\theta, \varphi)}{2} d\Omega$$

Effective area of an Isotropic Antenna

Antenna Side

$$P_f = \frac{kT_1}{\lambda^2} \int_{Sphere} A_e d\Omega$$

Black Body cavities



Resistor Side

The noise power (per Hz) generated by a resistor at temp, T, can be written

$$P_f = kT_2$$

At thermodynamic equilibrium, *T1=T2* and no current flows between antenna and resistor.

i.e. P_f(antenna side) = P_f(resistor side)

$$\therefore \frac{kT}{\lambda^2} \int_{Sphere} A_e d\Omega = kT \dashrightarrow \int_{Sphere} A_e d\Omega = \lambda^2$$

$$\langle A_e \rangle = \frac{1}{4\pi} \int_{Sphere} A_e d\Omega, \qquad \therefore \langle Ae \rangle = \frac{\lambda^2}{4\pi}$$

For an isotropic antenna

$$A_{iso} = \langle A_e \rangle = \frac{\lambda^2}{4\pi}$$

Effective Area ◄·····► Gain

From earlier results

$$A_{iso} = \langle A_e \rangle = \frac{\lambda^2}{4\pi} \quad \text{and} \quad \langle G \rangle = 1$$

$$\therefore \langle A_e \rangle = \langle G \rangle \frac{\lambda^2}{4\pi}$$

$$\therefore \frac{A_e(\theta, \varphi)}{A_{iso}} = G(\theta, \varphi)$$

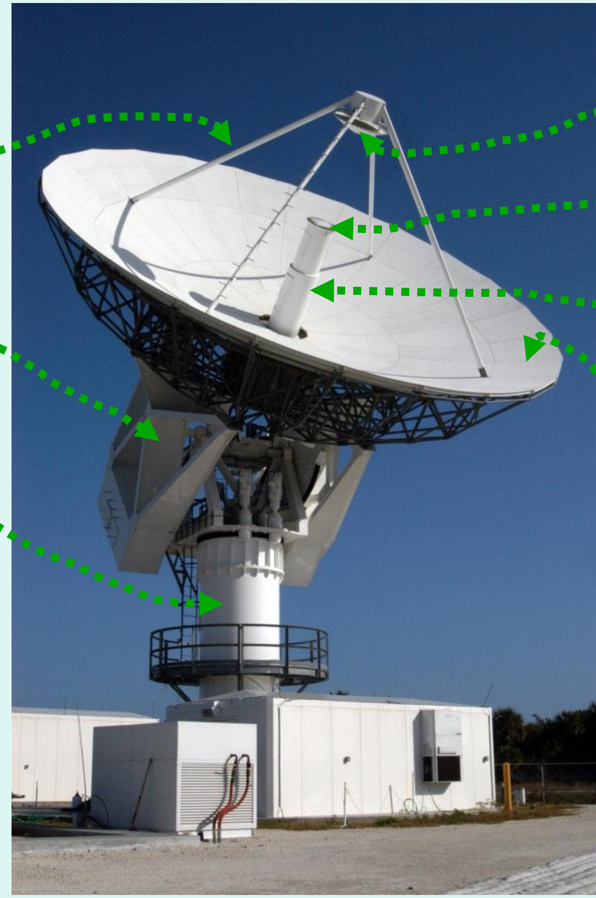
This allows us to calculate the receiving pattern from the transmitting pattern and vice versa.

Parabolic Reflector Antenna

Sub-reflector support legs

Antenna positioner

Pedestal
(aka antenna tower)



Secondary reflector
(aka Sub-reflector)

Feed Horn

Feed Horn Support Structure

Primary reflector

The ***antenna reflectors*** concentrate incoming E-M radiation into the focal point of the antenna.

The ***feed horn*** converts E-M radiation in free space to electrical currents in a conductor.

The ***antenna positioner*** points the antenna at the desired location on the sky.

Antenna Positioners – Alt-az

The antenna positioner is system that points the beam of the antenna toward the area of sky of interest. There are three main positioner systems: alt-az, equatorial, and X-Y.

Alt-az

This is the workhorse antenna mount for large radio telescopes. It has a fixed vertical axis, the azimuth axis, and a moving horizontal axis, the altitude (or elevation) axis that is attached to the platform that rotates about the azimuth axis. The azimuth motion is typically ±270° relative to either north or south and the elevation motion is typically 5° to 85°.

Advantages:

- Easy to balance the structure and hence optimum for supporting a heavy structure.

Disadvantages:

- Difficult to track through the zenith due to the coordinate singularity (key hole).
- Complications with cable management due to 540° of azimuth motion (cable wrap problem).

Antenna Positioners - Equatorial

Equatorial

This type of positioner is no longer used for large antenna's although it was in widespread use prior to the advent of high speed real-time computers for calculating coordinate transformations. It has a fixed axis in the direction of the celestial pole, the equatorial axis, and a moving axis at right angles to the equatorial axis, the declination axis. The declination axis is attached to the part of the antenna that rotates around the equatorial axis. The axis motion is somewhat dependent on latitude but is < ±180° in hour angle (equatorial) and < ±90° in declination.

Advantages:

- Can be used without computer control – just get on source and track at the sidereal rate.
- No cable wrap ambiguity

Disadvantages:

- Difficult to balance the structure and hence sub-optimal for large structures.
- Key hole problem at the celestial pole.

Antenna Positioners – X-Y Mount

X-Y Mount

This type of positioner is mainly used for high speed satellite tracking where key holes cannot be tolerated. The fixed axis points to the horizon and hence the only keyhole is at the horizon, which is too low for tracking. Full sky coverage can be achieved with ±90° motion in both axes.

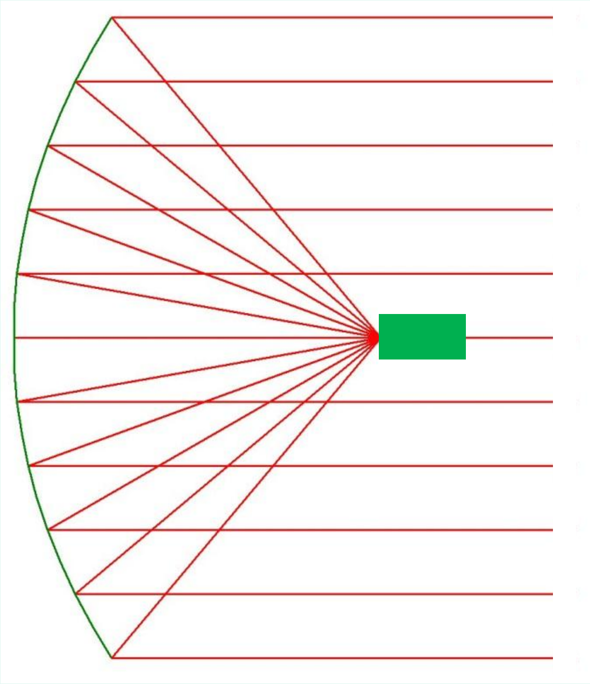
Advantages:

- No place where an object cannot be tracked (i.e. no key holes).
- No cable wrap ambiguity

Disadvantages:

- Structurally difficult to construct (compared with alt-az).

Parabolic Reflector Antenna



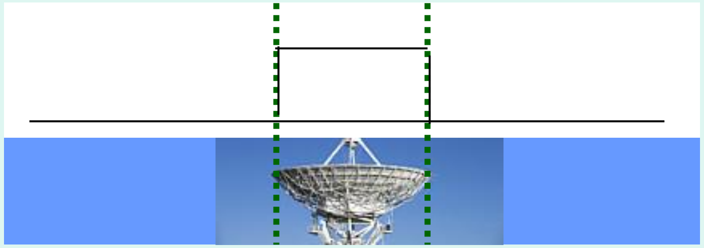
- A *parabolic antenna* takes the points on a plane wave front and reflects them such that they arrive simultaneously at a single point at the focus of the parabola.

- A '*Feed Horn*' is located at the antenna focus to take the concentrated wave and convert it to an electrical signal in a conductor.

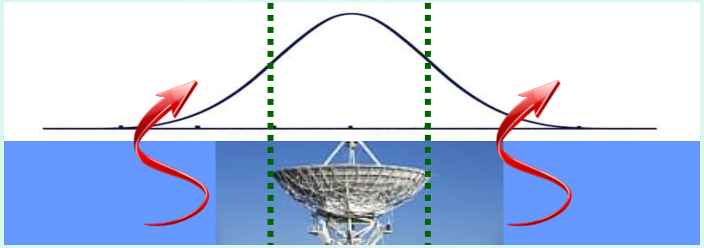
Aperture Illumination

The ‘*Feed Horn*’ is itself an antenna with a power pattern that ‘*illuminates*’ the reflector. The feed works equally well, in a radio telescope, as a receiving element.

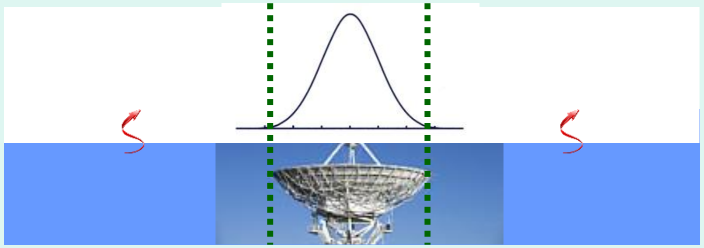
Ideal-illumination: The feed pattern is uniform across the reflector and zero everywhere else. A feed like this cannot be built.



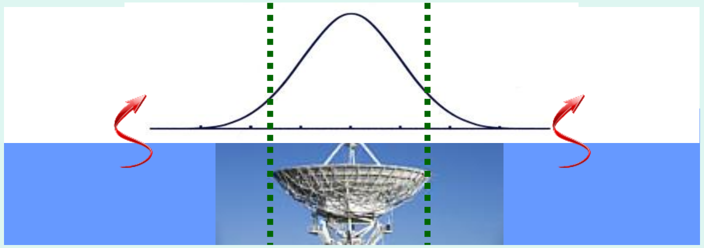
Over-illumination: The feed pattern extends well beyond the edge of the dish. Too much ground radiation is picked up from outside the reflector.



Under-illumination: The feed pattern is almost entirely within the dish. There is minimal ground pick-up but the dish appears smaller than it is.

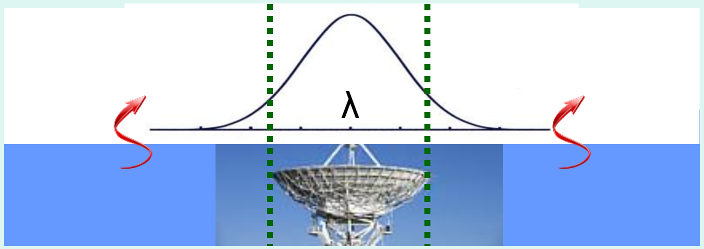


Optimal-illumination: This is the best balance between aperture illumination and ground pick-up. The power response is usually down about 10 dB (90%) at the edge of the dish.



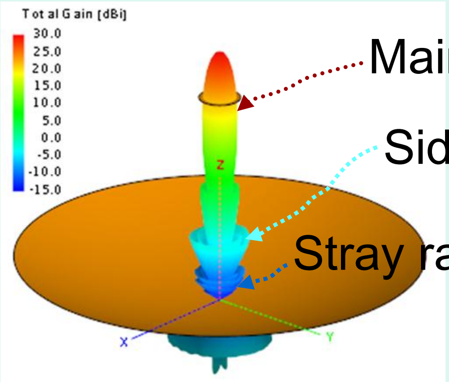
Aperture Illumination ◄····► Beam Pattern

The beam pattern of the antenna is the Fourier Transform of the aperture illumination (assuming that the aperture is measured in units of λ).



Aperture illumination

Fourier Transform



Beam pattern

Depending on the details of the aperture illumination, the Half Power Beam Width (HPBW) is approximately

$$HPBW \approx \frac{\lambda}{D}$$

where *D* is the diameter of the reflector.

The beam becomes narrower as dish becomes larger or λ becomes shorter. (λ becoming shorter is the same as the frequency becoming larger).

For a Parabolic Reflector Antenna with a Narrow Beam

If the source is smaller than the beam

$$P_f = \int A_e \frac{I_f}{2} d\Omega, \qquad \therefore P_f = \frac{A_e S_f}{2}$$

(i.e. all of the source flux is received)

If the source is larger than the beam

$$P_f = \int A_e \frac{I_f}{2} d\Omega, \qquad \therefore P_f = \frac{A_e S_f}{2} \frac{\Omega_{Beam}}{\Omega_{Source}}$$

(i.e. only part of the source flux is received)

Aperture efficiency

The antenna effective area, A_e , can be compared to the antenna geometric area with the ratio, η_A , being the antenna efficiency, i.e.

$$A_e = \eta_A A_{geo}$$

where, for a circular antenna, $A_{geo} = \frac{\pi}{4} D^2$.

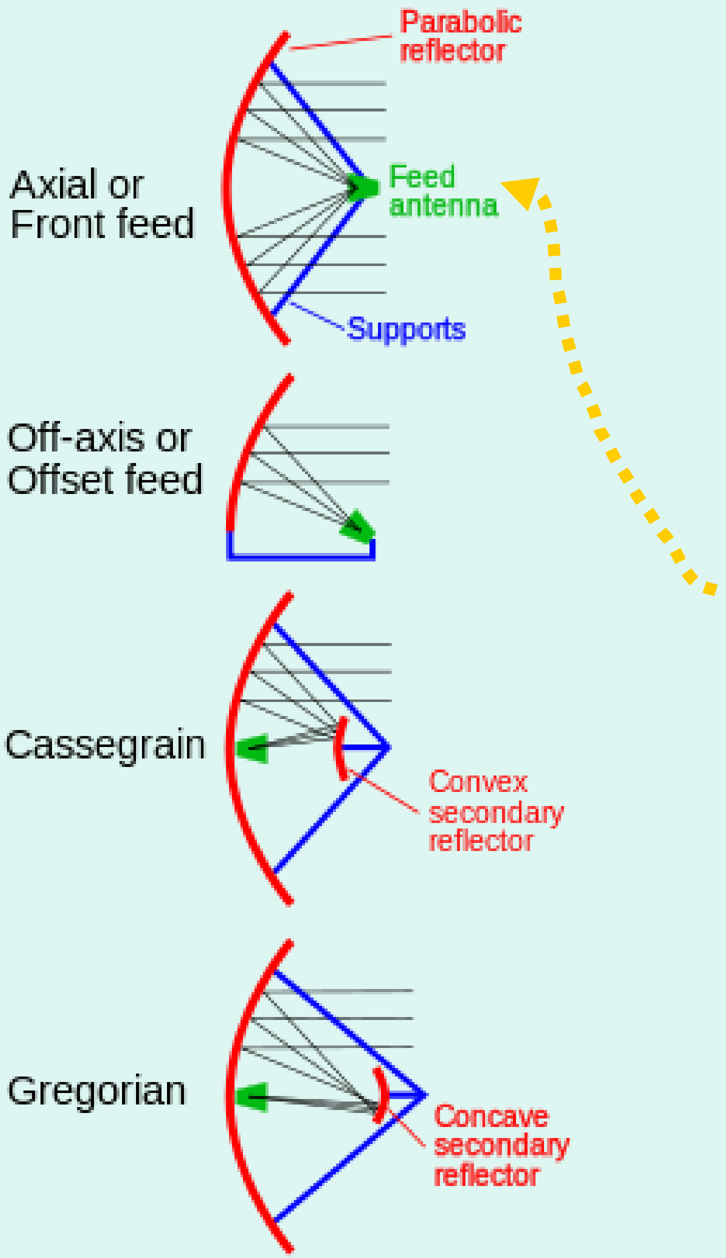
The antenna efficiency can be broken down into the product of a number of sub-efficiencies:

$$\eta_A = \eta_{sf} \times \eta_{bl} \times \eta_s \times \eta_t \times \eta_p \times \eta_{misc}$$

where

- η_{sf} Surface accuracy efficiency (both surface shape and roughness)
- η_{bl} Blockage efficiency
- η_s Spill-over efficiency
- η_t Illumination efficiency
- η_p Phase centre efficiency
- η_{misc} Miscellaneous efficiency, e.g. diffraction and other losses.

Antenna Optics – i.e. reflector configuration



The *purpose of the reflector system* is to concentrate the radiation intercepted by the full aperture (and from the boresite direction) into a single point.

Axial or Front Feed – aka Prime Focus

The front feed antenna uses a parabaloid primary reflector with the phase centre of the feed placed at the focal point of the primary reflector.

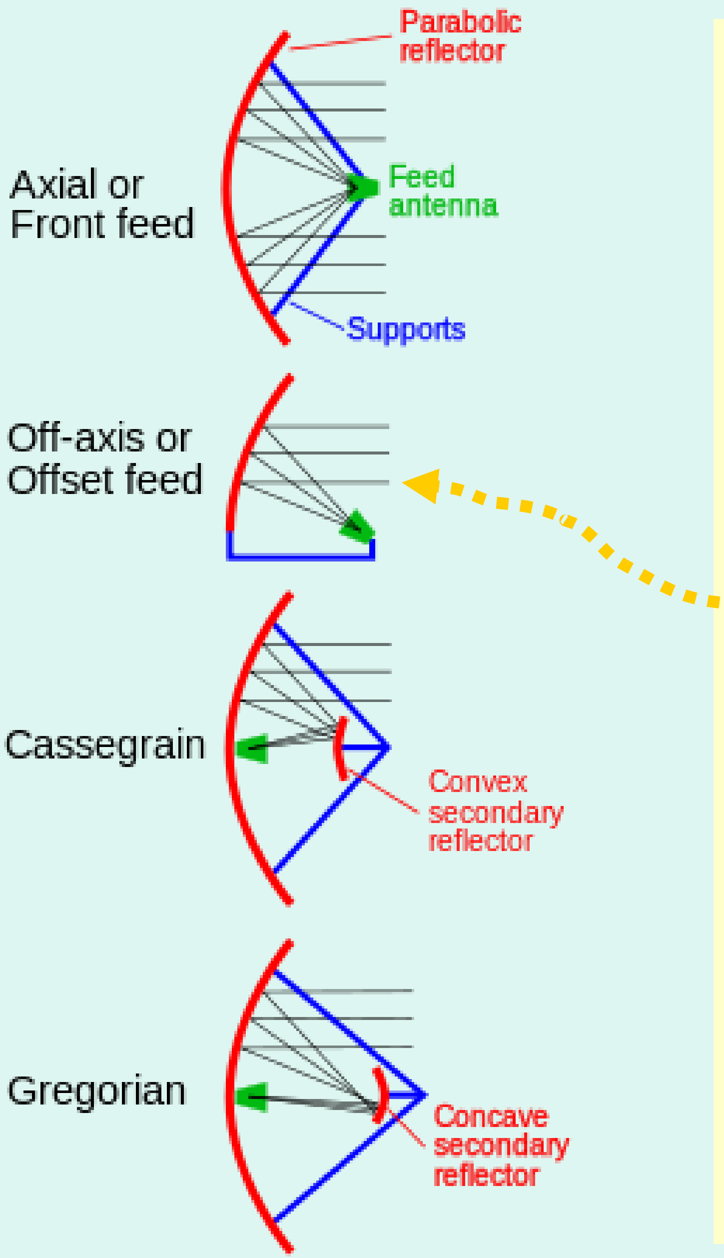
Advantages:

- Simple
- No diffraction loss at the sub-reflector (more important at lower frequencies)
- Only one reflection required leading to less loss and less noise radiated (minimal benefit if the reflector material is a good conductor).

Disadvantages:

- Spill-over looks directly at the warm ground.
- Added structural strength required to support feed plus front end receiver at the prime focus

Antenna Optics – i.e. reflector configuration



Off-axis or Offset Feed

The primary reflector is a section of paraboloid completely to one side of the axis, with the feed supported from one side of the reflector.

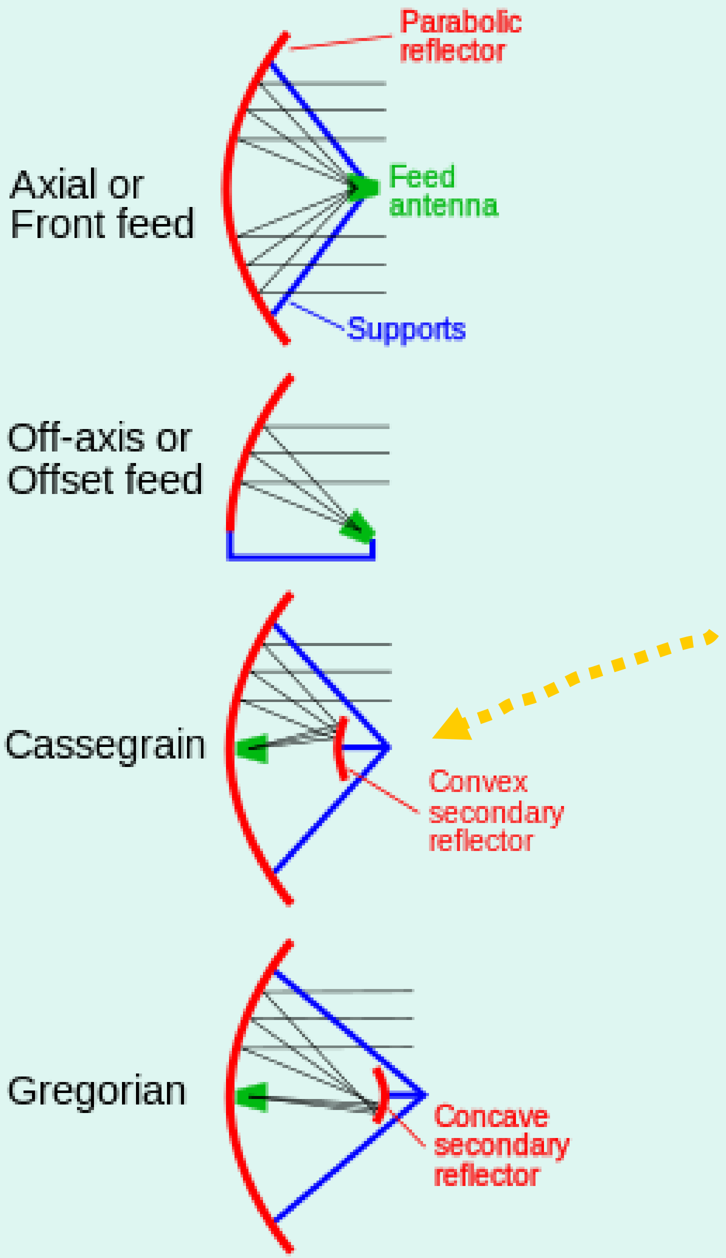
Advantages:

- No aperture blockage (leading to higher antenna efficiency)

Disadvantages:

- Spill-over looks preferentially toward the warm ground (especially for high-side feed support).
- Lack of symmetry.
- Added structural strength required to support feed plus front end receiver to one side of the reflector.
- Complications with all-sky positioner for low-side feed support
- Complications with feed/receiver access for high-side feed support

Antenna Optics – i.e. reflector configuration



Cassegrain

This is a two reflector system having a hyperboloid secondary reflector (sub-reflector) between the prime focus and the primary reflector. The sub-reflector focuses the signal to a point between the two reflectors.

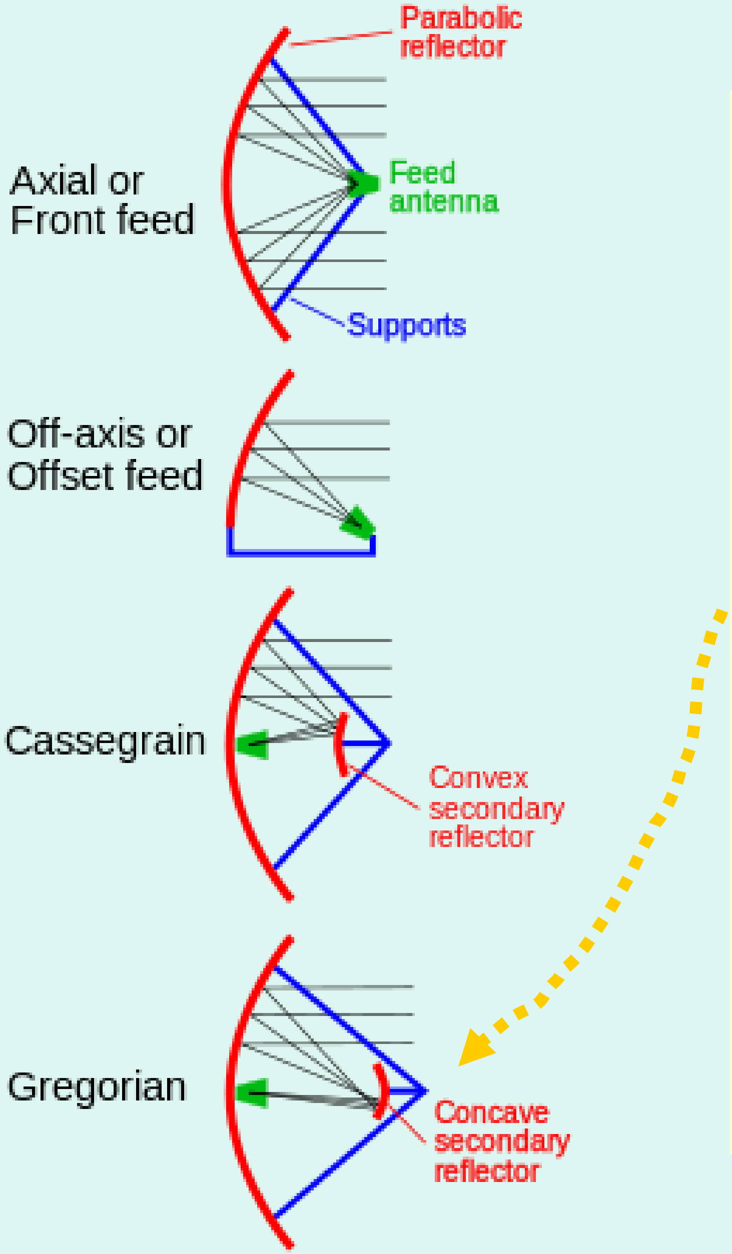
Advantages:

- Spill-over past the sub-reflector is preferentially toward cold sky.
- Minimal structural strength is required since the sub-reflector is located nearer the primary.

Disadvantages:

- The sub-reflector obscures the prime focus so it is difficult to achieve simultaneous operation with a prime focus feed.

Antenna Optics – i.e. reflector configuration



Gregorian

This is a two reflector system having a parabaloid secondary reflector (sub-reflector) located on the far side of the prime focus. The sub-reflector focuses the signal to a point between the prime focus and the primary reflector.

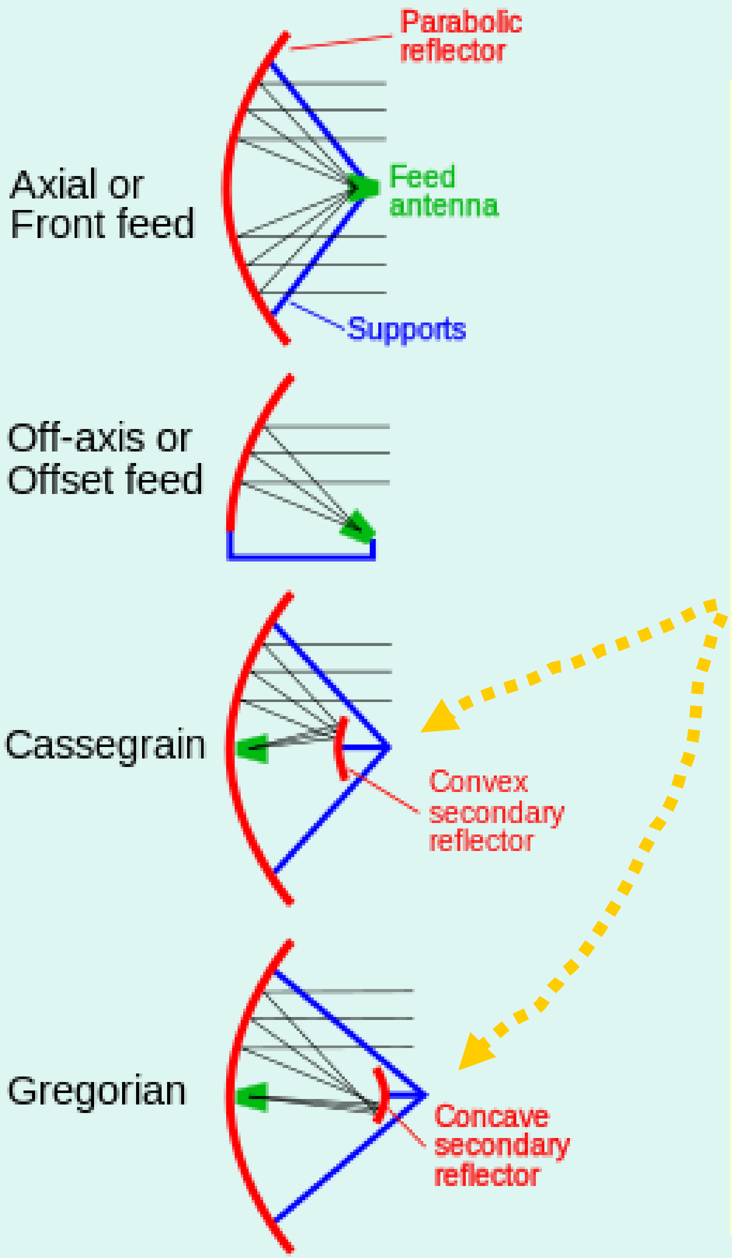
Advantages:

- Spill-over past the sub-reflector is preferentially toward cold sky.
- The sub-reflector does not obscure the prime focus so it is easier to achieve simultaneous operation with a prime focus feed.

Disadvantages:

- Greater structural strength is required since the sub-reflector must be supported further away from the primary.

Antenna Optics – i.e. reflector configuration



Shaped Reflector System

A shaped reflector system requires optics that involve more than one reflector, e.g. the Cassegrain or Gregorian systems. With a shaped reflector system, the shape of the secondary reflector is altered to improve illumination of the primary. To compensate for the distortion of the secondary, the shape of the primary must also be changed away from a pure parabaloid.

Advantages:

- Improved efficiency

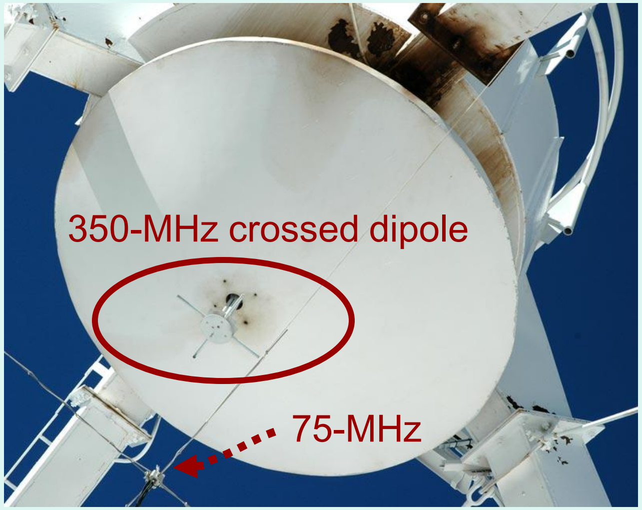
Disadvantages:

- The reflectors are no longer simple parabaloids or hyperbaloids but more complex mathematical shapes. [With the advent of readily available computer aided design and manufacture this is no longer a significant complication.]

Antenna Feed – crossed dipole

An antenna feed is itself an antenna. Whereas the reflector system concentrates radiation from a wide area into a single point, the feed converts the E-M radiation at the 'single point' into a signal in a conductor.

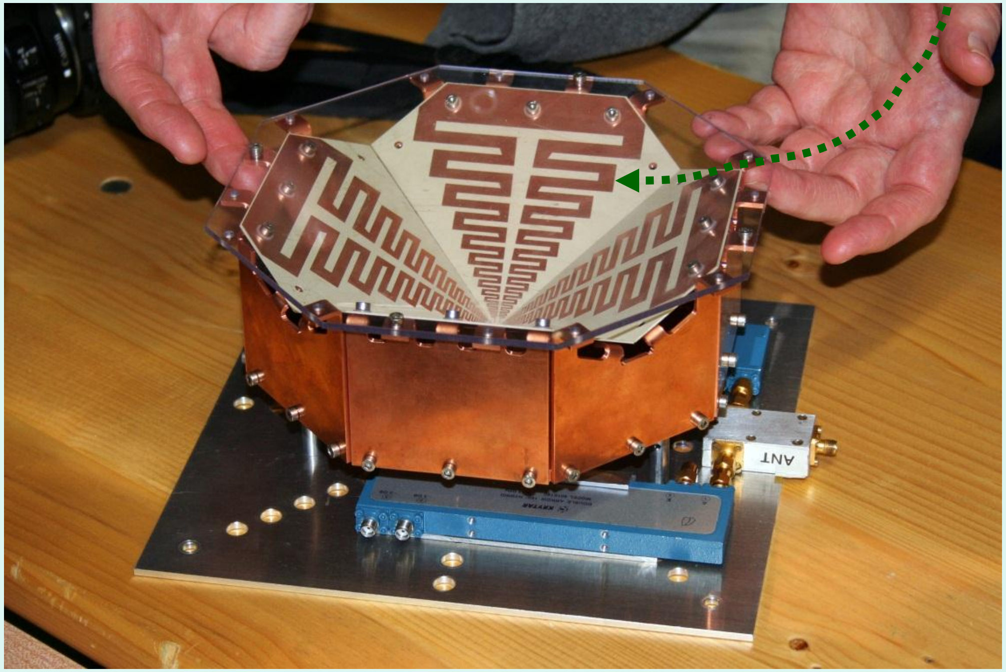
One of the simplest feeds is a crossed dipole, i.e. a pair of orthogonal ½-λ dipoles usually located ¼-λ above a ground plane, e.g. the VLA crossed dipoles.



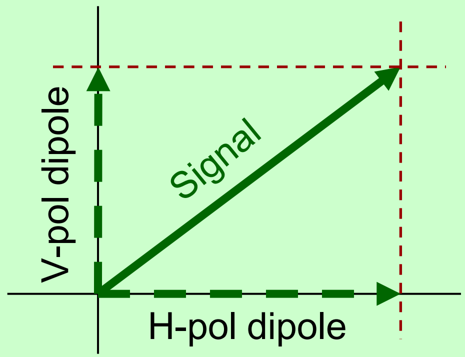
Antenna Feed - Broadband

A broadband feed can be designed using a series of log periodic dipoles. Log periodic means that the length and separation of the dipoles increases in a geometric ratio chosen so that all frequencies are covered. Here we see a version of the Eleven Feed developed at Chalmers University for VLBI2010. This version covers 2-12 GHz with a newer version covering 1-14 GHz.

Folded dipoles of the Eleven Feed

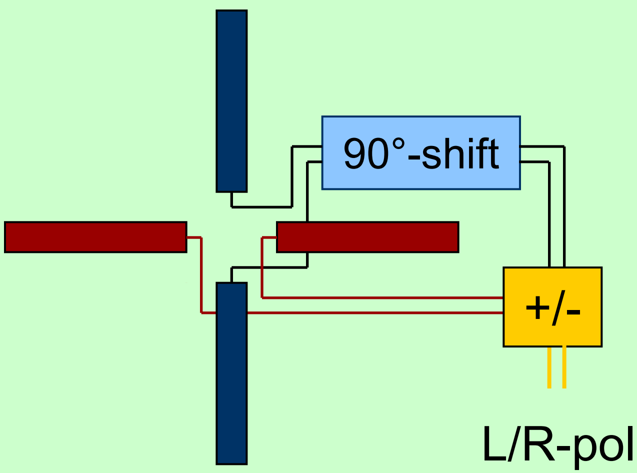


Crossed dipole - polarization



An arbitrary polarization vector decomposed into orthogonal components

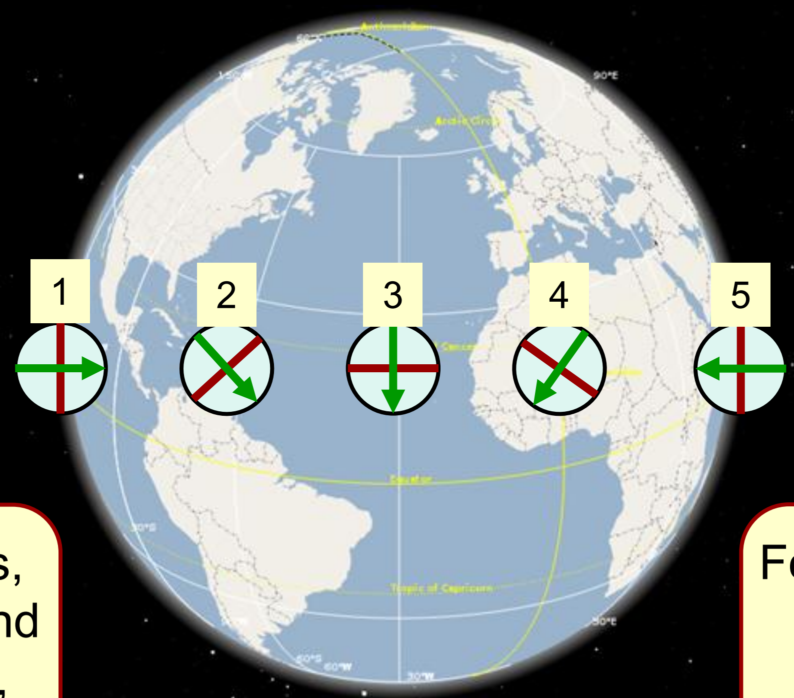
Each dipole of a crossed dipole is sensitive to signals with polarization vectors parallel to the dipole. Two orthogonal dipoles can receive all the power from an arbitrary signal.

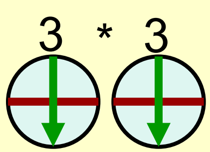


Circular polarization is formed by combining linear signals in quadradure (i.e. by adding and subtracting linear polarizations after one of them has been shifted by 90°). This works easily for narrow band signals – but the existence of broadband 90°-shifters (hybrids) also makes it applicable to broadband signals (like VLBI2010). For this to work well, the electronics must represent the mathematics accurately.

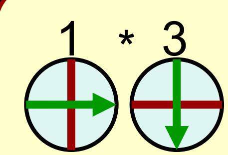
VLBI works best with circular polarization

As seen from above, the linear polarization orientation for alt/az antennas varies with geographic location





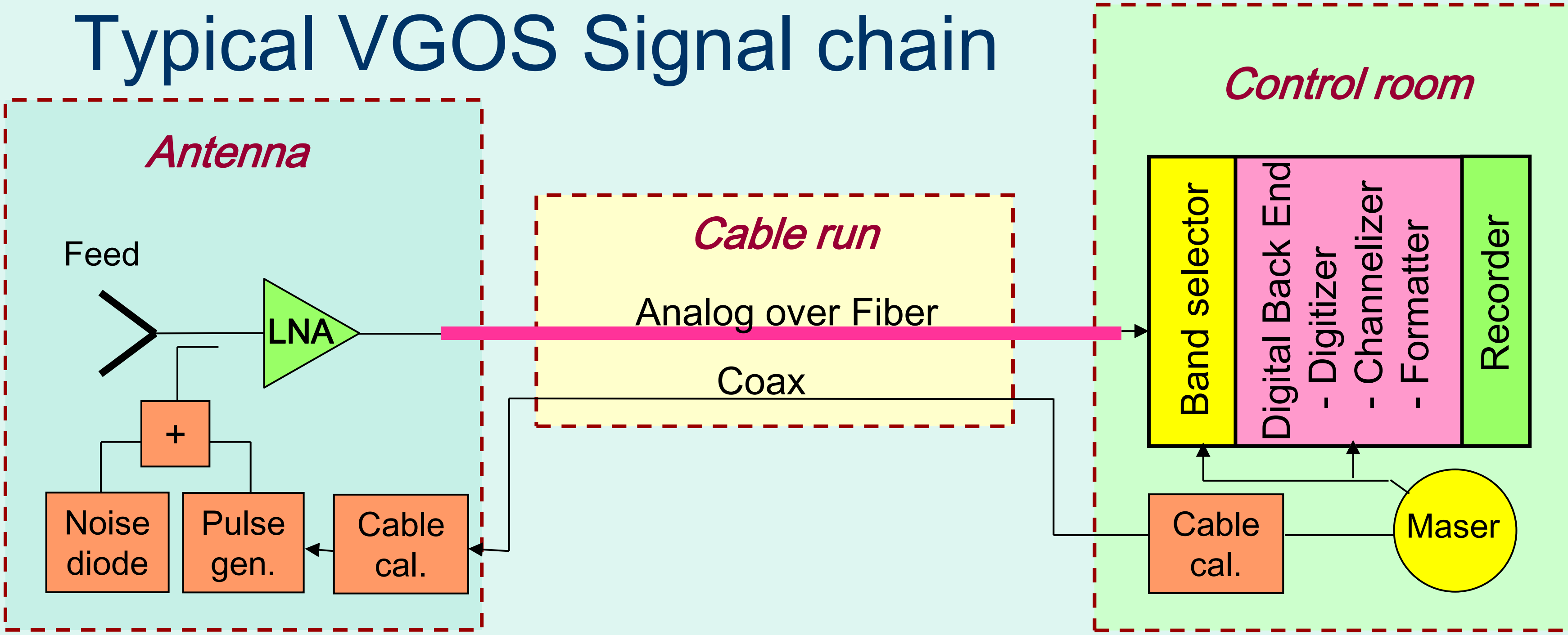
For parallel orientations, correlated signal is found in the co-pol products, e.g. v1*v2 and h1*h2



For orthogonal orientations, correlated signal shifts to the cross-pol products e.g. v1*h2 and h1*v2

To avoid the shifting of correlated amplitude between cross- and co-pol products , VLBI traditionally uses circular polarization, where correlated amplitude is independent of relative polarization orientation.

Typical VGOS Signal chain



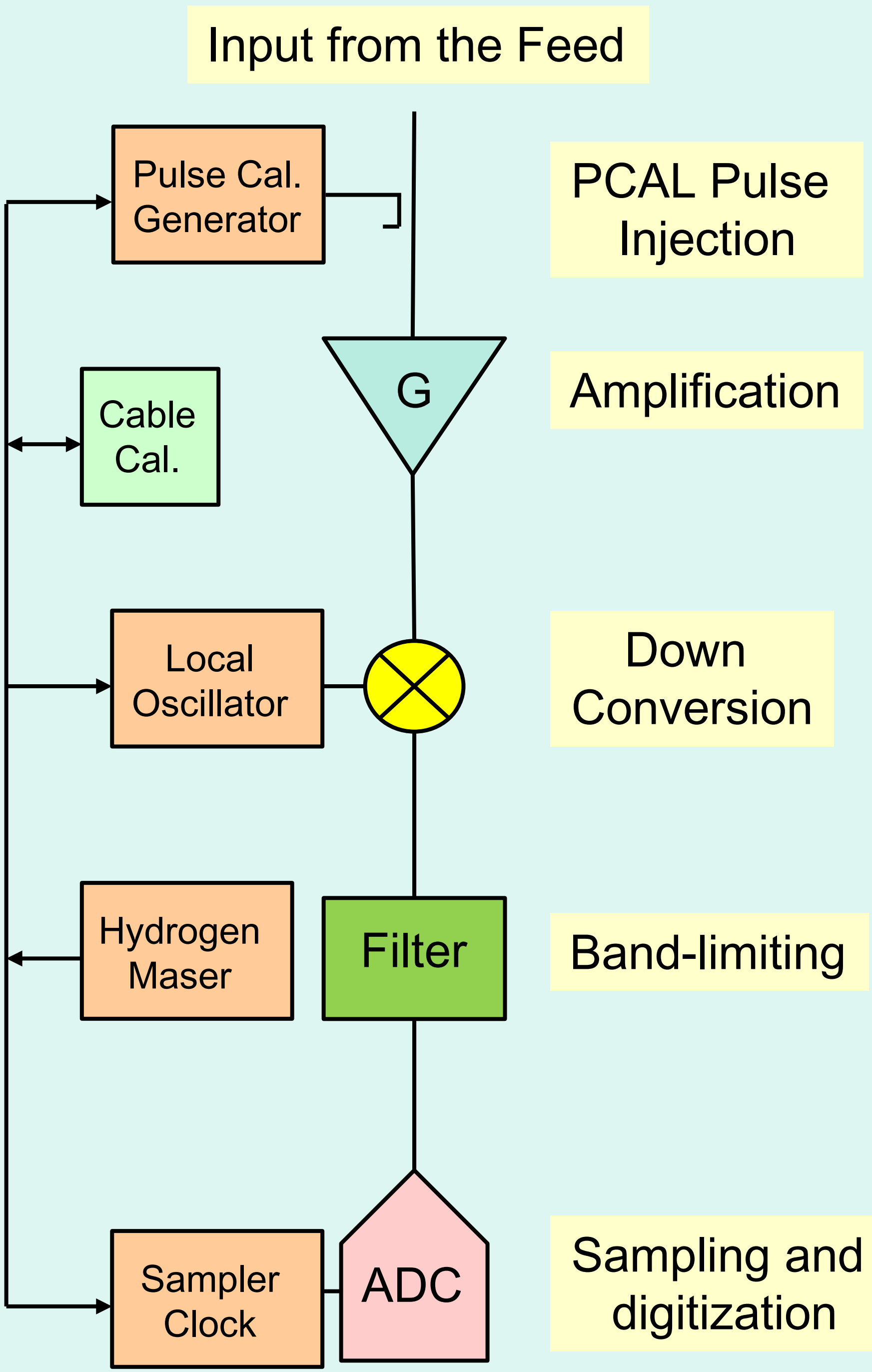
Front end receiver:

- Feed
- Low Noise Amplifier (LNA)
- Cryogenics
- Calibration equipment:
  - ~ Noise diode
  - ~ Phase Cal. (Pcal) Pulse Generator
  - ~ Cable Cal. Antenna Unit

Back end receiver:

- Band Selector (e.g. Up-Down Converter made up of analog amplifiers, mixers, filters, etc)
- Digital Back End (DBE)
  - ~ High rate/high resolution Digitizer/Sampler
  - ~ Channelizer:
    - * Polyphase Filter Bank (PFB), or
    - * Digital Baseband Converter
  - ~ Formatter
- Recorder
- Hydrogen Maser
- Cable Cal. Ground Unit

Simplified Signal Chain



A real VGOS signal chain contains multiple stages of gain, down conversion, filtering and even digitization. For analysis, the effect of the multiple stages can however be simplified into a single occurrence of each.

At any stage in the signal chain, the signal can, for convenience, be split into three components, i.e.:

$$X_i(t) = AST_i(t) + NOISE_i(t) + PCAL_i(t)$$

where

- $AST_i(t)$ is a broadband noise signal from the astronomy source. It is common to all stations and hence it is correlated between stations.
- $NOISE_i(t)$ is a local noise signal. Because of its independent origin, it is uncorrelated between stations.
- $PCAL_i(t)$ is the phase calibration signal.

Note: i is the station index.

Signal arrival at a station

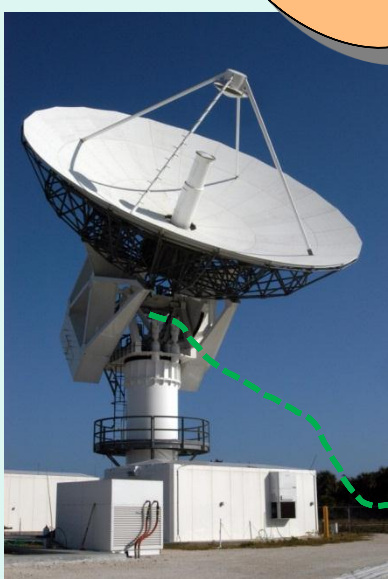
Plane waves arriving from an extragalactic point source in direction $\hat{k}$ and with flux S_f

Equation of a monochromatic plane wave in a vacuum:

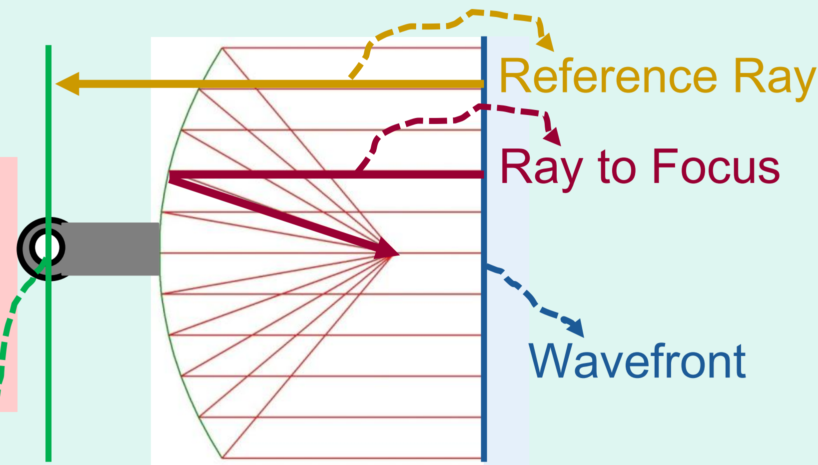
$$X(f,t) = \sqrt{S_f} \cdot e^{-j\left\{2\pi f\left(t - \frac{\hat{k}\cdot\vec{x}}{c}\right)\right\}}$$

Neutral atmosphere:
$$\varphi^{atm} = 2\pi f \tau^{atm}$$

Iosphere:
$$\varphi^{ion} = \frac{-2\pi K^{ion}}{f}$$

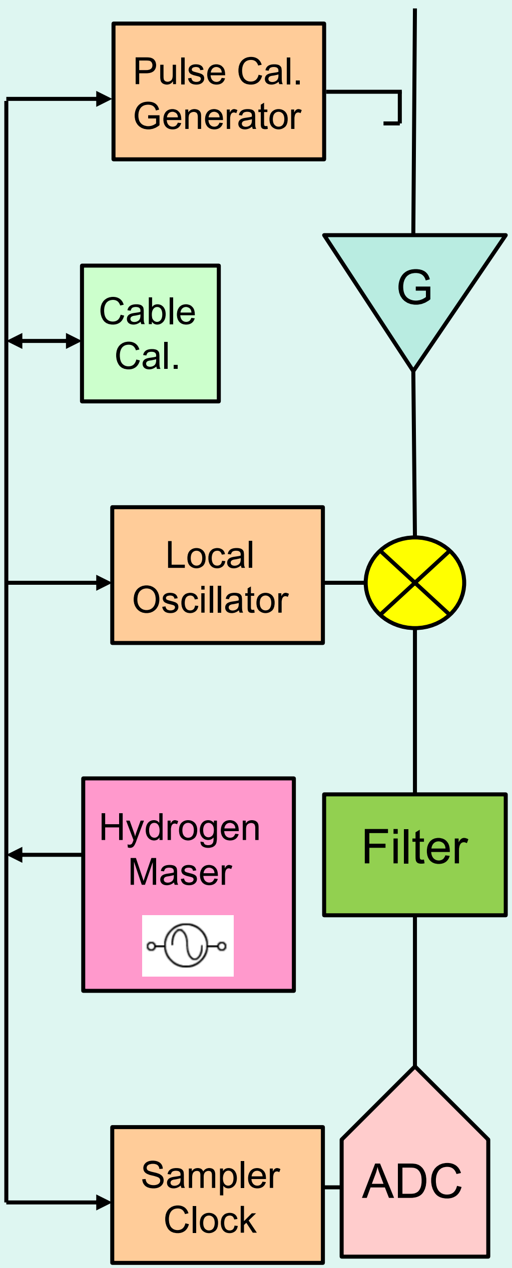


τ_i^{ant} - delay difference between the 'Reference Ray' and the 'Ray to Focus'.



Reference point for the antenna, $\vec{x}_i(t)$, is the intersection of axes

Input from Feed



Signal Input from the Feed

$$X_i(t) = AST_i(t) + NOISE_i(t),$$ where

$$AST_i(t) = \int_2^{14} A_i^{ast} \cdot e^{-j\{\theta_i^{ast}(f,t)\}} \cdot N^{ast} df$$

$$A_i^{ast} = \sqrt{\frac{S_f}{2} A_i^e R}$$

$$P = \frac{V^2}{R} \quad \therefore V = \sqrt{PR}$$

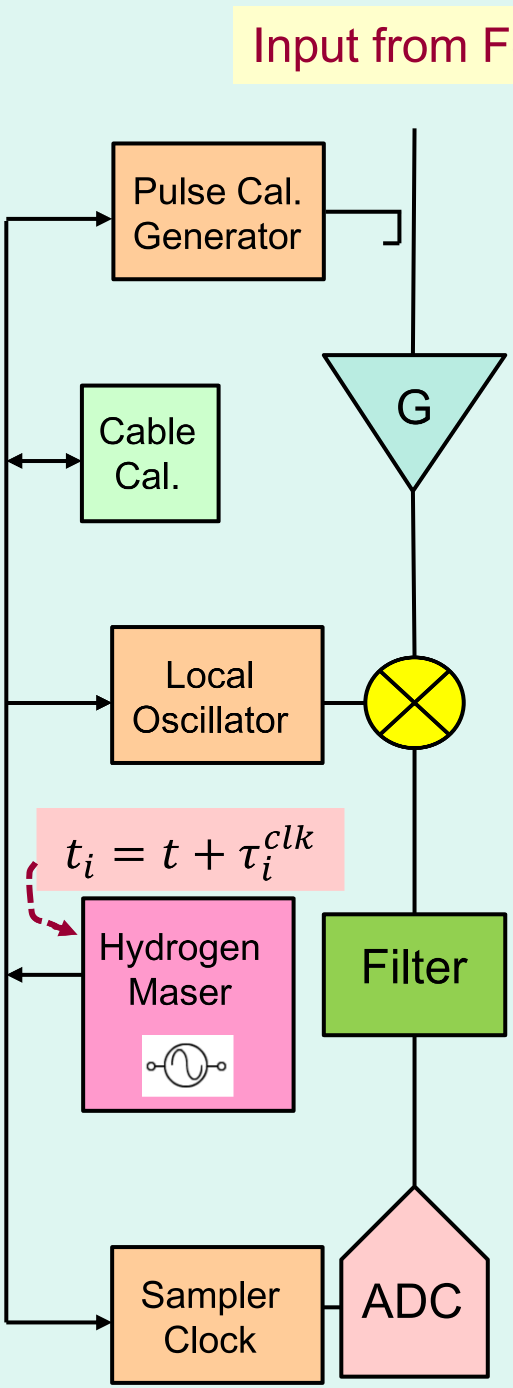
$$\theta_i^{ast}(f,t) = 2\pi f \left(t - \tau_i^{ast}(t)\right) + \frac{2\pi K_i^{ion}}{f}$$

$$\tau_i^{ast}(t) = \frac{\hat{k} \cdot \vec{x_i}}{c} + \tau_i^{atm} + \tau_i^{ant}$$

$$NOISE_i(t) = \int_2^{14} A_i^{Sys} \cdot N_i^{Sys} df$$

$$A_i^{Sys} = \sqrt{k T_i^{Ant} R}$$

Typically, $A_i^{Sys} \gg A_i^{ast}$



Signal In Terms of H-maser Time

All signals at a station are reference to the time kept by the local H-maser frequency reference, which is offset from t by τ_i^{clk}, $i.e.$

$$t^{Hm} = t + \tau_i^{clk}$$

This can be accounted for in the delay term of the astronomy signal, i.e.

$$\tau_i^{ast}(t) = \frac{\hat{k} \cdot \vec{x_i}}{c} + \tau_i^{atm} + \tau_i^{ant} + \tau_i^{clk}$$

[Note 1: $\vec{x}\left(t - \tau_i^{clk}\right)$ is affected by the clock offset since the station is moving, but this will not be shown explicitly in this derivation.]

[Note 2: The noise term is also affected by the clock offset, but this is of no consequence since the local noise is uncorrelated between stations. Hence this effect will be ignored.]

Phase Calibration

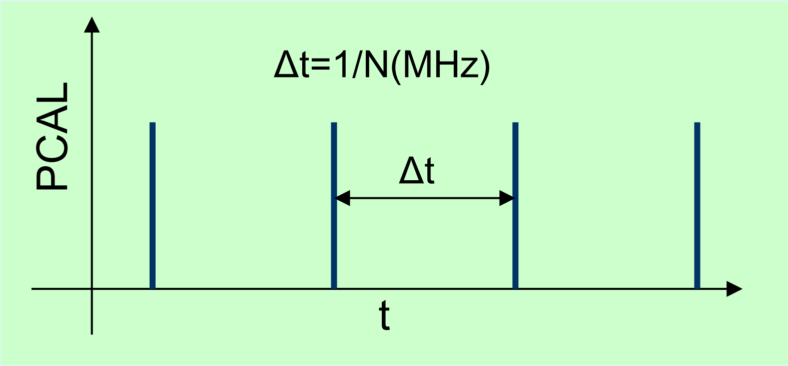
The ***phase calibration*** (PCAL) system measures changes in system phase/delay.

A train of narrow pulses is injected near the start of the signal chain.

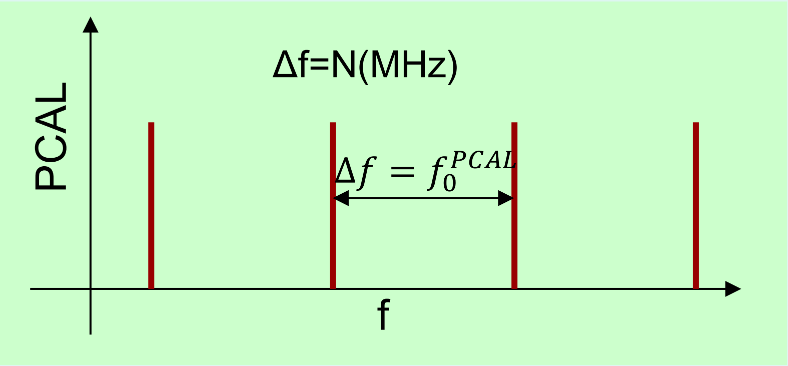
- Since the PCAL signal follows exactly the same path (from the point of injection onward) as the astronomical signal, any changes experienced by the astronomical signal are also experienced by the PCAL signal.

Pulses of width t_{pulse} with a repetition rate of N MHz correspond to a series of frequency tones spaced N MHz apart from DC up to a frequency of $\sim 1/t_{pulse}$

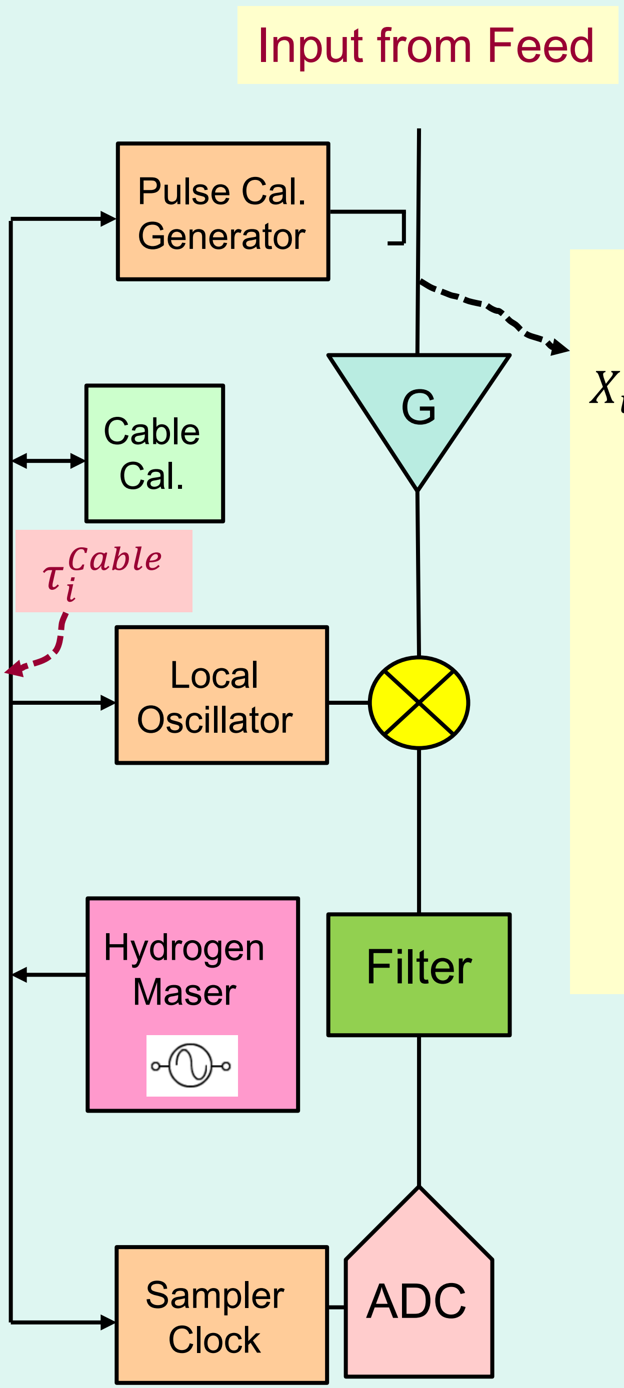
- e.g., pulses of width ~50 ps yield tones up to ~20 GHz
- Typical pulse rate will be 5 or 10 MHz for VGOS (to reduce the possibility of pulse clipping).



Time domain representation



Frequency domain representation



Signal After PCAL Injection

$$X_i(t) = AST_i(t) + NOISE_i(t) + PCAL_i(t),$$ where

$$PCAL_i(t) = \sum_{2}^{14} A_{ik}^{PCAL} \cdot e^{-i\{\theta_i^{PCAL}(k,t)\}}$$

$$\theta_i^{PCAL}(k,t) = 2\pi k f_0^{PCAL}\left(t - \tau_i^{cable}\right)$$

Amplification - Low Noise Design

The signal received from a radio source is very weak, e.g. using

$$P_f = A_e S_f = 2kT_a$$

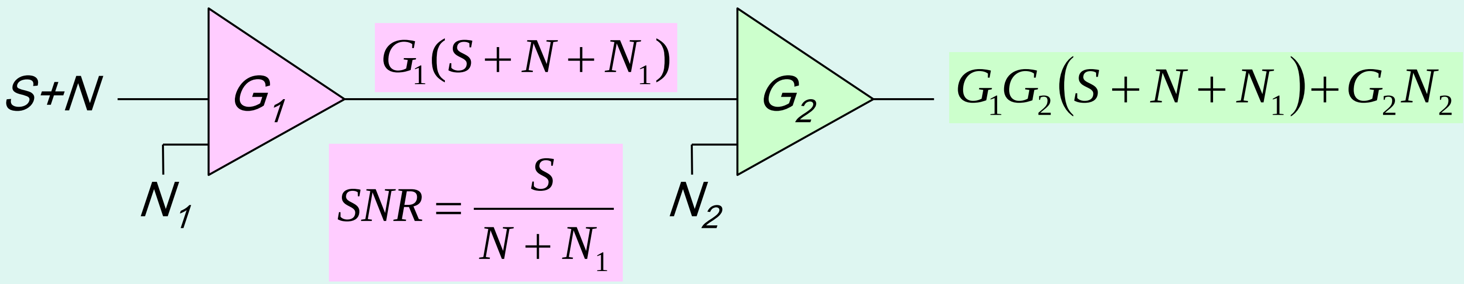
a 1 Jy source observed by a 12 m antenna with 50% efficiency will produce an antenna temperature, T_A=0.02° K about 1000 times smaller than typcial system noise. [See the table below for a breakdown of system noise components.]

Source of noise	*Typical antenna temperature (°K)*	*Major dependencies*
Cosmic microwave background	3	
Milky Way Galaxy	0-1	frequency, direction
Ionosphere	0-1	time, frequency, elevation
Troposphere	3-30	elevation, weather
Antenna radome	0-10	
Antenna	0-5	
Ground spillover	0-30	elevation
Feed	5-30	
Cryogenic LNA	5-20	
Total	16-130	

Amplification - Low Noise Design

It is important that good low noise design strategies be used, i.e. that the first amplifier in the signal chain (the one immediately after the feed) has:

- very low input noise, i.e. that it is a cryogenically cooled Low Noise Amplifier (LNA).
- high gain to dilute the noise contribution of later stages.



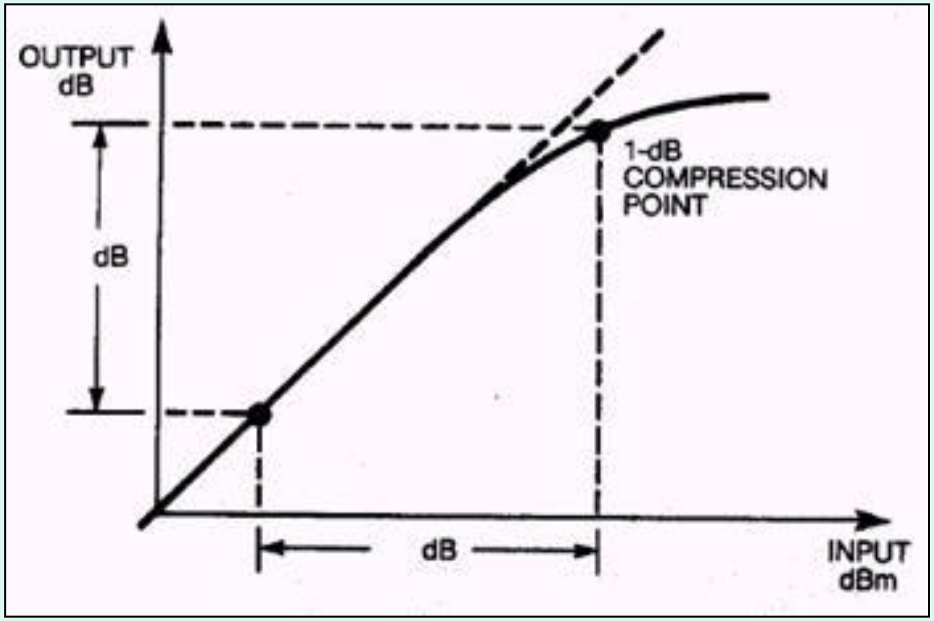
Hence, $$SNR = \frac{G_1G_2S}{G_1G_2(N+N_1)+G_2N_2} = \frac{S}{N+N_1+\frac{N_2}{G_1}}$$

The second noise contribution has been reduced by the first gain.

For example, if G_1=3000 (35-dB) and N_2=200°K, N_2/G_1=0.07°K.

Amplification - Gain Compression

It is important that amplifiers operate in the linear range, i.e. output is simply a multiple of the input (e.g. if input doubles the output must also double).

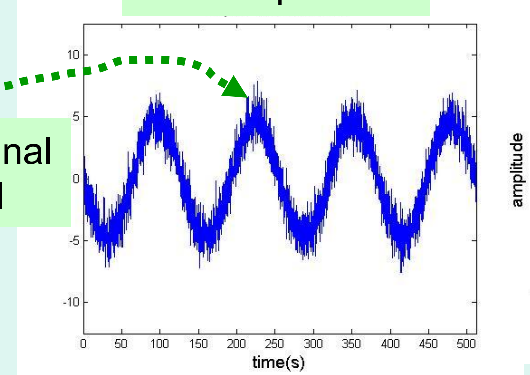


The *1 dB compression point* is an important amplifier specification. It is a measure of how large a signal can be input to an amplifier before significant non-linear behaviour begins. It occurs at the input signal level where output increases 1 dB less than the input. To guarantee linear operation, systems are usually designed to operate at least 10 dB below the 1 dB compression point.

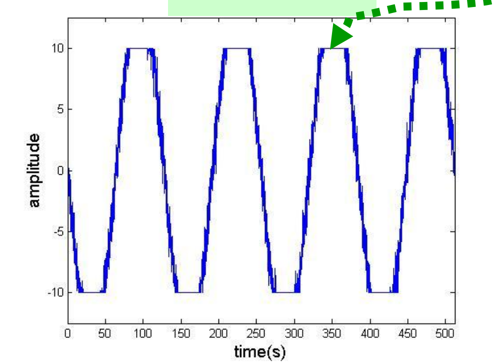
Consequenses of non-linear behaviour

VLBI noise signal plus cw RFI

Linear operation



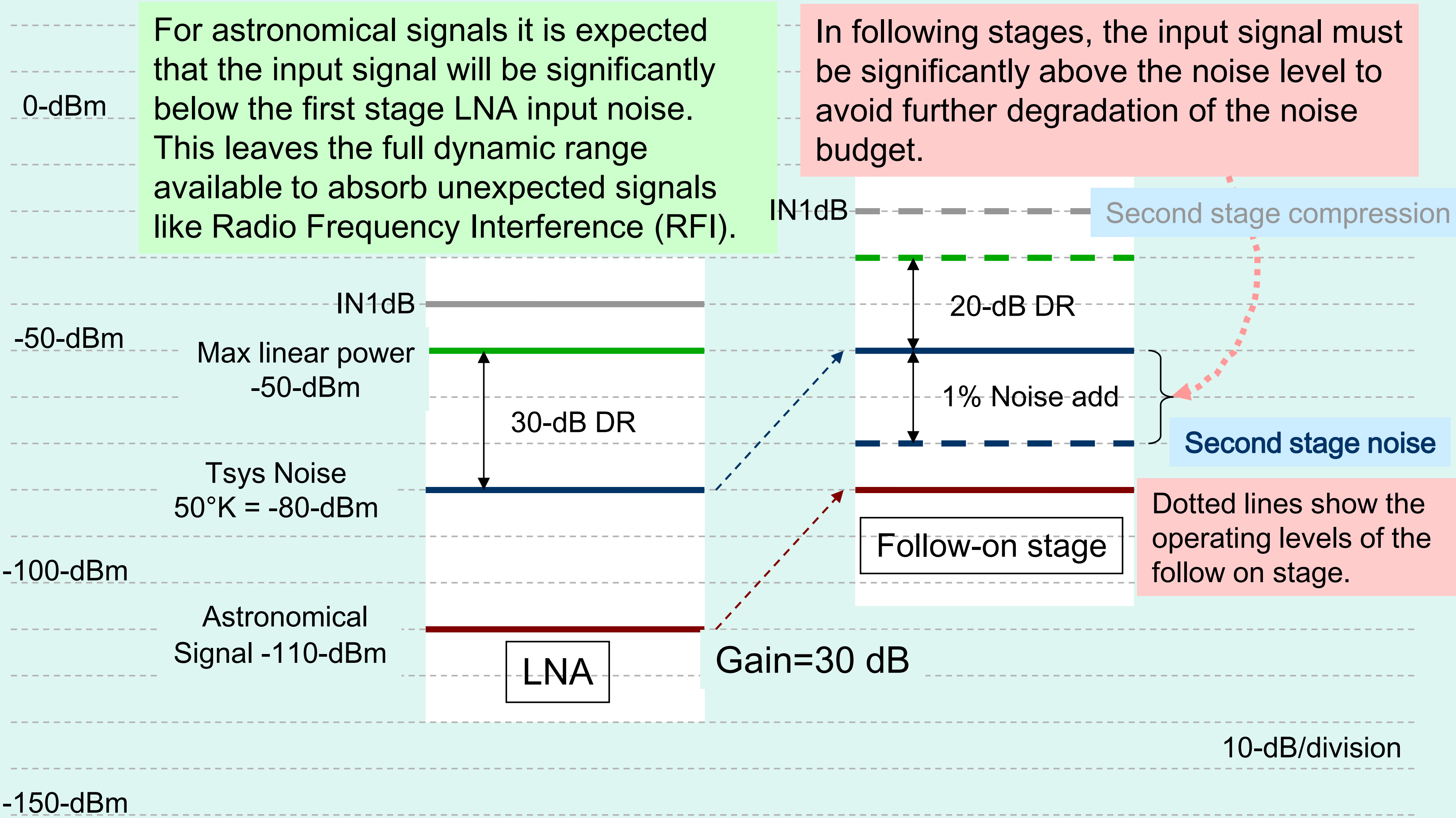
Saturation

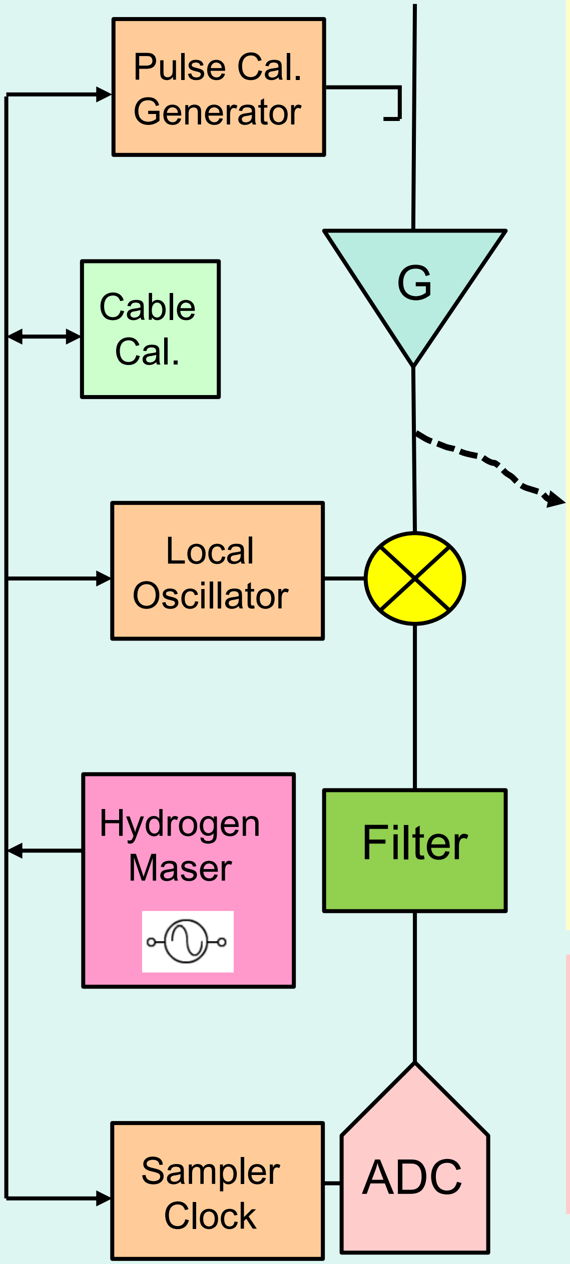


- VLBI signal disappears at saturation.
- Amplitude modulation shifts frequencies

Dynamic Range Example

Dynamics Range is the range of amplitudes in which an amplifier can operate. The lower end is limited by noise performance of the amplifier and the upper end is limited by gain compression.





Signal After Amplifier

The complex gain of the system affects the following elements of the signal equation

$$A_i^{ast} = G\sqrt{\frac{S_f}{2} A_i^e R}$$

$$\tau_i^{ast}(t) = \frac{\hat{k} \cdot \vec{x_i}}{c} + \tau_i^{atm} + \tau_i^{ant} + \tau_i^{clk} + \tau_i^{Inst}$$

$$A_i^{Sys} = G\sqrt{k\left(T_i^{Ant} + T_i^{Rec}\right)R}$$

$$A_{ik}^{PCAL} = G \cdot A_i^{PCAL}$$

$$\theta_i^{PCAL}(k,t) = 2\pi k f_0^{PCAL}\left(t - \tau_i^{cable} - \tau_i^{Inst}\right)$$

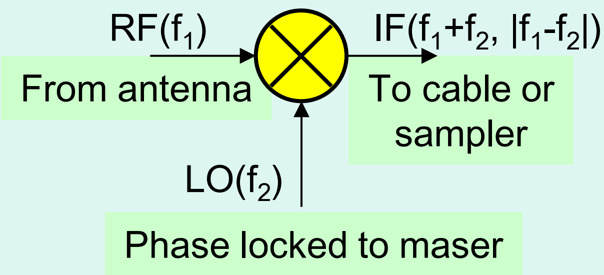
- τ_i^{Inst} ~ is the cumulative delay of all instrumentation in the signal path including cables, components, etc.
- G ~ is the cumulative gain of the signal path
- T_i^{Rec} ~ is the noise temperature of the receiver.

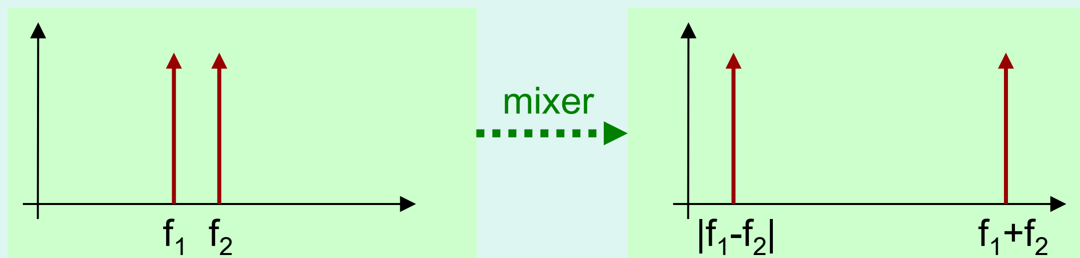
Down converter: mixer operation

A down converter translates a signal downward in frequency.

An important element of a down converter is a mixer. Conceptually, a mixer can be considered a multiplier producing outputs at the sum and difference frequencies of the two inputs:

$$\cos(f_1 t)\times\cos(f_2 t)=\frac{\cos(f_1+f_2)t+\cos(f_1-f_2)t}{2}$$

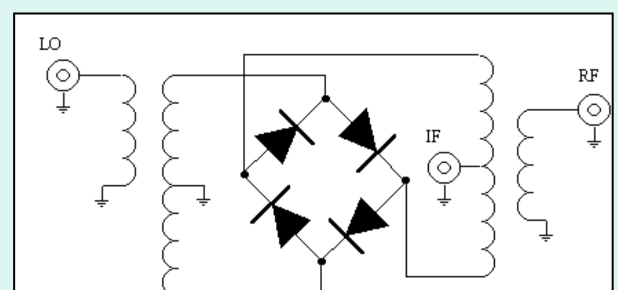




If the frequency ***sum*** is isolated using a filter this is referred to as an ***up converter.***

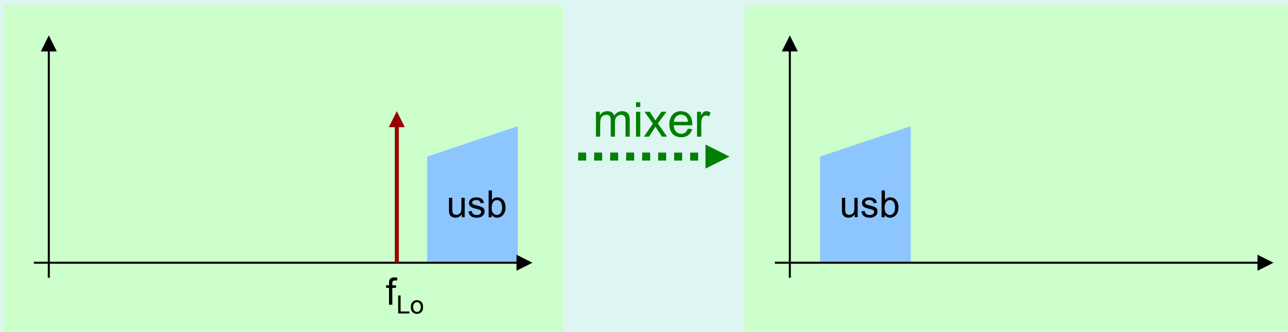
If the ***difference*** is isolated using a filter this is referred to as a ***down converter.***

Mixers used at RF frequencies are typically double or triple balanced ring diodes and not pure multipliers. As a result, other (usually unwanted) mixer products can be found in the output, e.g. at frequencies f_1, $2f_1$, $3f_1$, f_2, $2f_2$, $3f_2$, $2f_1-f_2$, $2f_1+f_2$, $(nf_1 \pm mf_2)$, …..

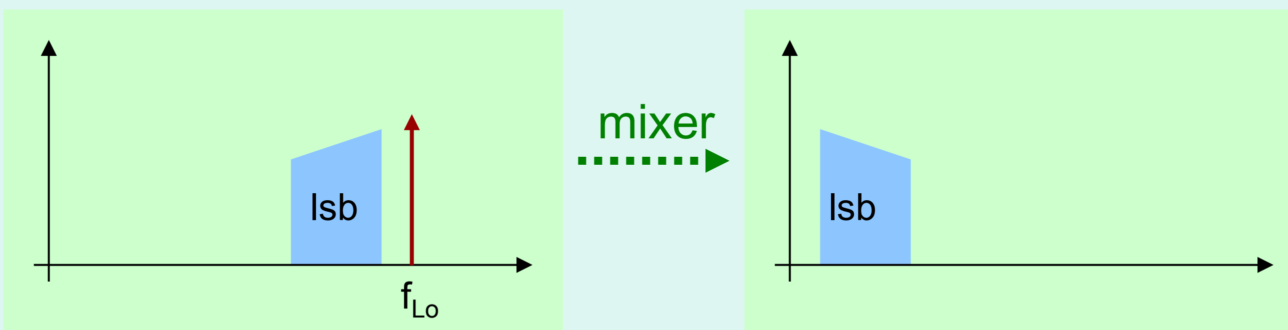


Down converter: sidebands

Signals, at the input to a down converter, with frequencies higher than the Local Oscillator (LO) frequency are referred to as *upper sideband (usb)* signals.

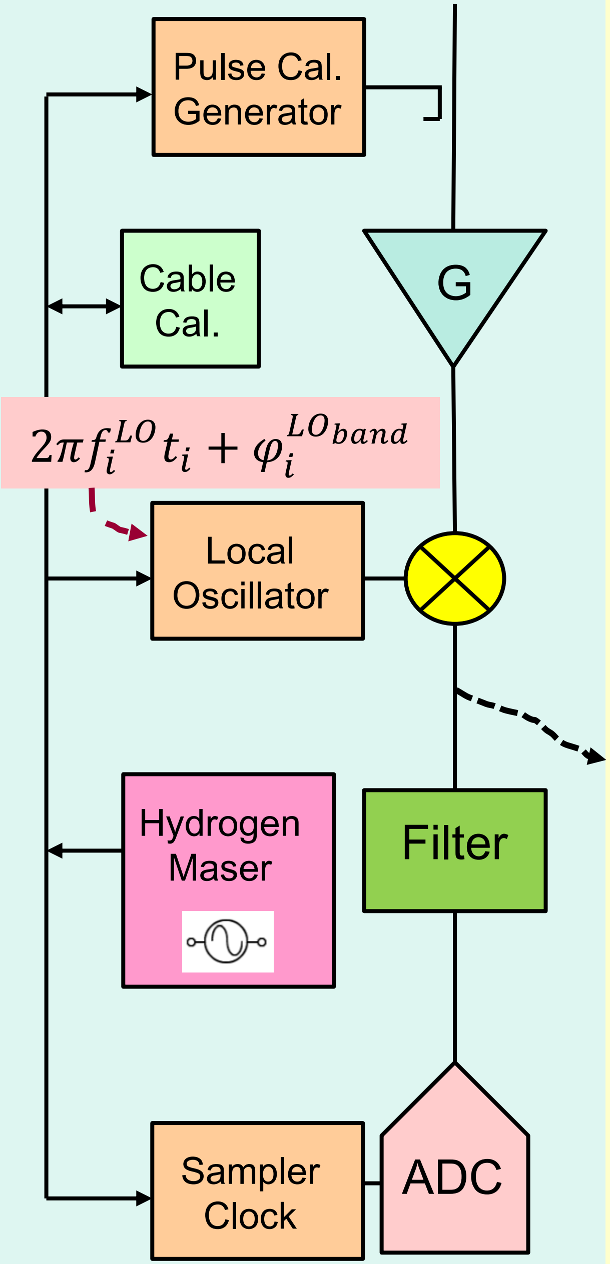


Signals with frequencies lower than the Local Oscillator (LO) frequency are referred to as *lower sideband (lsb)* signals.



Note that in the lower sideband (lsb) output, the ordering of the frequencies is reversed.

Signal After Down Converter Mixer



The LO mix affects the phase of the $AST(t)$ and $PCAL(t)$ signals, i.e.

$$\theta_i^{ast}(f,t) = 2\pi f\left(t - \tau_i^{ast}(t)\right) + \frac{2\pi K_i^{ion}}{f}$$

$$-\left(2\pi f_i^{LO_{band}} t + \varphi_i^{LO_{band}}\right)$$

$$\theta_i^{PCAL}(k,t) = 2\pi k f_0^{PCAL}\left(t - \tau_i^{cable} - \tau_i^{Inst}\right)$$

$$-\left(2\pi f_i^{LO_{band}} t + \varphi_i^{LO_{band}}\right)$$

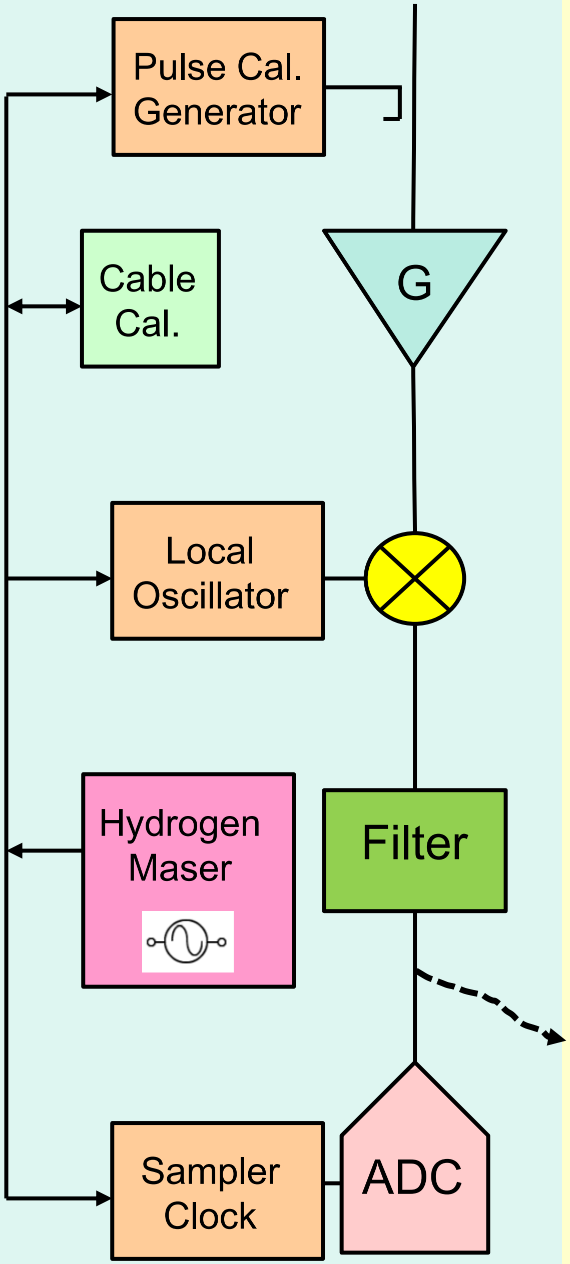
To better reflect the frequency shift caused by the LO mix, these can be re-expressed as

$$\theta_i^{ast}(f,t) = 2\pi\left(f - f_i^{LO_{band}}\right)\left(t - \tau_i^{ast}(t) - \frac{K_i^{ion}}{\left(f_i^{LO}\right)^2}\right)$$

$$-2\pi f_i^{LO_{band}} \tau_i^{ast}(t) + \frac{K}{f_i^{LO}} - \varphi_i^{LO_{band}}$$

$$\theta_i^{PCAL}(k,t) = 2\pi\left(k f_0^{PCAL} - f_i^{LO_{band}}\right)t$$

$$-2\pi k f_0^{PCAL}\left(\tau_i^{cable} + \tau_i^{Inst}\right) - \varphi_i^{LO_{band}}$$



Signal After Down Converter Filter

For the astronomical signal it is possible to remove frequency independent terms from the integral and integrate over the shifted frequency, $\acute{f} = f - f_i^{LO_{band}}$ to get

$$AST_i(t) = A_i^{ast} \cdot e^{-j\varphi_i^{ast}(t)} \cdot \int_0^{BW} e^{-j\theta_i^{ast}(t)} N_i^{ast} df$$

where

$$\varphi_i^{ast}(t) = -2\pi f_i^{LO_{band}} \cdot \tau_i^{ast}(t) + \frac{K}{f_i^{LO}} - \varphi_i^{LO_{band}}$$

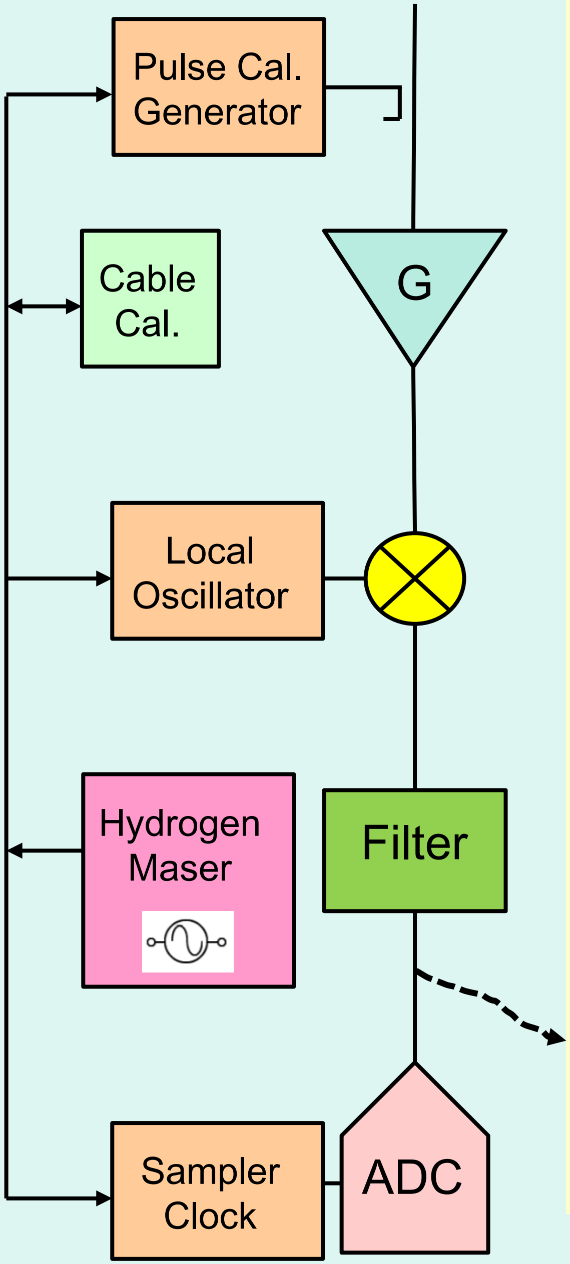
$$\theta_i^{ast}(t) = 2\pi\left(f - f_i^{LO_{band}}\right)\left(t - \tau_i^{ast}(t) - \frac{K_i^{ion}}{\left(f_i^{LO}\right)^2}\right)$$

$$\tau_i^{ast}(t) = \frac{\hat{k} \cdot \overrightarrow{x_i}}{c} + \tau_i^{atm} + \tau_i^{ant} + \tau_i^{clk} + \tau_i^{Inst}$$

$AST_i(t)$ can be rewritten

$$AST_i(t) = A_i^{ast} \cdot e^{-j\varphi_i^{ast}(t)} \cdot s\left(t - \tau_i^{ast}(t) - \frac{K_i^{ion}}{\left(f_i^{LO}\right)^2}\right)$$

where $s(t)$ is a unity amplitude band limited (BW) baseband noise signal.



Signal After Down Converter Filter (cont'd)

The local noise signal, after down conversion and filtering, can be written

$$NOISE_i(t) = \int_0^{BW} A_i^{Sys} \cdot N_i^{Sys} df = A_i^{Sys} \cdot n_i(t)$$

where $n_i(t)$ is a unity amplitude band limited (BW) baseband noise signal.

Finally, the PCAL signal can be rewritten,

$$PCAL_i(t) = \sum_0^{BW} A_{ik}^{PCAL} \cdot e^{-i\left\{2\pi\left(kf_0^{PCAL} - f_i^{LO_{band}}\right)t + \varphi_i^{PCAL}(k)\right\}}$$

where

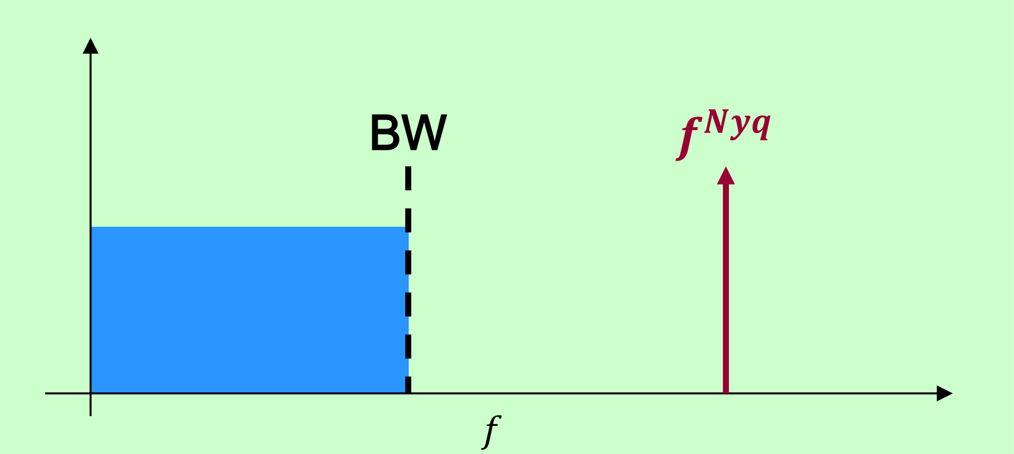
$$\varphi_i^{PCAL}(k) = 2\pi k f_0^{PCAL}\left(\tau_i^{cable} + \tau_i^{Inst}\right) - \varphi_i^{LO_{band}}$$

and $kf_0^{PCAL} - f_i^{LO_{band}}$ is the PCAL tone detection frequency.0

Sampling

- ***Sampling*** is the process of freezing and extracting signal values at specified times, often regular intervals defined by a sampling clock, e.g. $X_m = X(m\Delta t_s)$, where $\Delta t_s = \frac{1}{f_s}$ is the sampling interval and f_s is the sampling frequency.

- The ***Nyquist Frequency***, f^{Nyq}, is the minimum sampling frequency that extracts all information from a signal: $\boldsymbol{f^{Nyq} = 2BW}$

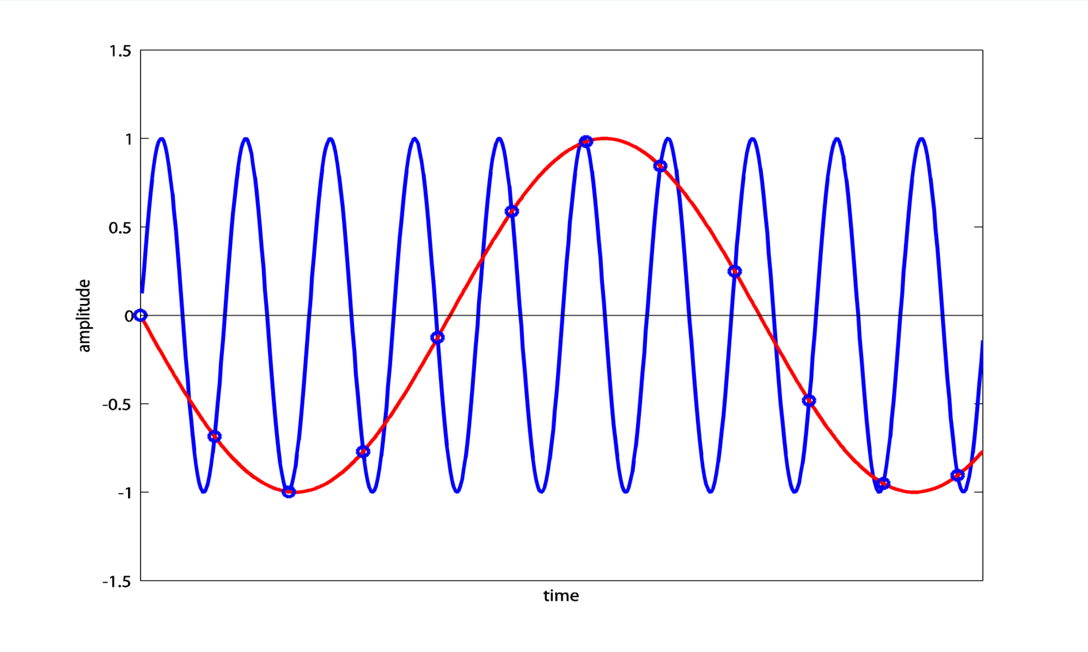


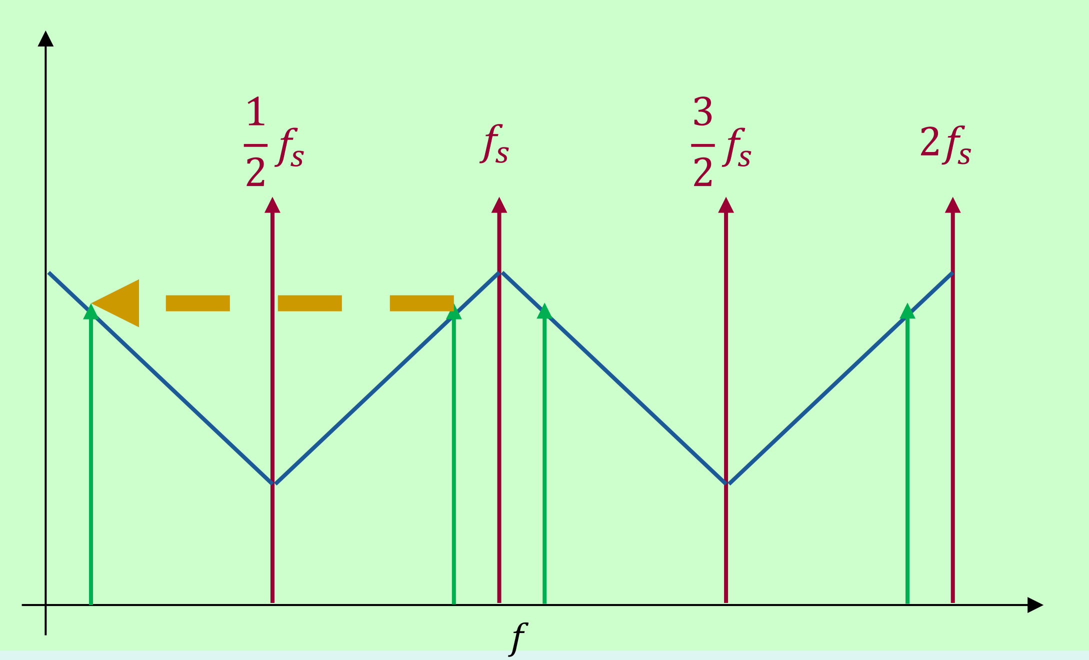
Aliasing

It is impossible to distinguish sampled signals that have characteristic frequency relationships, e.g. a sinusoid with frequency f is impossible to distinguish from sinusoids with frequencies, $f_s \pm f$, $2f_s \pm f$, $3f_s \pm f$, ... where f_s is the sampling frequency. This overlaying of signals is referred to as ***aliasing***.

This is generally avoided through the use of ***anti-alias filters.***

The apparent downward translation of the frequency of a signal through aliasing is equivalent to a down conversion.



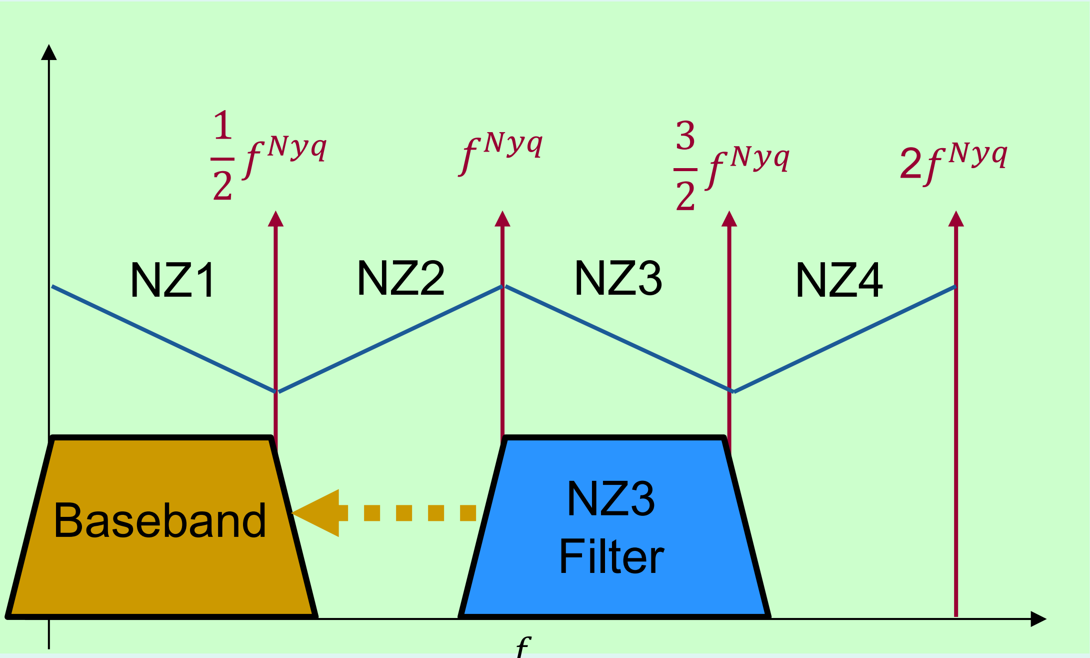


Nyquist zones

Frequencies of a sampled signal can be separated into Nyquist Zones, e.g. $NZ1 = 0 \rightarrow \frac{1}{2} f^{Nyq}$; $NZ2 = \frac{1}{2} f^{Nyq} \rightarrow f^{Nyq}$; $NZ3 = f^{Nyq} \rightarrow \frac{3}{2} f^{Nyq}$; $etc.$

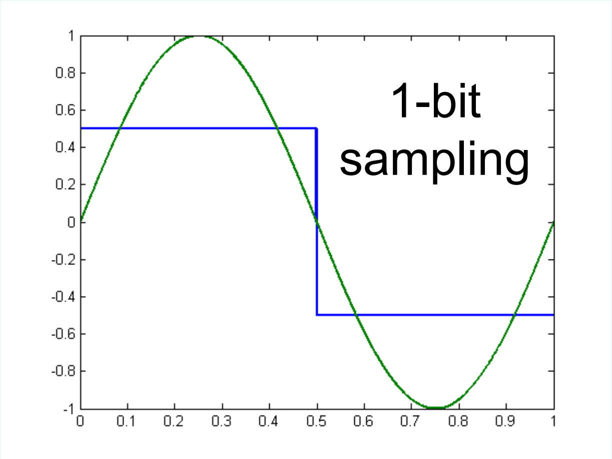
If a filter is placed over a full Nyquist Zone, all the info in that zone is captured and nothing is aliased into it. The zone is effectively translated to baseband.

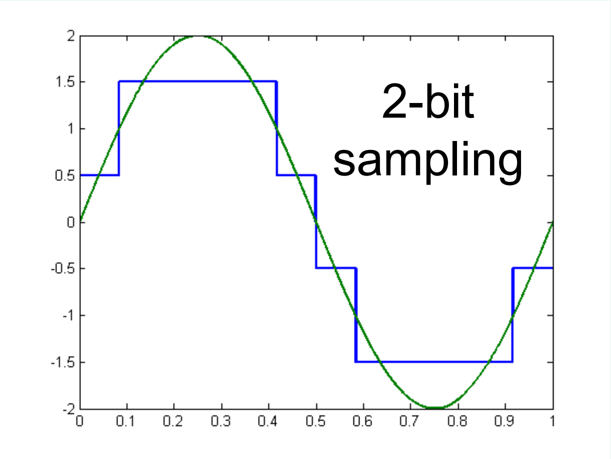
For odd zones there is no frequency inversion so these appear as upper sideband (USB) down conversions; for even zones the frequencies are reversed so these appear as lower sideband (LSB) down conversions.

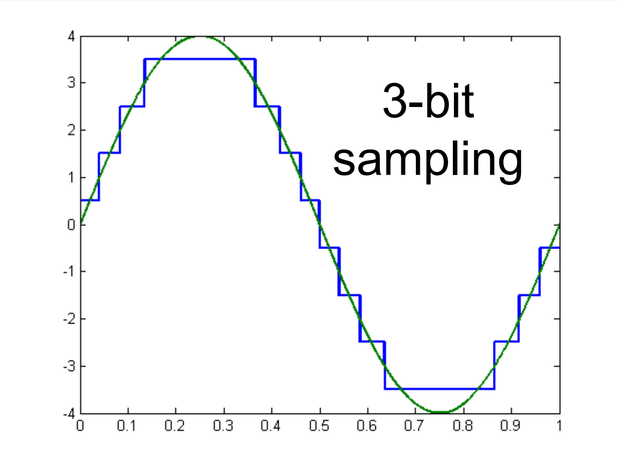


Digitization

- Digitization is the conversion of an analog voltage into a number.
- It is usually done at the same time as sampling.
- The greater the number of bits used, the closer the digital representation is to the analog voltage.
- In modern VLBI acquisition systems, digitization is done comparatively early in the system:
  - This allows many of the analog functions (gain balancing, down conversion, filtering, channelization, etc) to be done digitally, which is now more efficient and at the same time more stable and accountable.
  - To ensure that losses and artifacts (in the digital processing) are minimized (and that dynamic range is maximized) sampling is done using at least 8-bits.





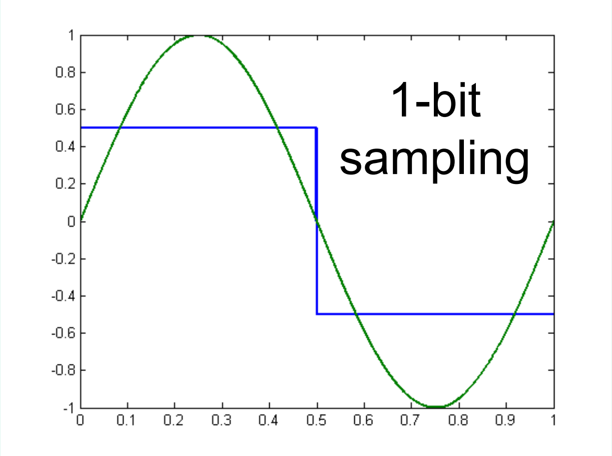


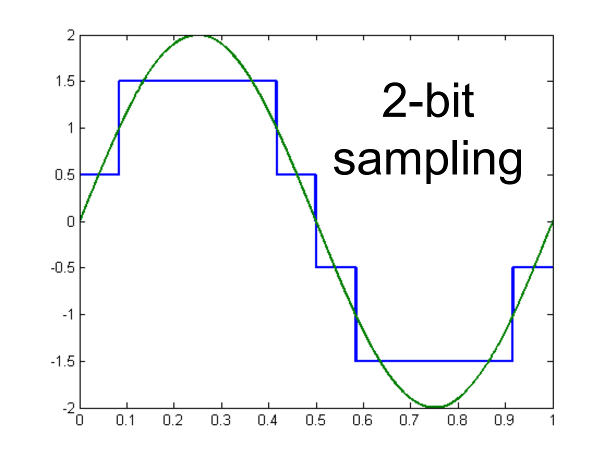
Requantization

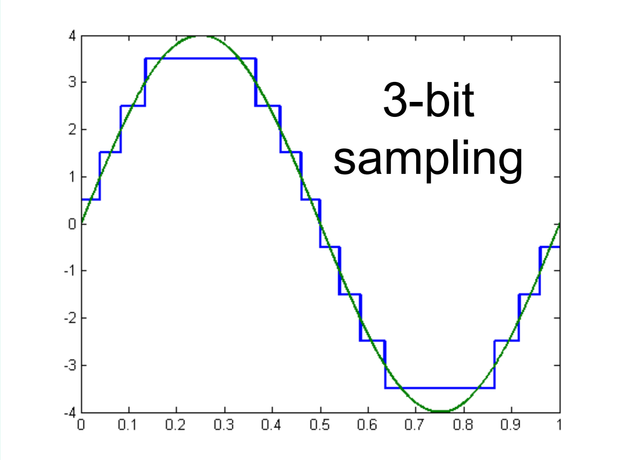
- Just before transmission, the data is requantized to maximize data transmission efficiency, i.e. maximum SNR per bit transmitted.
- The options are to either increase the number of bits per sample or to increase the sample rate since $\boldsymbol{Bit\ rate = Sample\ rate * bits\ per\ sample}$.

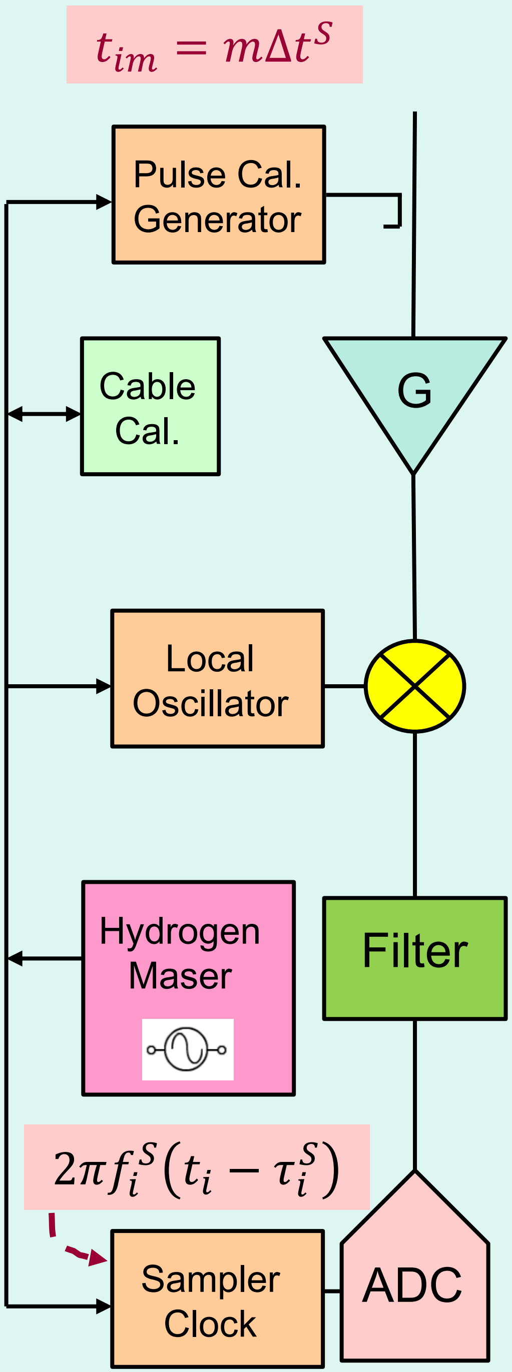
# of bits	η – bit rate	η – bits per sample
1	64%	64%
2	90%	88%
3	110%	94%
4	128%	97%

In geodetic VLBI, the most commonly used number of bits per sample is 2









Sampled Output Signal

$$X_{im} = AST_{im} + NOISE_{im} + PCAL_{im}, \quad \text{where}$$

$$AST_{im} = G\sqrt{\frac{S_f}{2}A_i^e \cdot BW \cdot R} \cdot e^{-j\varphi_{im}^{ast}} \cdot s\left(m\Delta t^s - \tau_{im}^{ast} - \frac{K_i^{ion}}{\left(f_i^{LO}\right)^2}\right)$$

$$\varphi_{im}^{ast} = -2\pi f_i^{LO_{band}} \cdot \tau_{im}^{ast} + \frac{K}{f_i^{LO}} - \varphi_i^{LO_{band}}$$

$$\tau_{im}^{ast} = \frac{\hat{k} \cdot \vec{x_i}(m\Delta t^s)}{c} + \tau_i^{atm} + \tau_i^{ant} + \tau_i^{clk} + \tau_i^{Inst} + \tau_i^S$$

$$NOISE_{im} = G\sqrt{k\left(T_i^{Ant} + T_i^{Rec}\right) \cdot BW \cdot R \cdot n_i(m\Delta t^s)}$$

$$PCAL_{im} = \sum_{0}^{BW} A_{ik}^{PCAL} \cdot e^{-i\left\{2\pi\left(k f_0^{PCAL} - f_i^{LO_{band}}\right)m\Delta t^s + \varphi_i^{PCAL}(k)\right\}}$$

$$\varphi_i^{PCAL}(k) = 2\pi k f_0^{PCAL}\left(\tau_i^{cable} + \tau_i^{Inst} + \tau_i^{s}\right) - \varphi_i^{LO_{band}}$$

System Equivalent Flux Density (SEFD)

SEFD is an excellent measure of the sensitivity of the system. It is defined as the input flux density (S_f) that produces a power from the antenna ($P_{Ant} = \frac{S_f}{2} A_e$) that equals the power of the system noise ($P_{Sys} = kT_{Sys}$), so $S_f = SEFD$ when $P_{Ant} = P_{Sys}$, i.e.

$$\frac{SEFD}{2} A_e = kT_{Sys} \quad \text{and} \quad SEFD = \frac{2kT_{Sys}}{A_e}$$

Finally, expanding A_e gives, $$SEFD = \frac{8kT_{Sys}}{\eta_A \pi D^2}$$

Note: SEFD decrease as sensitivity increases.

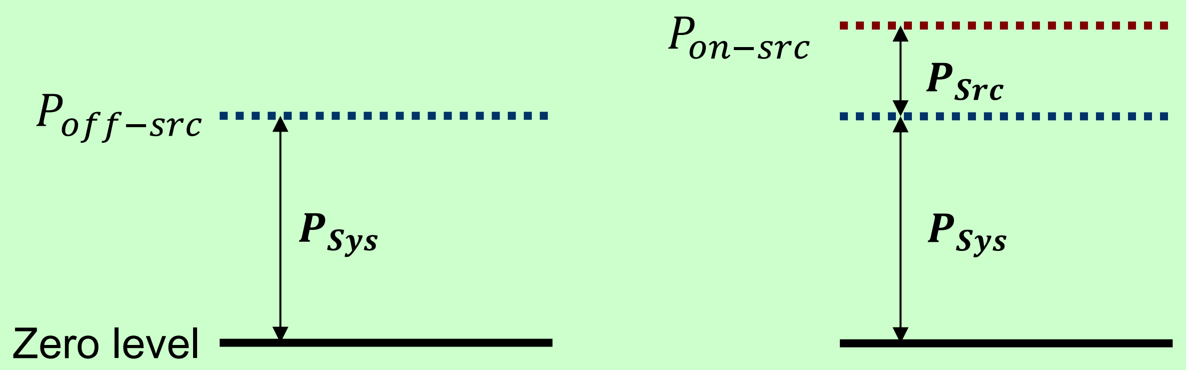
SEFD is very useful in VLBI as a measure of system sensitivity and for predicting the correlated amplitude and SNR, i.e.

$$Amp = \frac{\eta_c S_f}{\sqrt{SEFD_1 \times SEFD_2}} \qquad SNR = Amp\sqrt{2 \times BW \times T}$$

where η_c is the correlator digital processing efficiency and *2xBWxT* is the number of independent samples. *Amp* is typically ~10^{-4} so *2xBWxT* must be very large to get a good SNR. [Note: The SEFD spec for VLBI2010 is 2500.]

Measurement of SEFD

Operationally, SEFD is determined by measuring the power on and off a source with calibrated flux density.



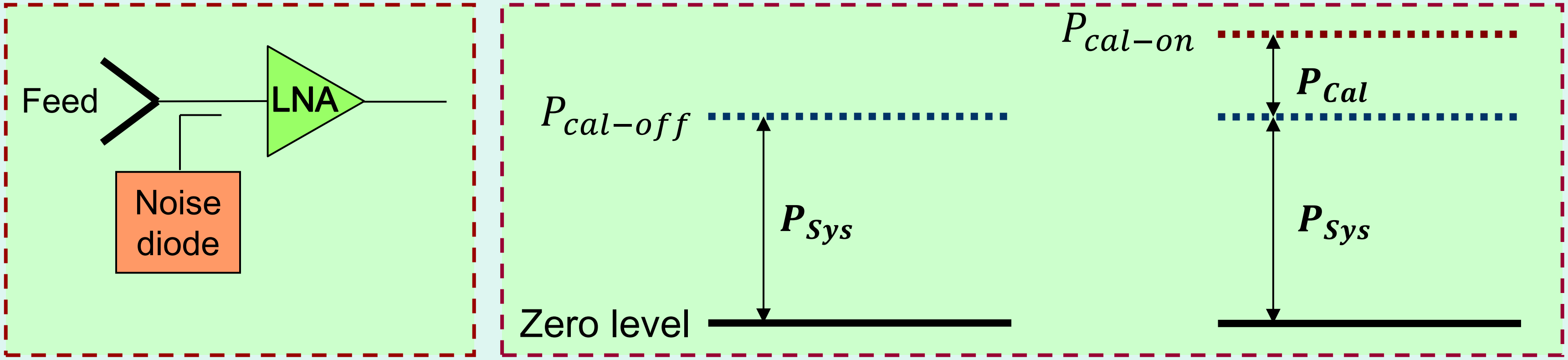
Using P_{on-src}, $P_{off-src}$, and the flux, S_f, of the calibration source, SEFD can be determined according to:

$$SEFD = \frac{S_f}{\left(\frac{P_{on-src}}{P_{off-src}} - 1\right)}$$

Note: The units of P_{on-src} and $P_{off-src}$ are irrelevant (provided they are both the same) since it is only their ratio that is used in the equation.

Noise Calibration - T_{Sys}

- Noise calibration measures T_{sys}, the system temperature.
- A signal of known strength is injected ahead of, in, or just after the feed, and the fractional change in system power is measured.

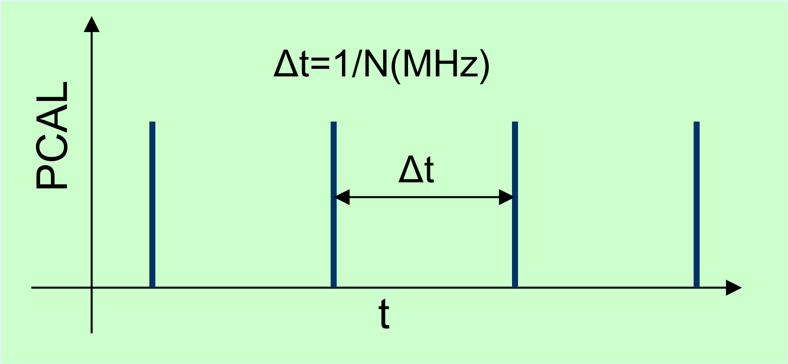


- The system temperature is then calculated from the known cal diode signal strength as

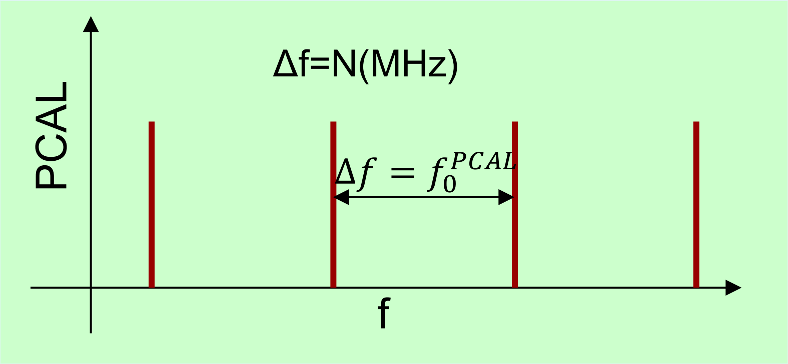
$$T_{Sys} = \frac{T_{cal}}{\left(\frac{P_{calon}}{P_{caloff}} - 1\right)}$$

- If T_{cal} is small (≤5% of T_{Sys}), continuous measurements can be made by firing the cal signal periodically (VLBA uses an 80 Hz rep rate) and synchronously detecting the level changes in the backend.

Phase Calibration (PCAL) Detection



Time domain representation



Frequency domain representation

The PCAL signal is detected in the digital output of the receiver (usually at the correlator where it is used). The detection can be either in the time domain or the frequency domain:

- In the frequency domain, a quadrature function at the frequency of the tone stops the tone so that it can be accumulated thus implementing the ***tone extractor***. In baseband, the frequency of the kth tone is $2\pi\left(kf_0^{PCAL} - f_i^{LO_{band}}\right)$
- In the time domain, averaging of the repetitive pulse periods implements the ***pulse extractor*** with an FFT transforming the result to the frequency domain.

The phase extracted from the kth tone can be written

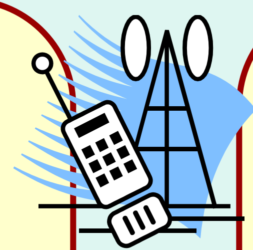
$$2\pi k f_0^{PCAL}\left(\tau_i^{cable} + \tau_i^{Inst} + \tau_i^{S}\right) + \varphi_i^{LO_{band}}$$

RFI - Sources (2-14 GHz)

Entire frequency range is already fully allocated by international agreement

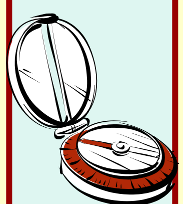
Sources internal to VLBI and co-located space geodetic techniques (e.g. SLR, DORIS, GNSS)

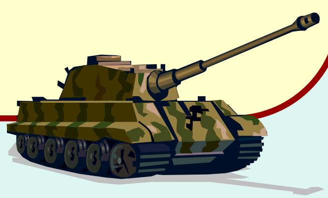
- Local oscillators, clocks, PCAL pulses, circuits
- DORIS beacon at ~2 GHz
- SLR aircraft avoidance radar at ~9.4 GHz

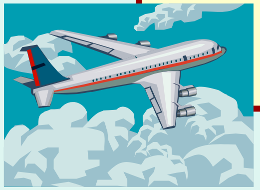


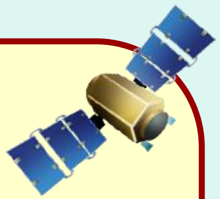
Terrestrial Sources

- General communications, fixed and mobile – land, sea, air
- Personal communications cell phones, wifi
- Broadcast
- Military
- Navigation
- Weather
- Emergency







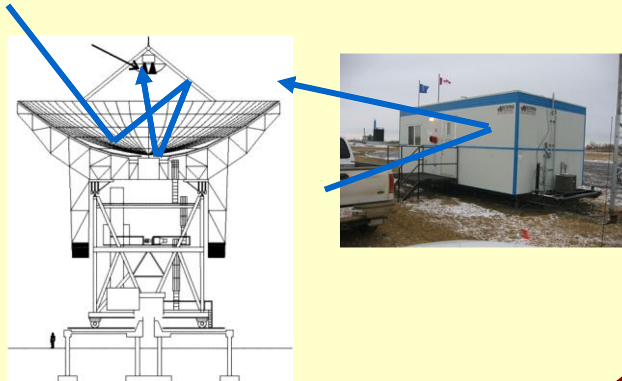


Space Sources

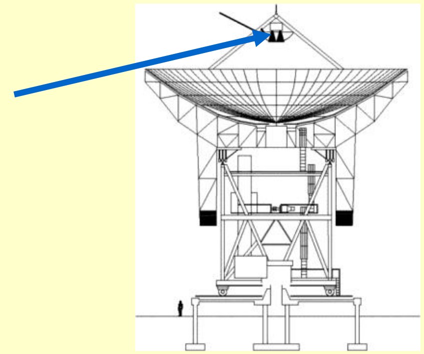
- Communications
- Broadcast (C-, Ka-band; in Clarke belt at ±8° dec)
- Military
- Exploration
- Navigation
- Weather
- Emergency

How does RFI enter the receiver chain?

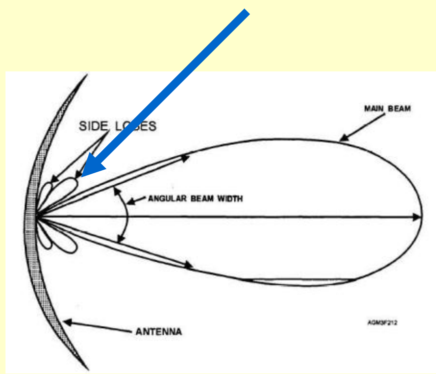
Multipath off objects and antenna structure



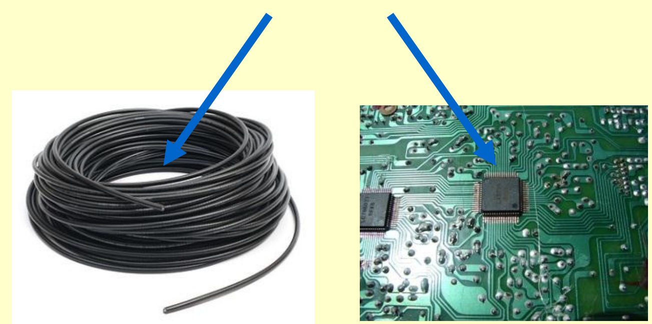
Spillover direct into the feed



Antenna sidelobes



Direct coupling into cables and circuits



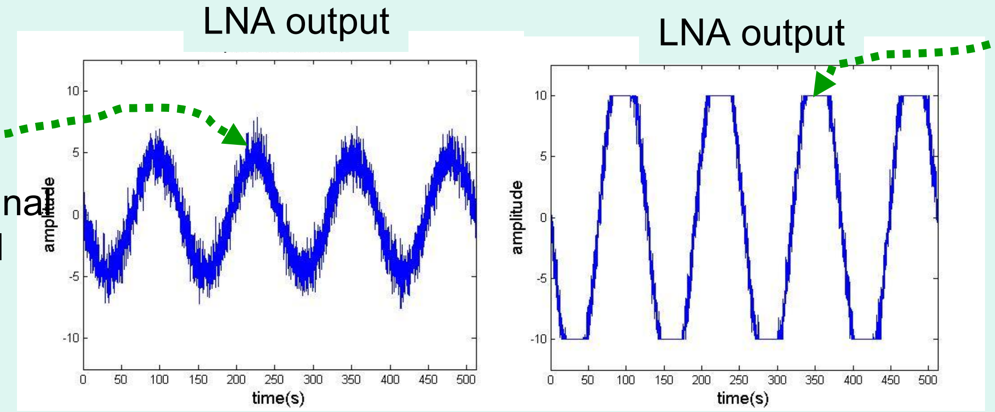
RFI - Negative Impacts

Increasing severity

Small RFI appears as added noise
- Reduces performance of the system
- Only impacts frequencies where RFI occurs
- Undesirable but can be tolerated within limits

Larger RFI can saturate the signal chain
- Impacts entire band, not just frequencies where RFI occurs
- Must be avoided (observation is lost)

VLBI noise signal plus cw RFI

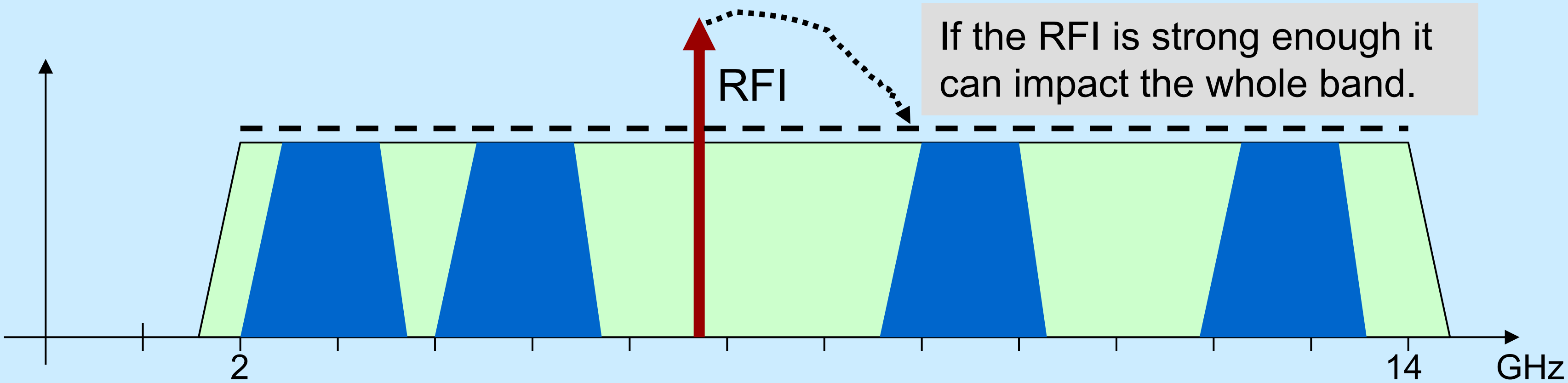


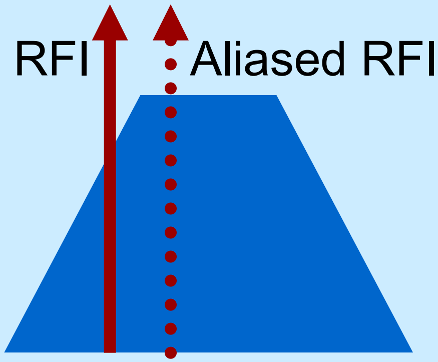
- VLBI signal disappears at saturation.
- Amp modulation shifts frequencies

Even larger RFI can damage the VLBI receiver
- Typically LNA is most vulnerable
- Must be protected against (leads to expense and down time)

Impacts of out-of-band RFI

Strong out-of-band RFI (even if in a very narrow band) that saturates the signal chain prior to the point where bands are separated will destroy the whole input range and hence destroy all bands.





If the band select filters do not cut off sharply enough, strong RFI can penetrate the wings of the filter and be aliased into the band during Nyquist sampling.

RFI Mitigation Strategies

Avoidance mask

Physical barrier as attenuator

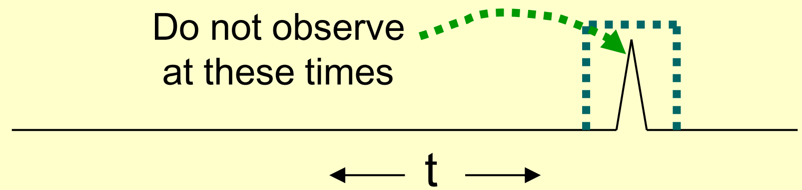
Design improvements

-Diode protection for LNA`s

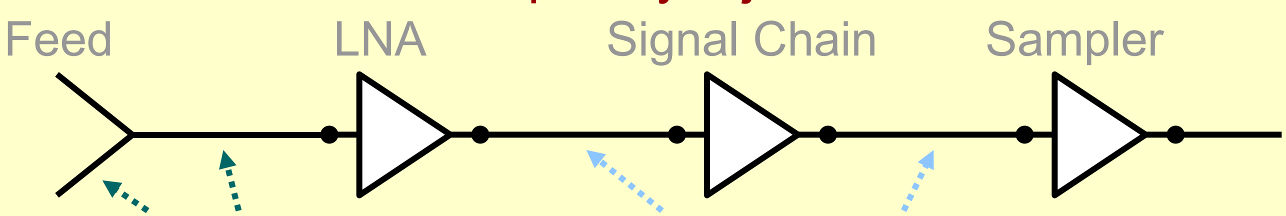
- Higher dynamic range components

- Lower antenna sidelobes

Time windowing for pulsed signals



Frequency reject filters



Frequency selective feeds or cryogenic IF filters are technically challenging and expensive

Less flexible

IF filters are technically easier and less expensive

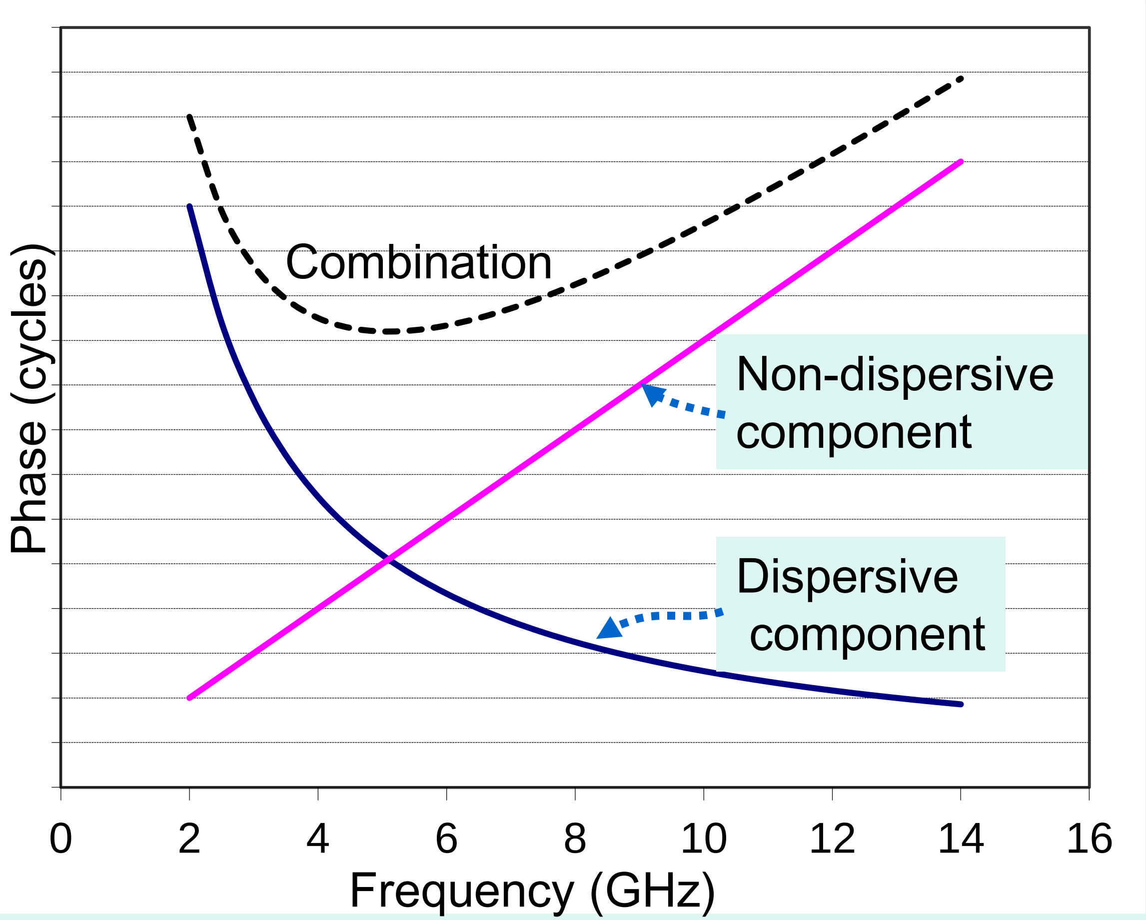
More flexible

Correlator Output

The correlator output for each channel has both a dispersive and non-dispersive element, e.g.

$$\delta\varphi^{cor} = 2\pi f^{LO_{Chan}}\left(\delta\tau^{geo} + \delta\tau^{atm} + \delta\tau^{ant} + \delta\tau^{clk}\right) + \frac{2\pi K}{f^{LO_{Chan}}}$$

Broadband Delay requires that the dispersive and non-dispersive elements be separated at the time of fringe detection



Non-dispersive delay.
Delay is independent of frequency (i.e. phase is linear wrt frequency)

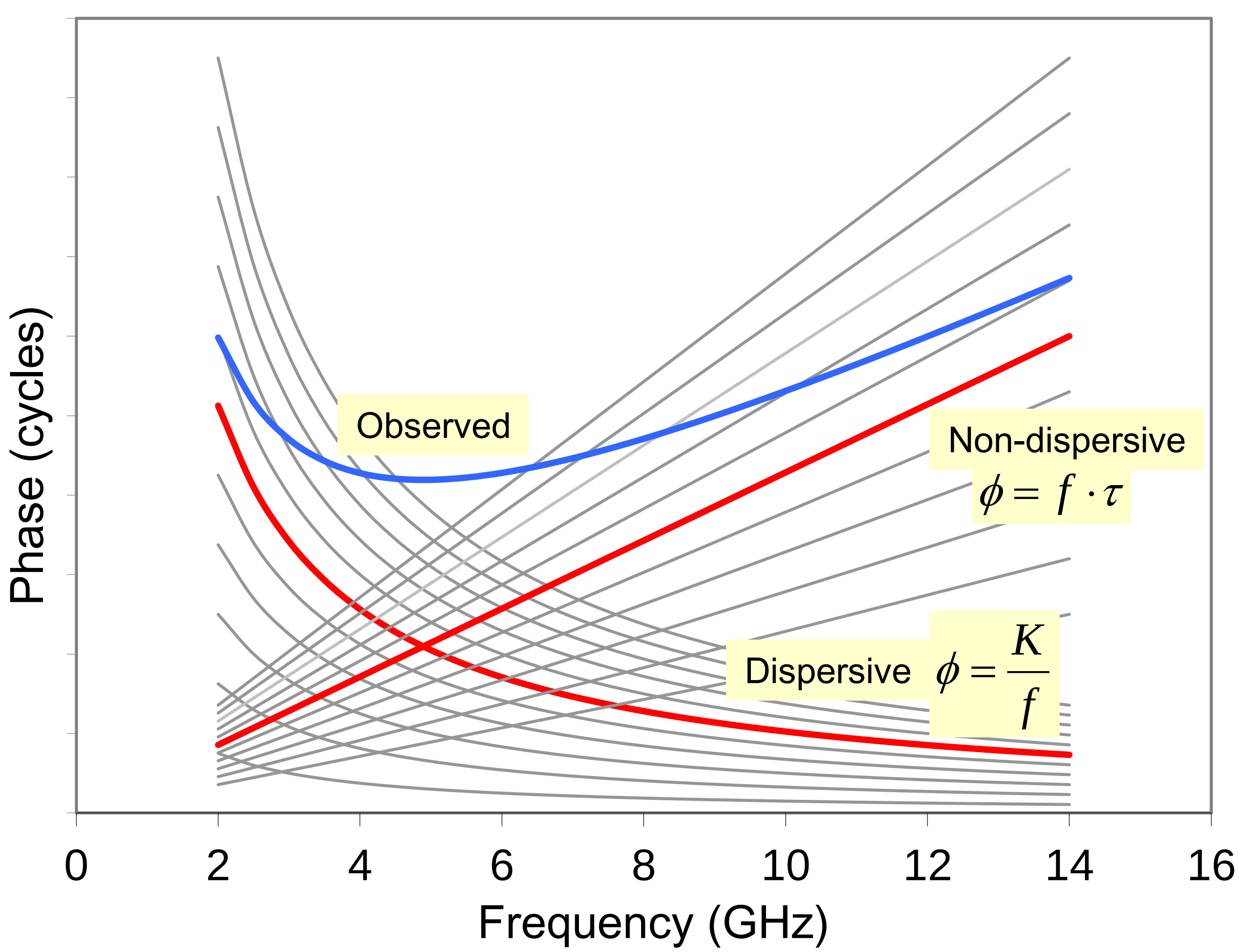
$$\phi = f \cdot \tau$$

$$\phi = f \cdot \left(\tau_g + \tau_{clk} + \tau_{atm} + \ldots\right)$$

Dispersive delay.
Delay varies with frequency. Variation is due to the Ionosphere.

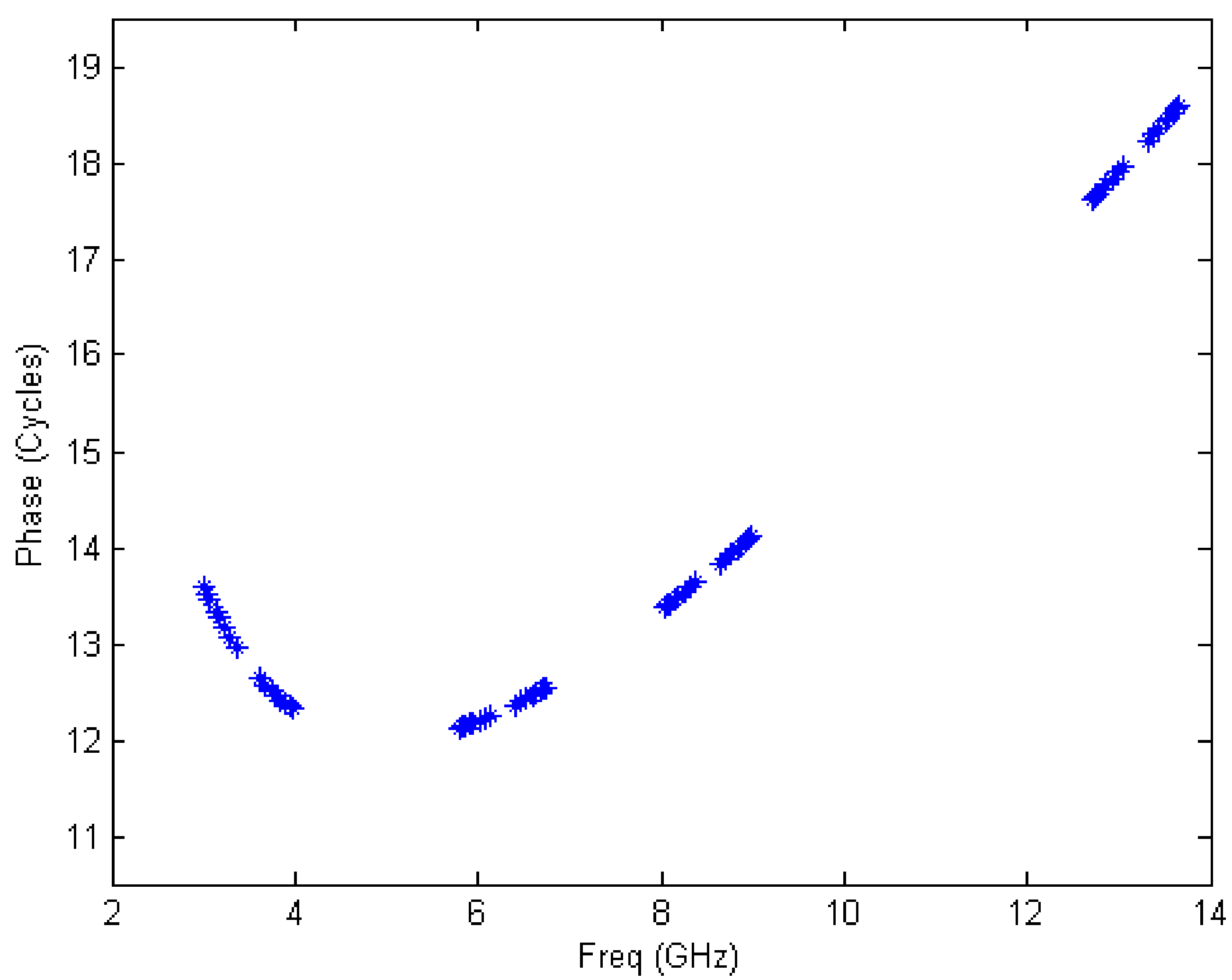
$$\phi_{Ion} = \frac{K}{f} \qquad \tau_{Ion} = -\frac{K}{f^2}$$

In practice, a search algorithm is used to determine τ and K



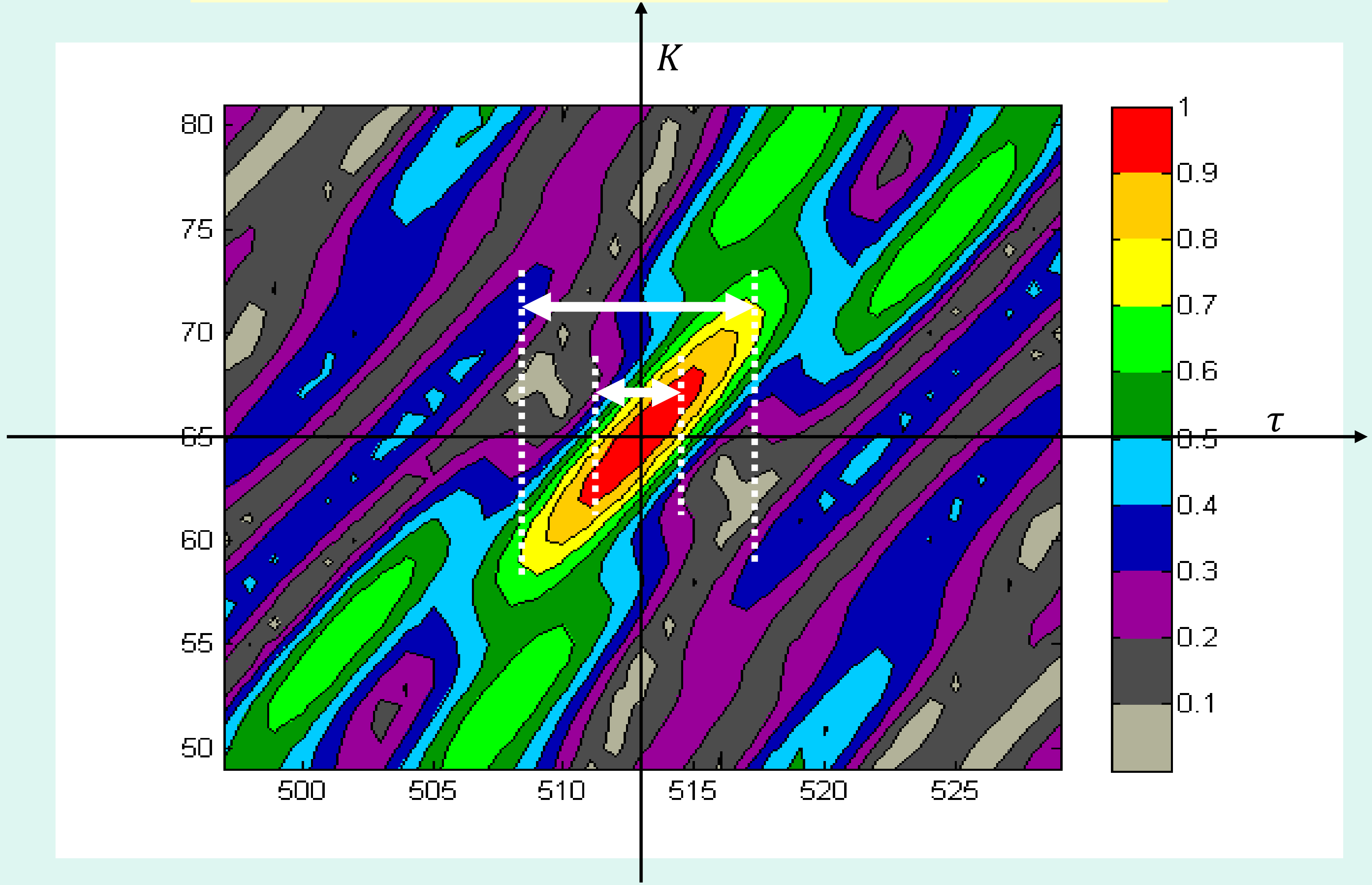
A search is undertaken to find values of τ *and* K that flatten the observed phase response and hence maximize the coherent sum. These are the maximum likelihood values of τ and K.

The frequency coverage is not continuous, which can result in integer cycle phase errors between bands.



2-D (τ, K) Delay Resolution Function

The correlation of τ *and* K increases their error values.




Questions?