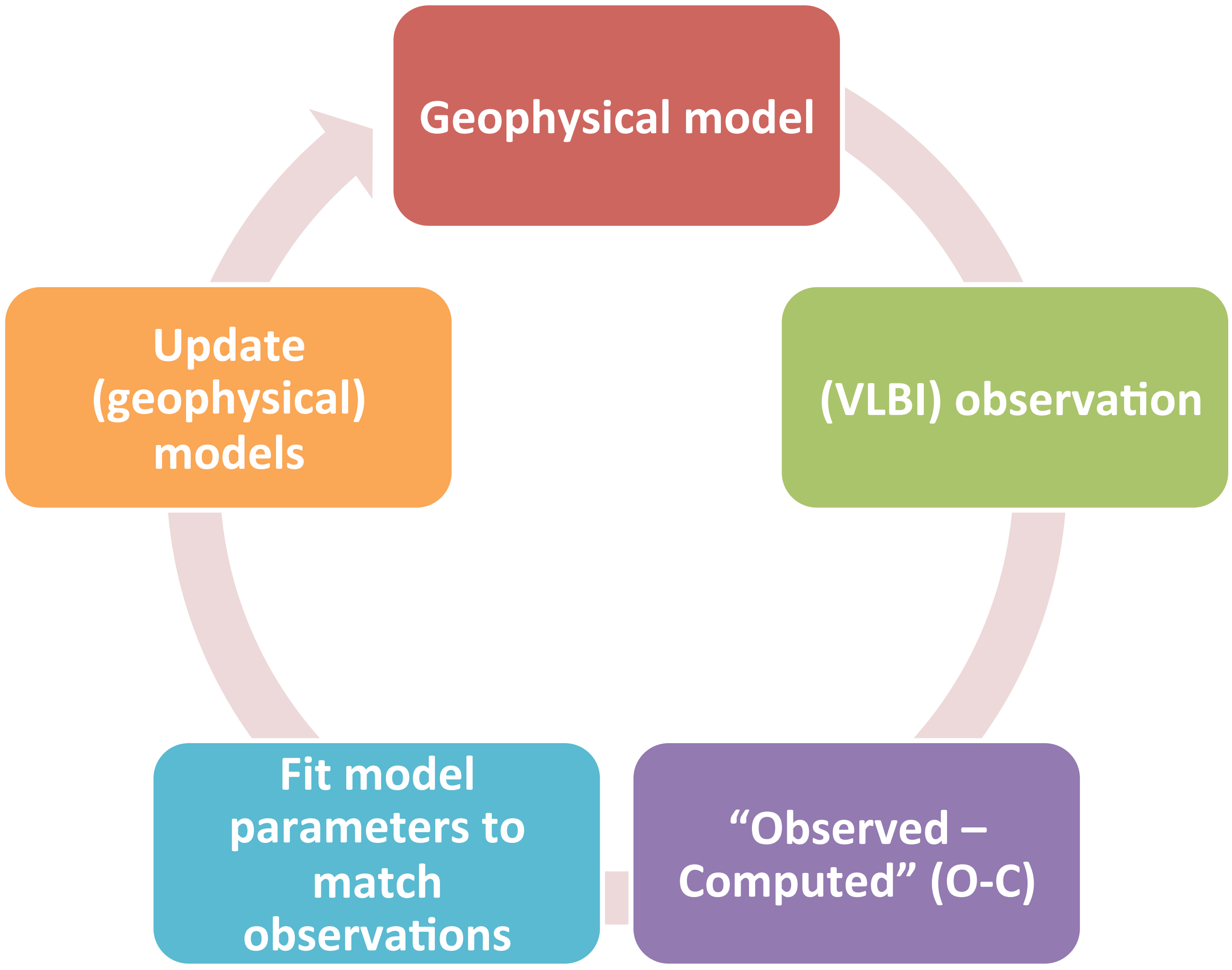
Geophysical modelling (in geodetic VLBI)

Thomas Hobiger

Onsala Space Observatory

Chalmers University of Technology

The (ideal) geophysical model cycle



The main tasks of geophysical modelling

1. Compute a-priori station positions
   a. Get coordinates in TRF at observation epoch
   b. Add (model) "deformations" at observation epoch
2. Transform TRF -> CRF
   1. Precession/Nutation
   2. Polar Motion
   3. UT1

this lecture

3. Compute (theoretical) VLBI delay in Barycentric Celestial Reference System (BCRS)

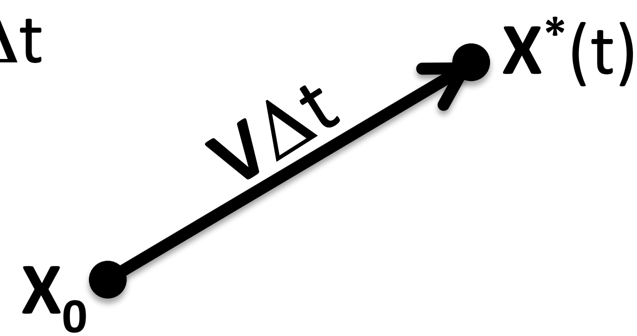
→ **see L13 "Data analysis for geodesy" by Thomas Artz**

1a) Computing a TRF station position at a given epoch

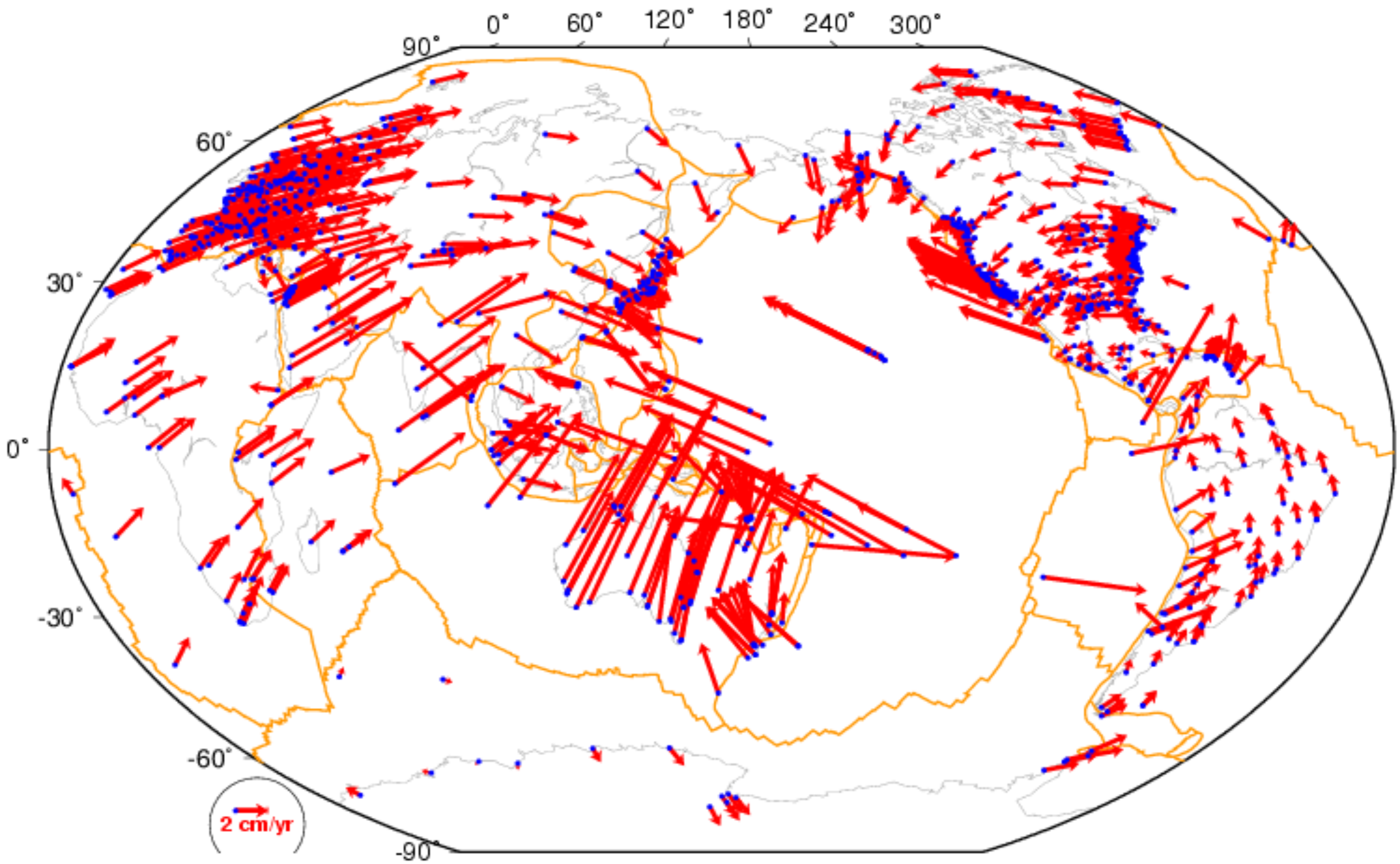
Simple recipe

1. Obtain coordinates $\mathbf{X_0}$ and velocities $\mathbf{V}$ for a given site
2. Compute time difference Δt between observation epoch t and the reference epoch t_0 of the TRF
3. Get TRF coordinates at observation epoch

$$\mathbf{X}^*(t) = \mathbf{X_0} + \mathbf{V}\,\Delta t$$

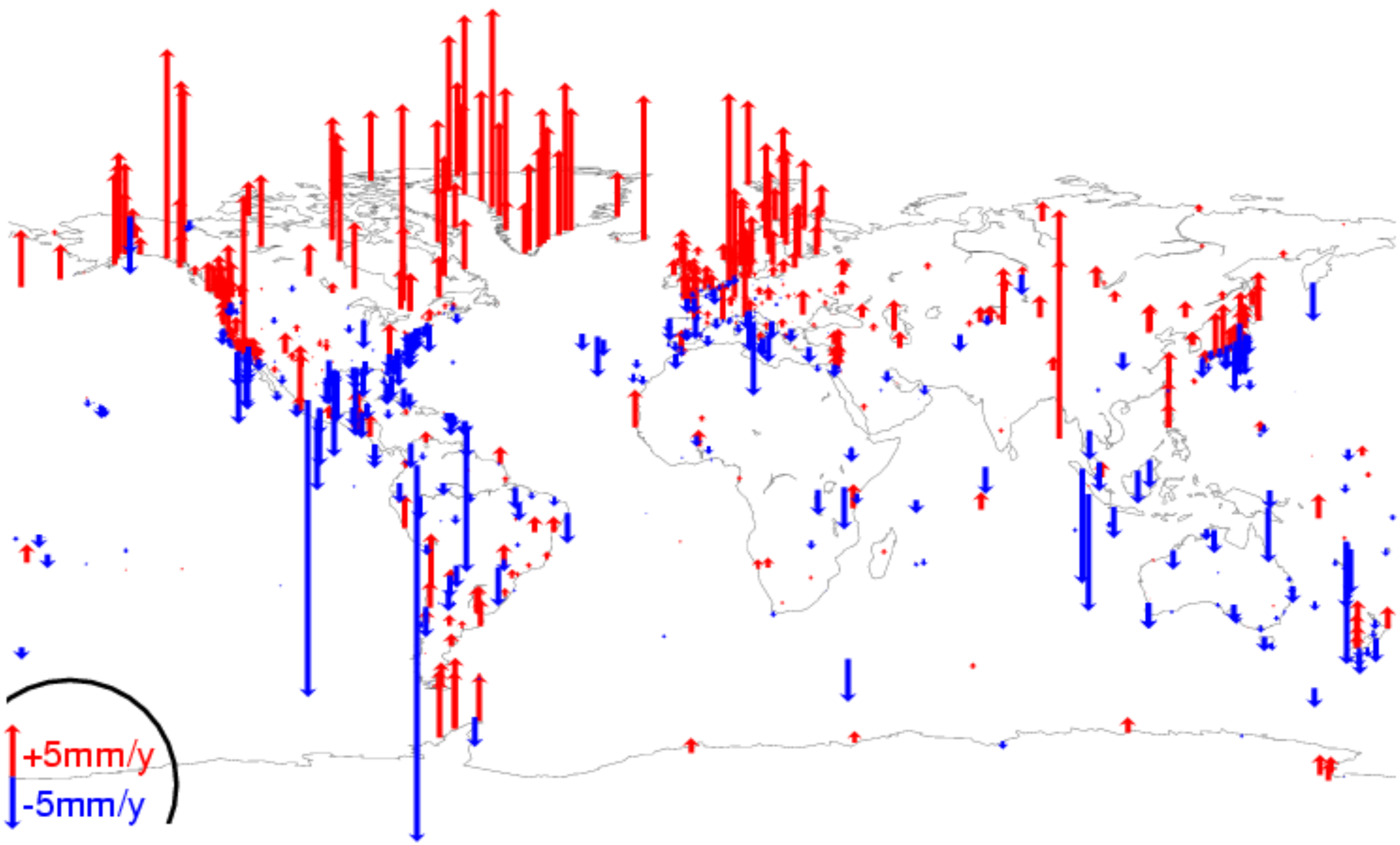


ITRF2014P: Horizontal Velocities



(courtesy of Z. Altamimi, IGN)

ITRF2014P: Vertical Velocities



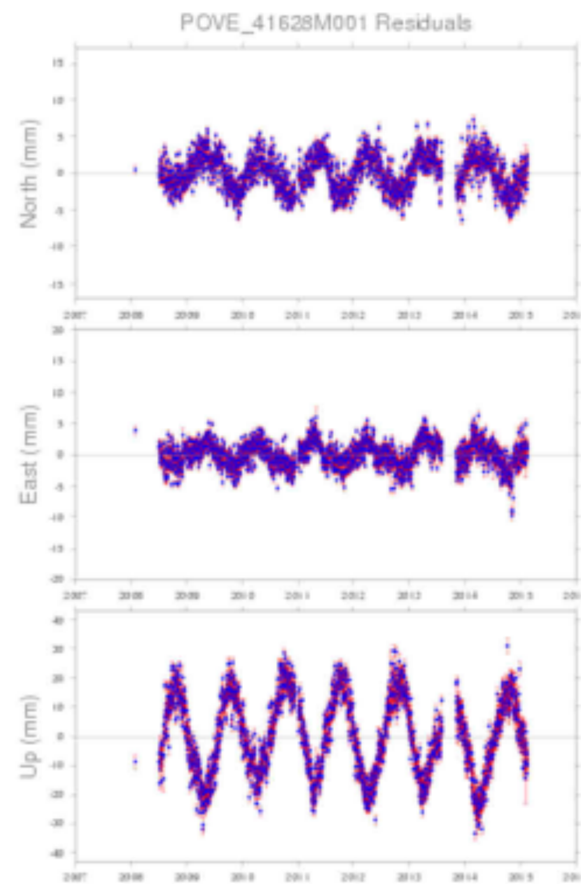
(courtesy of Z. Altamimi, IGN)

More “features” of the latest ITRF(14)

Annual & semi-annual terms estimated, using:

$$\sum a \cos \omega t + b \sin \omega t$$

Removing draconitics in addition to annuals and semi-annuals has no impact on site velocities



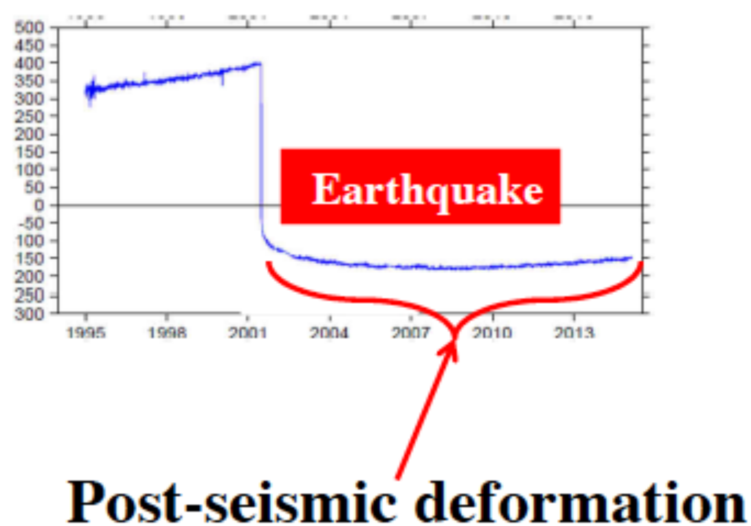
(courtesy of Z. Altamimi, IGN)

More “features” of the latest ITRF(14)

Post-Seismic Deformations

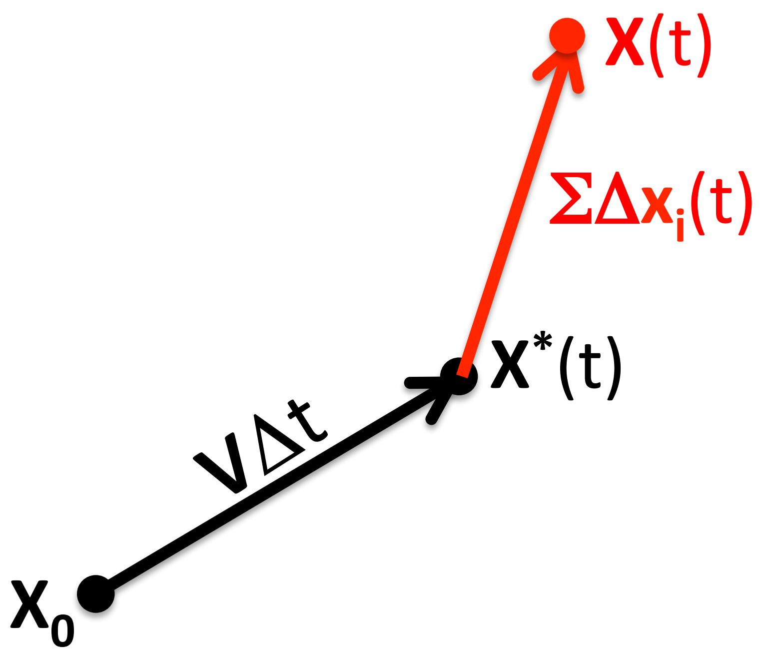
- **Fitting parametric models using GNSS/GPS data**
  - at major GNSS/GPS Earthquake sites
  - Apply these models to the 3 other techniques at Co-location EQ sites

- **Parametric models:**
  - Logarithmic
  - Exponential
  - Log + Exp
  - Two Exp



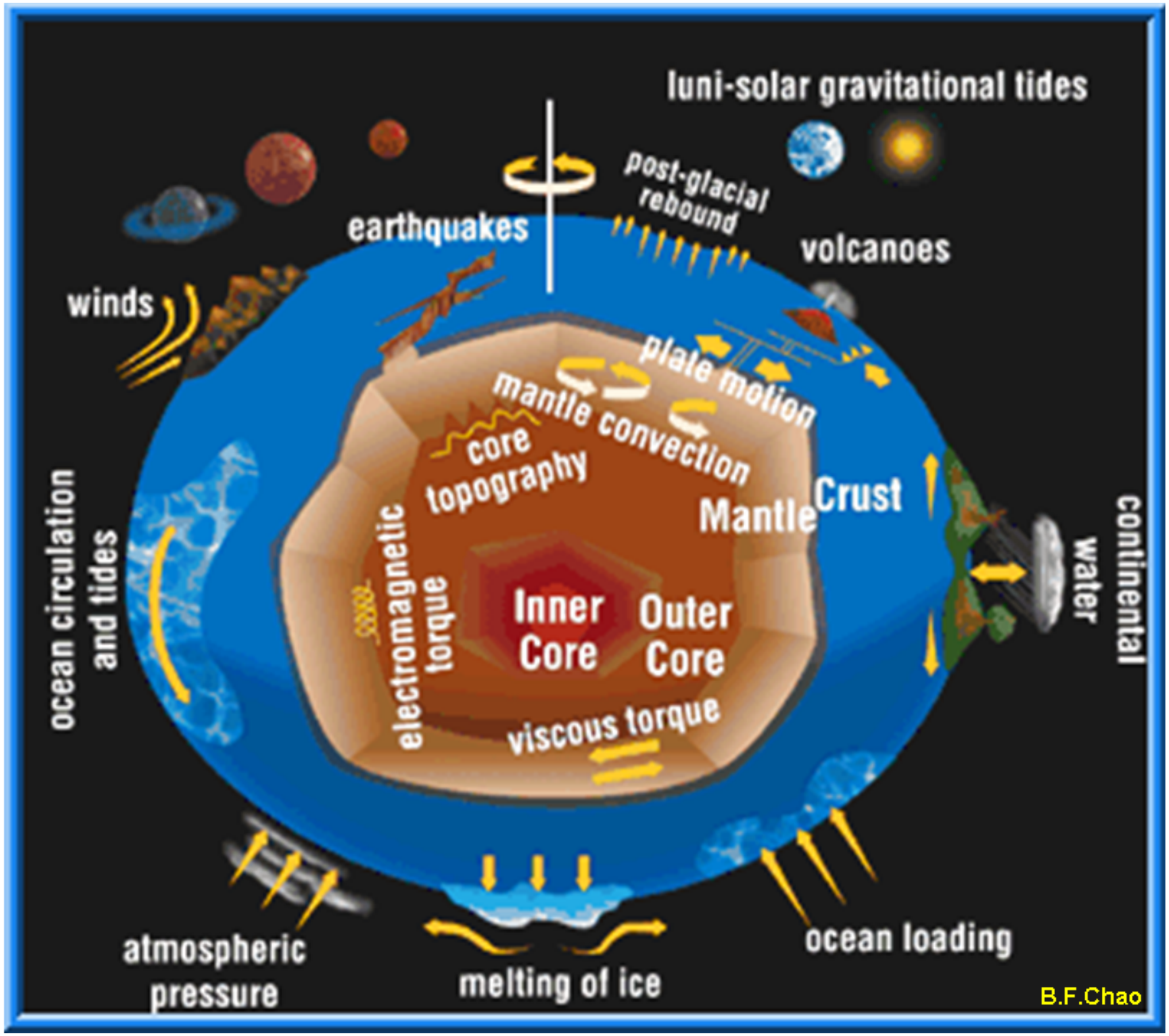
(courtesy of Z. Altamimi, IGN)

Once we have the ITRF position at observation epoch we can add displacement models to that 3D coordinate



Displacement models to be considered

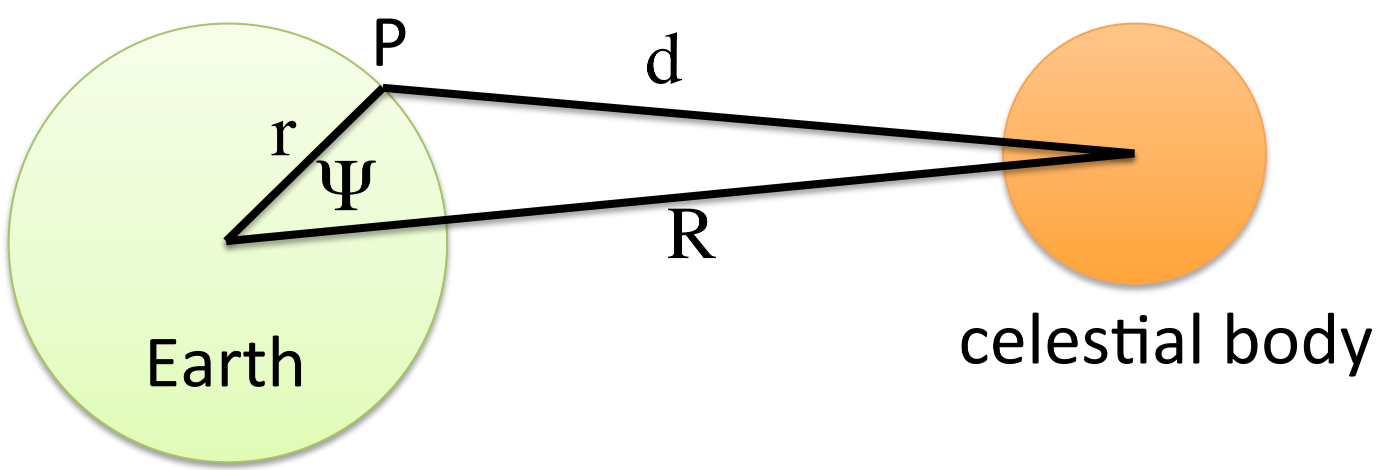
- Solid Earth Tide (largest for 12h band, up to 40 cm vertical)
- Ocean Tidal Loading (largest for 12h, mm-cm vertical)
- S1-S2 atmosphere loading (a few mm)
- Pole Tide (12, 14 mo., mm-cm)
- Ocean Pole Tide Loading (mm)
- Non-tidal loading
  - Atmosphere Pressure Loading
  - Non-tidal Ocean Loading
  - Hydrology loading, …



(From: http://geodesy.agu.org)

Solid Earth tides

Solid Earth tides are deformation of the Earth caused (mainly) by the attraction of the Sun and the Moon.



Tide generating potential

Potential U at point P: $U = GM_{body} / d$

We can expand $1/d$ as

$$\frac{1}{d} = \frac{1}{\sqrt{R^2 - 2Rr\cos\Psi + r^2}} = \sum_{n=0}^{\infty} \frac{r^n}{R^{n+1}} P_n(\cos\Psi)$$

- n = 0: constant (GM_{body} / R)
- n = 1: independent on the position on the Earth

➔ Only terms of degree 2 and higher need to be considered for the tide generating potential

Tide generating potential U_T

This means we get

$$U_T = \frac{GM_{body}}{R} \sum_{n=2}^{\infty} \left(\frac{r}{R} \right)^n P_n\left(\cos\Psi\right)$$

Or as “vector tide model”

$$U_T = GM_{body} \left(\frac{1}{d} - \frac{1}{r} - \frac{\vec{R} \cdot \vec{r}}{R^3} \right)$$

Solid Earth tides

The work of Love showed that the response(deformation of the (spherical) Earth can be related to the tide generating potential by

radial $$d_r = \frac{h_n}{g} U_T \vec{e}_r$$

N/S E/W

tangential $$d_t = \frac{l_n}{g} \nabla U_T \vec{e}_t = \frac{l_n}{g} \frac{\partial U_T}{\partial \theta} \vec{\theta} + \frac{l_n}{g \sin\theta} \frac{\partial U_T}{\partial \lambda} \vec{\lambda}$$

Solid Earth tides

- h_n and l_n are called Love (and Shida) numbers
- In case of the Moon r/R=60, but in case of the Sun r/R=1/23,000.
- 2nd degree harmonics h_2=0.6078 and l_2=0.0847
- We can expand the 2nd harmonic term as a function of (θ,λ) and (θ',λ') of the celestial body

2nd harmonic Solid Earth tides

$$U_T = \frac{GM_{body} r}{R} \left\{ \left[\frac{3}{2}\cos^2\theta - \frac{1}{2} \right] \left[\frac{3}{2}\cos^2\theta' - \frac{1}{2} \right] \right.$$

Long period

$$+ \cos\theta \sin\theta \cos\theta' \sin\theta' \left[\cos\left(\lambda - \lambda'\right) \right]$$

diurnal

$$\left. + \frac{1}{4}\sin^2\theta \sin^2\theta' \left[\cos 2\left(\lambda - \lambda'\right) \right] \right\}$$

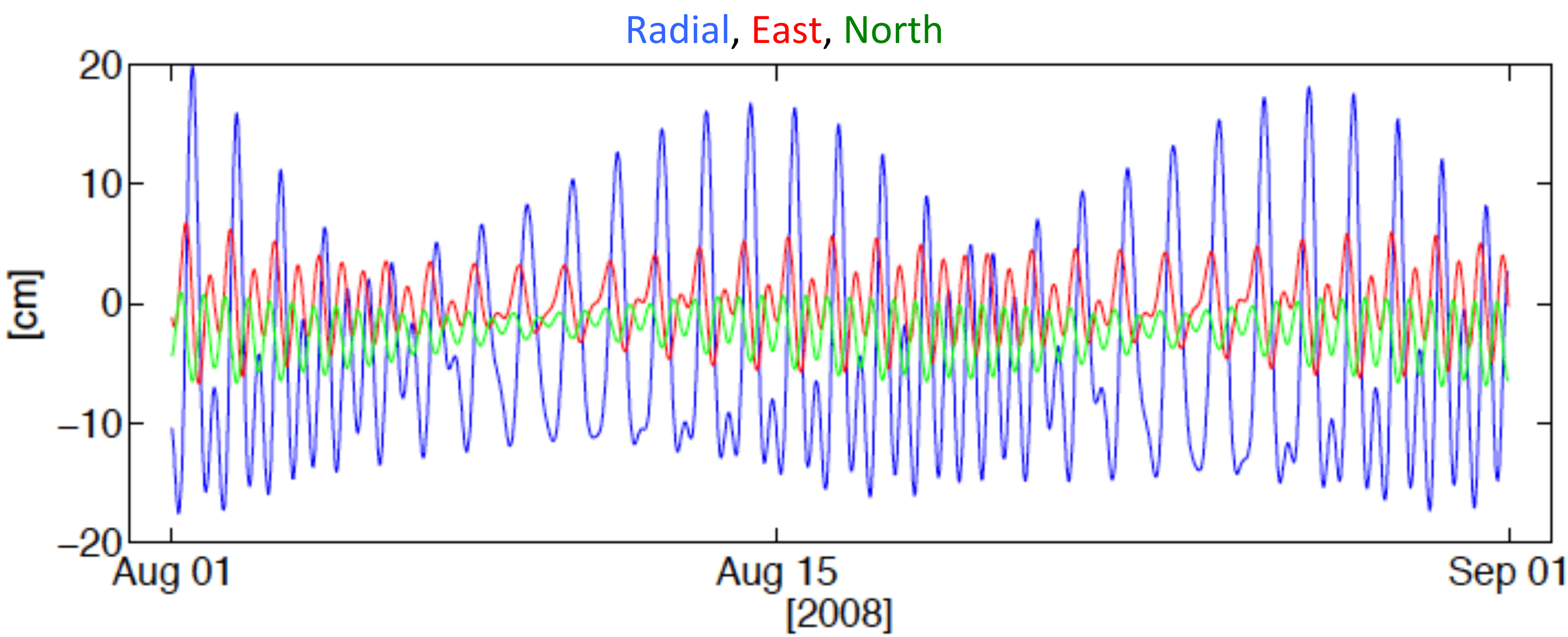
semi-diurnal

Solid Earth tides …

- Need to consider Earth's anelasticity
  - Love/Shida numbers become complex numbers (allows to consider also a phase lag of the displacement w.r.t. U_T)
- Which terms do we need to consider?

degree	Moon	Sun
2	425 mm	173 mm
3	7.5 mm	0.01 mm
4	0. 13 mm	0.00 mm

Solid Earth tides (example for Wettzell, Germany)



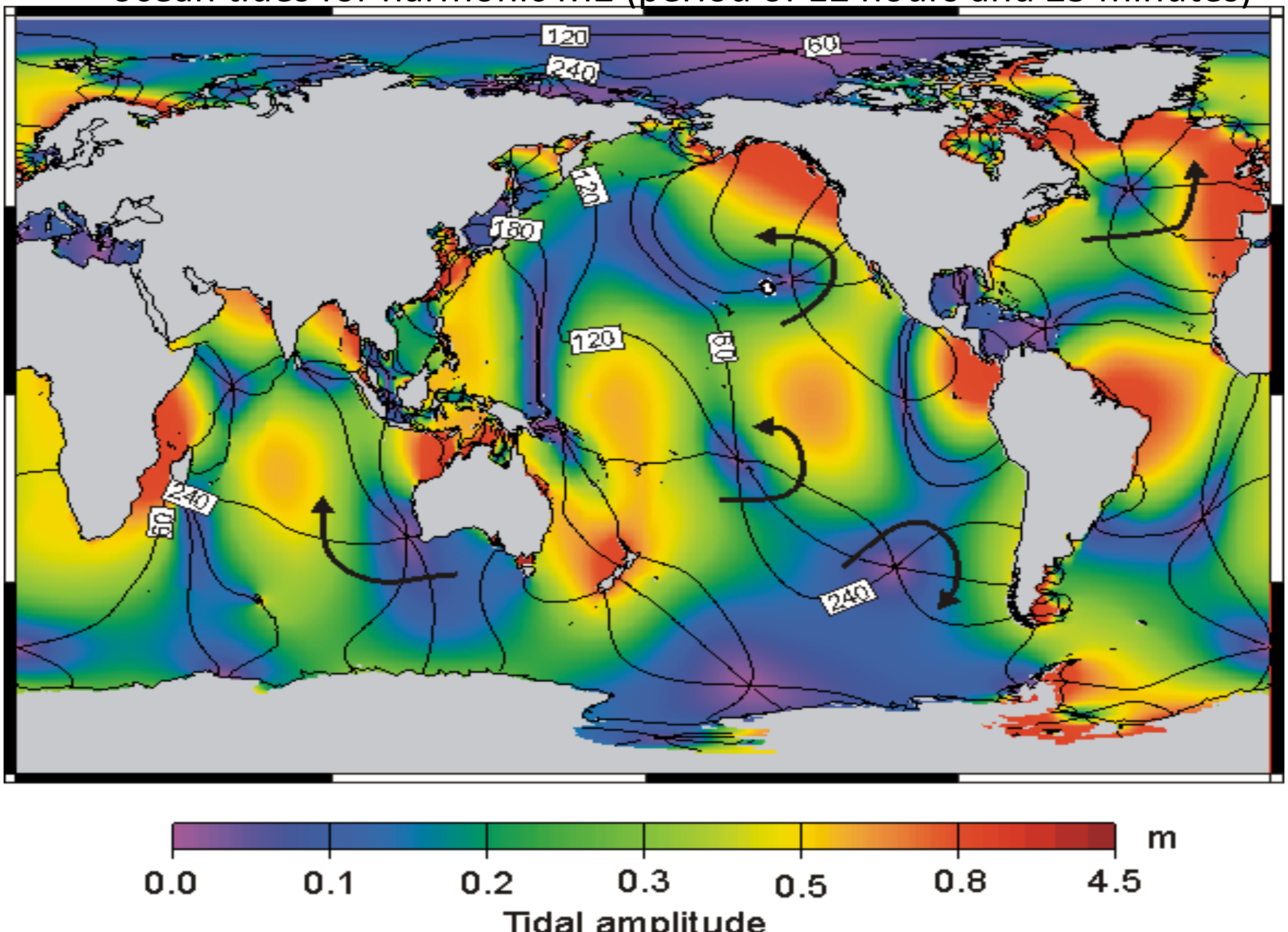
(from Krasna, 2013, https://geo.tuwien.ac.at/fileadmin/editors/GM/GM91_krasna.pdf)

Ocean Tidal Loading

- Tide generating potential forced oceans tides (similar to solid Earth tides)
- Redistribution of water mass cause a load on the Oceanic crust which deforms under this load
- Response of the crust depends on
  - Location (proximity to the coast)
  - Regional condition (shape of coast lines, etc…)

Ocean Loading Tides

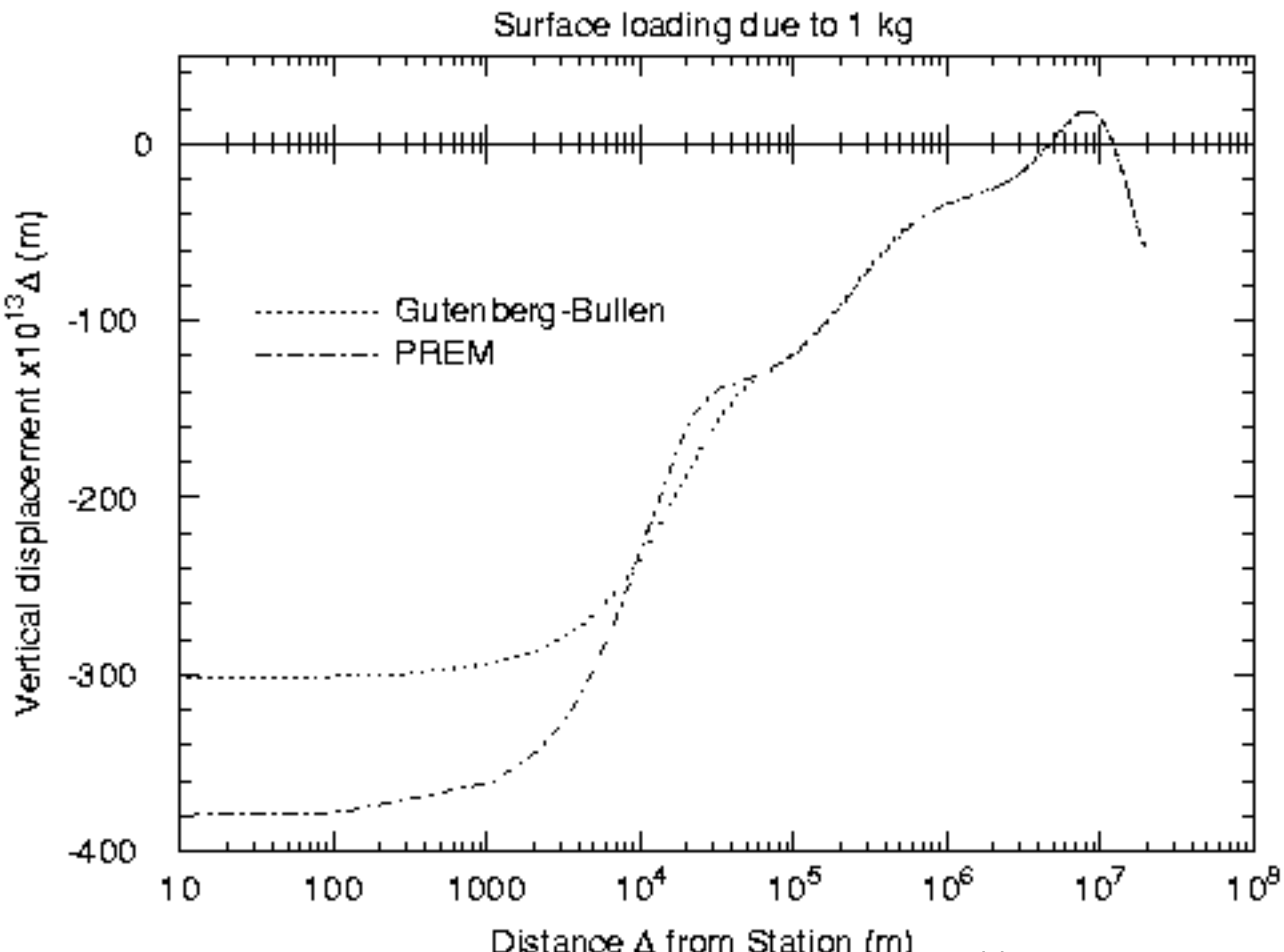
ocean tides for harmonic M2 (period of 12 hours and 25 minutes)



(from http://holt.oso.chalmers.se/loading/)

Response to Ocean Tides

Computed by globally integrating the loading Green's function over the tide elevation mass for each tidal constituent -> time consuming !!



(from http://holt.oso.chalmers.se/loading/)

Ocean Tidal Loading

- Displacements computed for you at any given site (ocean tide loading provider, e.g. http://holt.oso.chalmers.se/loading)
- Example for Onsala, Sweden

```
  ONSALA60
$$ FES2004_PP ID: 2010-03-24 09:27:25
$$ Computed by OLMPP by H G Scherneck, Onsala Space Observatory, 2010
$$ ONSALA60,                              lon/lat:   11.9264   57.3958    0.00
  .00304 .00118 .00071 .00029 .00240 .00124 .00080 .00014 .00083 .00049 .00043
  .00145 .00037 .00032 .00009 .00047 .00046 .00016 .00008 .00013 .00007 .00007
  .00069 .00025 .00021 .00006 .00033 .00019 .00012 .00004 .00003 .00002 .00002
   -63.3  -40.1 -105.0  -47.9  -51.1 -117.0  -53.5  176.1   16.2   10.8    2.0
    88.6  125.6   56.6  114.8  103.5   36.6   98.9   -6.2 -168.2 -170.5 -177.7
   108.5  140.5   82.6  147.7   44.6  -40.7   44.0 -118.7   45.2   33.3    3.7
$$
```

Contains the UEN amplitudes and phases for 11 main tides (M_2,S_2,N_2,K_2, K_1, O_1, P_1, Q_1, M_f, Mm, Ssa)

Ocean Tidal Loading

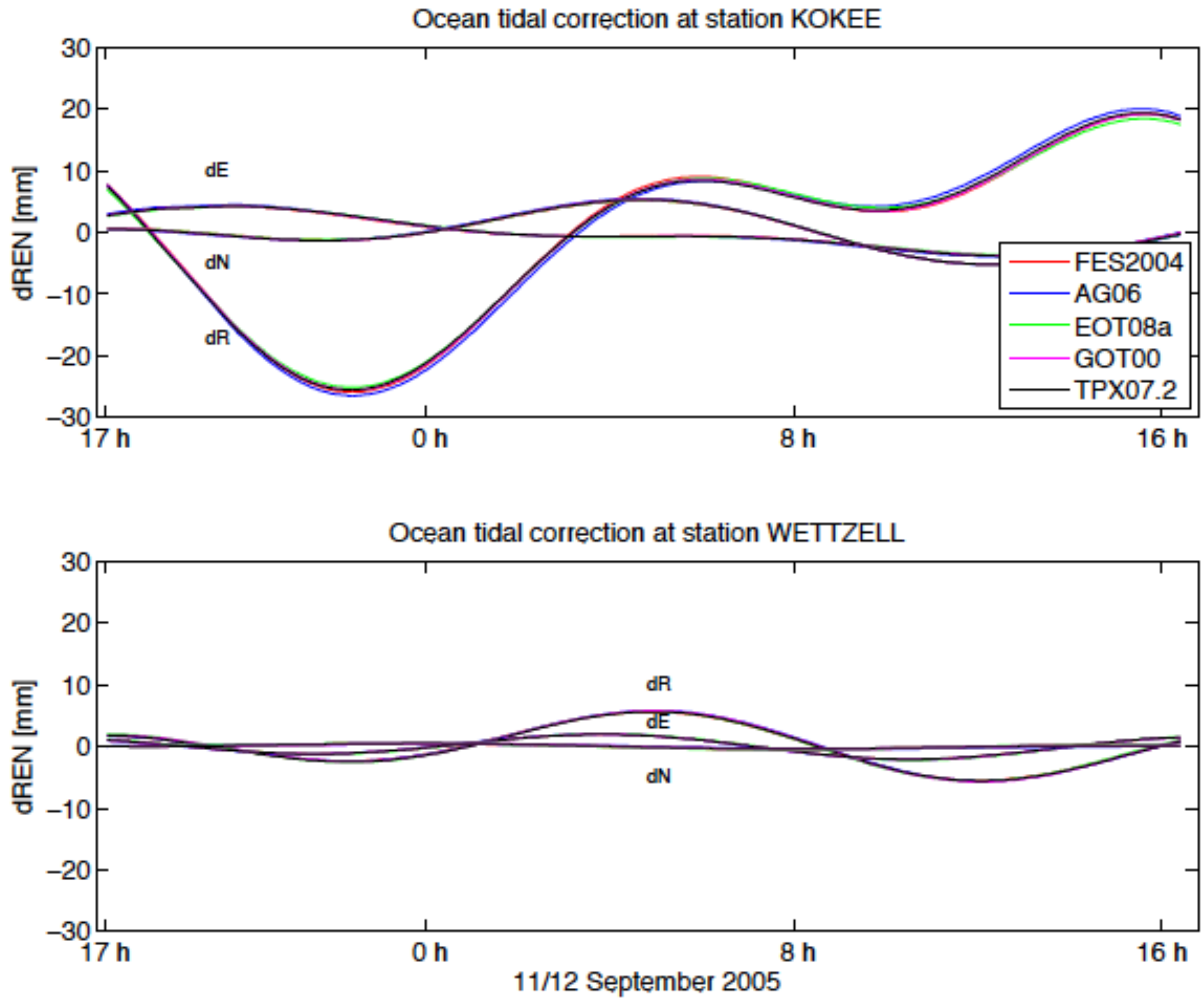
Side displacement

$$\Delta x_i(t) = \sum_{j=1}^{11} A_{ij} \cos\left(\chi_j(t) - \underline{\Phi_{ij}}\right) \qquad (i=U,E,N)$$

Astron. argument of the tide

However, it is recommended to compute loading based on 342 constituents found by interpolating tidal admittances based on the 11 main tides. ("HARDISP" routine)

Ocean tidal loading (example)

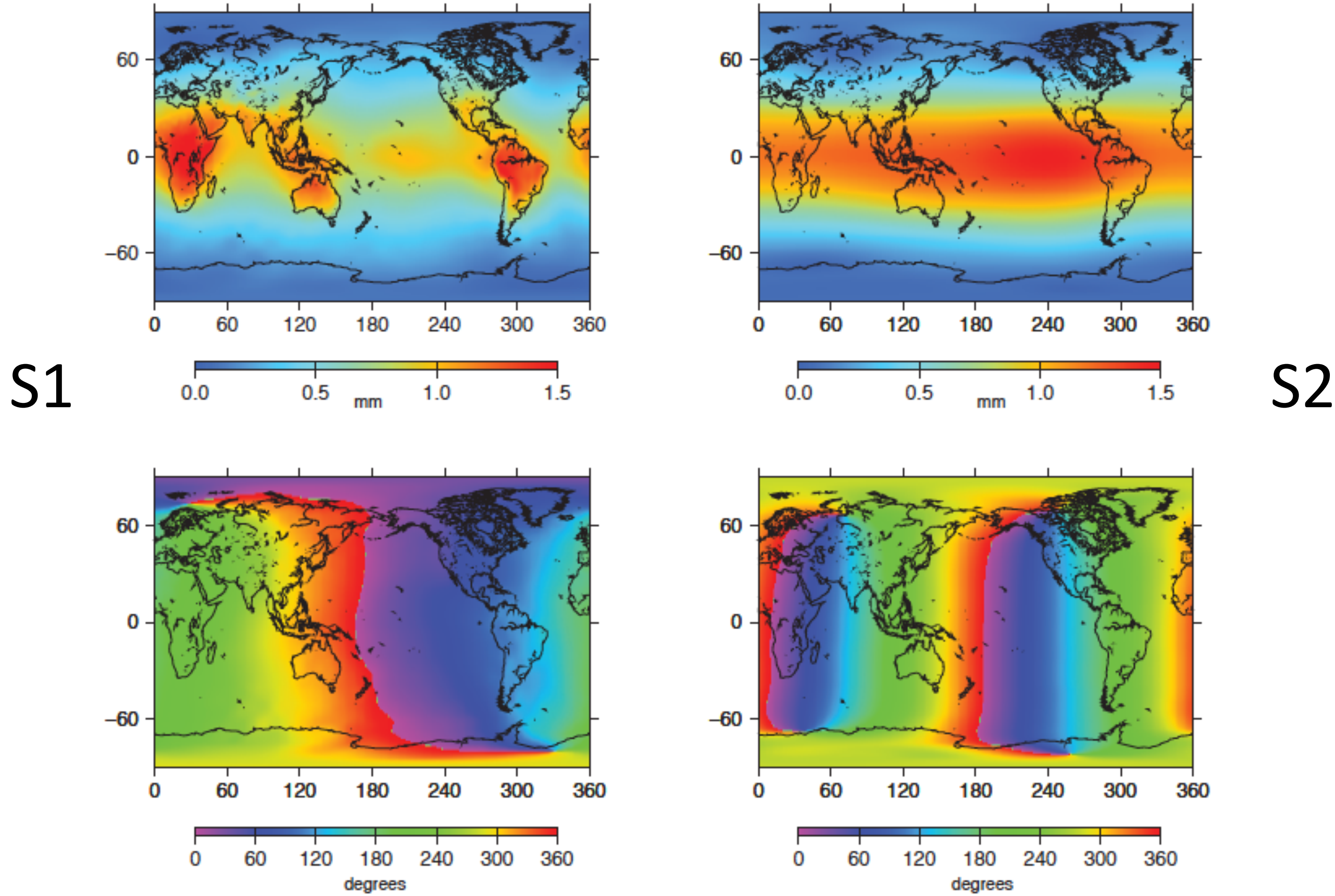


(from Krasna, 2013, https://geo.tuwien.ac.at/fileadmin/editors/GM/GM91_krasna.pdf)

Atmosphere loading

- Redistribution of air masses causes a site displacement
- Tidal loading (both available as gridded correction)
  - S1 (diurnal)
  - S2 (semi-diurnal)
- Non-tidal APL (time-series for station, see "non-tidal loading" slide)

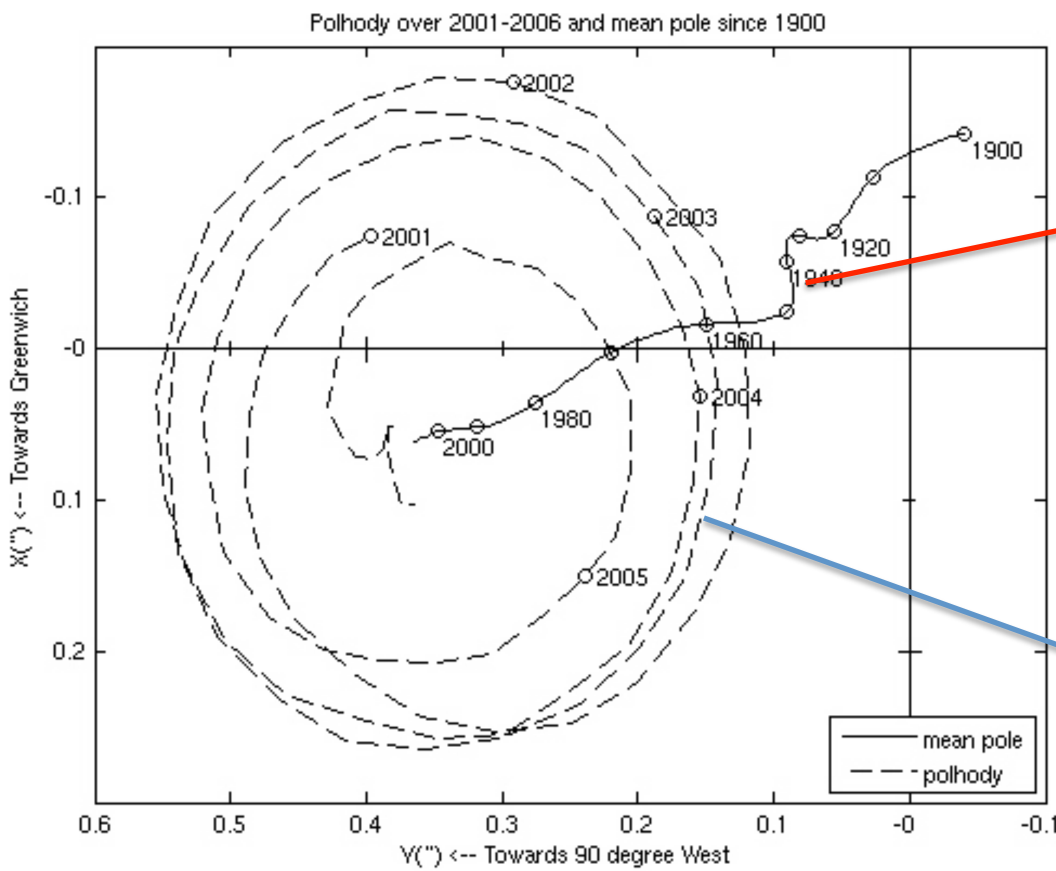
Atmosphere loading (S1/S2)



(from IERS Conv. 2010)

Pole tide

The Earth’s mean rotation pole has a secular variation, i.e. a long-term drift of pole w.r.t. the Earth’s surface (we will come to that again later)



mean pole

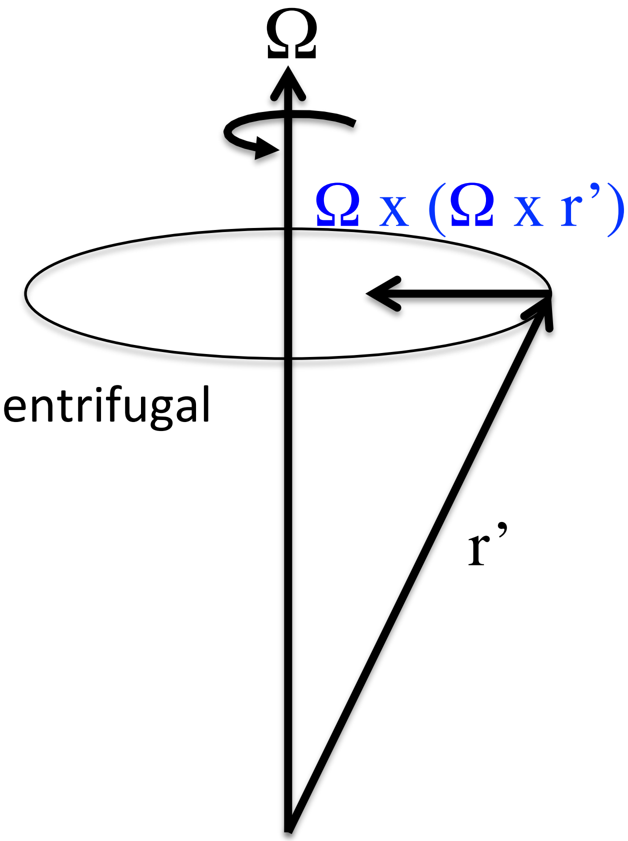
“Chandler wobble” (~435 days)

(From https://iers.org)

Pole tide - theory

Acceleration in fixe frame vs. rotating primed system

$$\vec{a} = \vec{a}' + \underbrace{2\vec{\Omega} \times \vec{v}'}_{\text{Coriolis acceleration}} + \underbrace{\vec{\Omega} \times \left(\vec{\Omega} \times \vec{r}' \right)}_{\text{Centrifugal acc. } a_c}$$



We can express the centrifugal acceleration from the centrifugal Potential V_c

$$V_c = \frac{1}{2} \left| \vec{\Omega} \times \vec{r}' \right|$$

i.e.

$$\vec{a}_c = -\nabla V_c$$

Pole tide – theory contd.

Angular rotation of the Earth

$$\vec{\Omega} = \Omega_0 \left[m_x \vec{e}_x + m_y \vec{e}_y + (1 + m_z) \vec{e}_z \right]$$

Time-dependent pole offsets

Fractional variation of rotation rate (m_z about 1 /100 of m_x or m_y)

$$V(\theta, \lambda) = -\frac{1}{2} \Omega_0^2 r^2 \left[\sin 2\theta \left(m_x \cos\lambda + m_y \sin\lambda \right) \right]$$

Guess what? We have a disturbing potential so we can compute the displacement by using the Love (and Shida) numbers like in the case of solid earth tides!

Pole tide

$$S_r = -33\sin 2\theta\,(m_1\cos\lambda + m_2\sin\lambda)$$

$$S_\theta = -9\cos 2\theta\,(m_1\cos\lambda + m_2\sin\lambda)$$

$$S_\lambda = 9\cos\theta\,(m_1\sin\lambda - m_2\cos\lambda)$$

displacements in mm

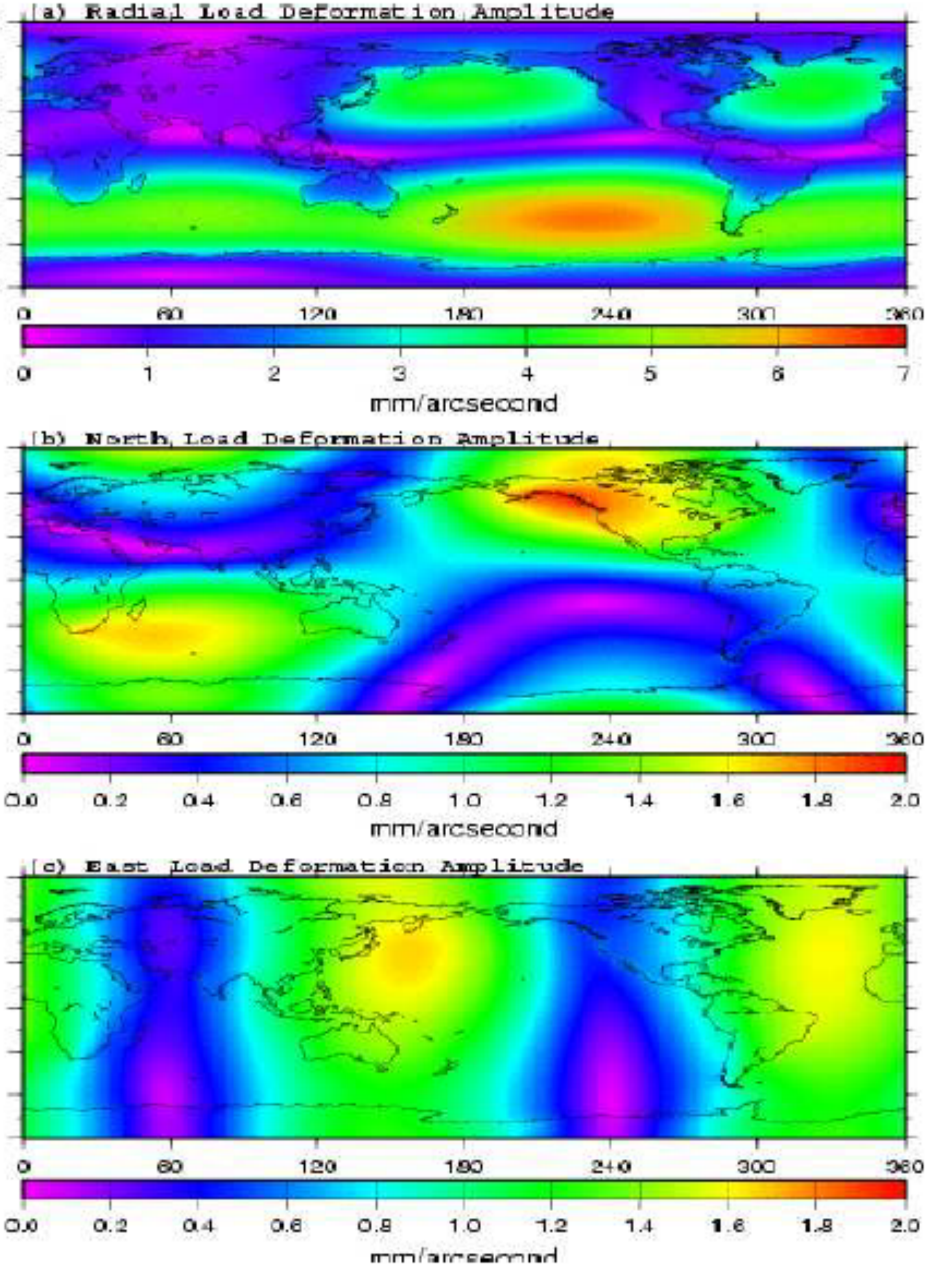
$$m_1 = x_p - \bar{x}_p, \qquad m_2 = -(y_p - \bar{y}_p).$$

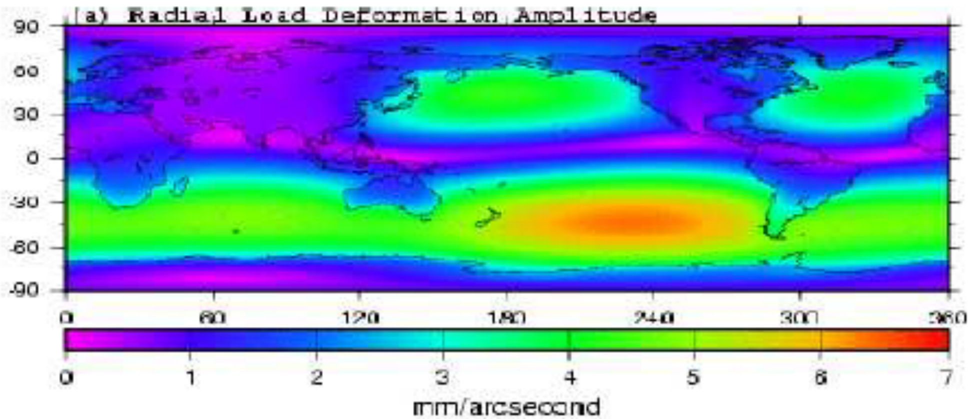
polar motion

Mean pole (recent model used tabulated values)

Ocean Pole Tide Loading

- Similar idea to pole tide loading
  - Centrifugal potential causes ocean height to change ➔ leads to a slightly different load on the crust ➔ site displacement
- Computed also via Green functions approach
- Make sure that all calculations are mass conservative (subtract average ocean height at each epoch)
- Provided as a grid


(a) Radial Load Deformation Amplitude
90
60
30
0
-30
-60
-90
0
60
120
180
240
300
360
0
1
2
3
4
5
6
7
mm/arcsecond
(b) North Load Deformation Amplitude
0.0
0.2
0.4
0.6
0.8
1.0
1.2
1.4
1.6
1.8
2.0
mm/arcsecond
(c) East Load Deformation Amplitude
mm/arcsecond



$$\begin{bmatrix} u_r(\phi,\lambda) \\ u_n(\phi,\lambda) \\ u_e(\phi,\lambda) \end{bmatrix} = K \left\{ (m_1\gamma_2^R + m_2\gamma_2^I) \begin{bmatrix} u_r^R(\phi,\lambda) \\ u_n^R(\phi,\lambda) \\ u_e^R(\phi,\lambda) \end{bmatrix} + (m_2\gamma_2^R - m_1\gamma_2^I) \begin{bmatrix} u_r^I(\phi,\lambda) \\ u_n^I(\phi,\lambda) \\ u_e^I(\phi,\lambda) \end{bmatrix} \right\}$$

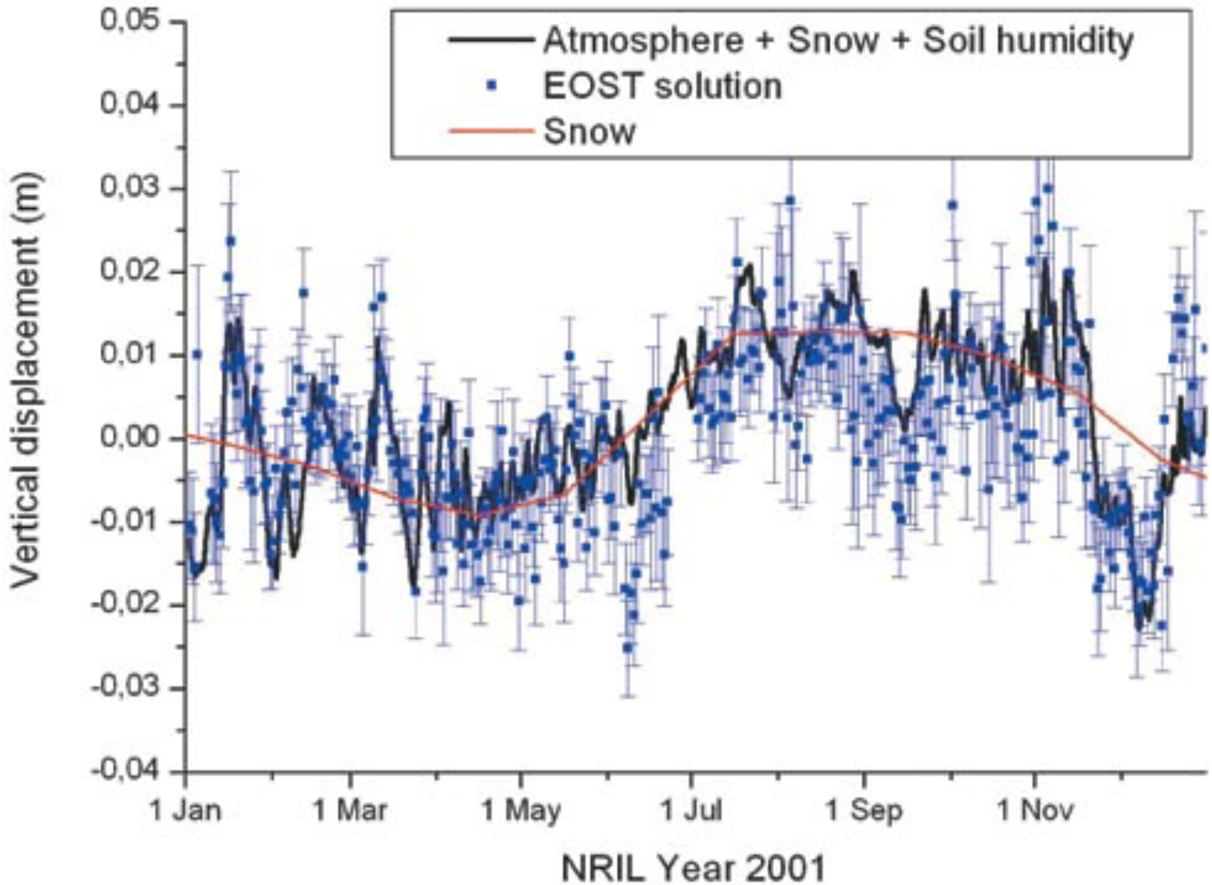
$(m_1, m_2) = (m_x, m_y)$, c.f. pole tide calculations!

Non-tidal (or mass) loading

- Atmosphere pressure loading
- Hydrology loading
- Non-tidal ocean loading
- Others (snow, ….)

Non-tidal loading - why should we look into it?

A GNSS site in Northern Siberia



(From http://dx.doi.org/10.1590/S0102-261X2007000100004)

Atmospheric pressure loading

- Simple idea

$$\Delta x_i = \alpha_{pressure} (P-P_{ref})$$

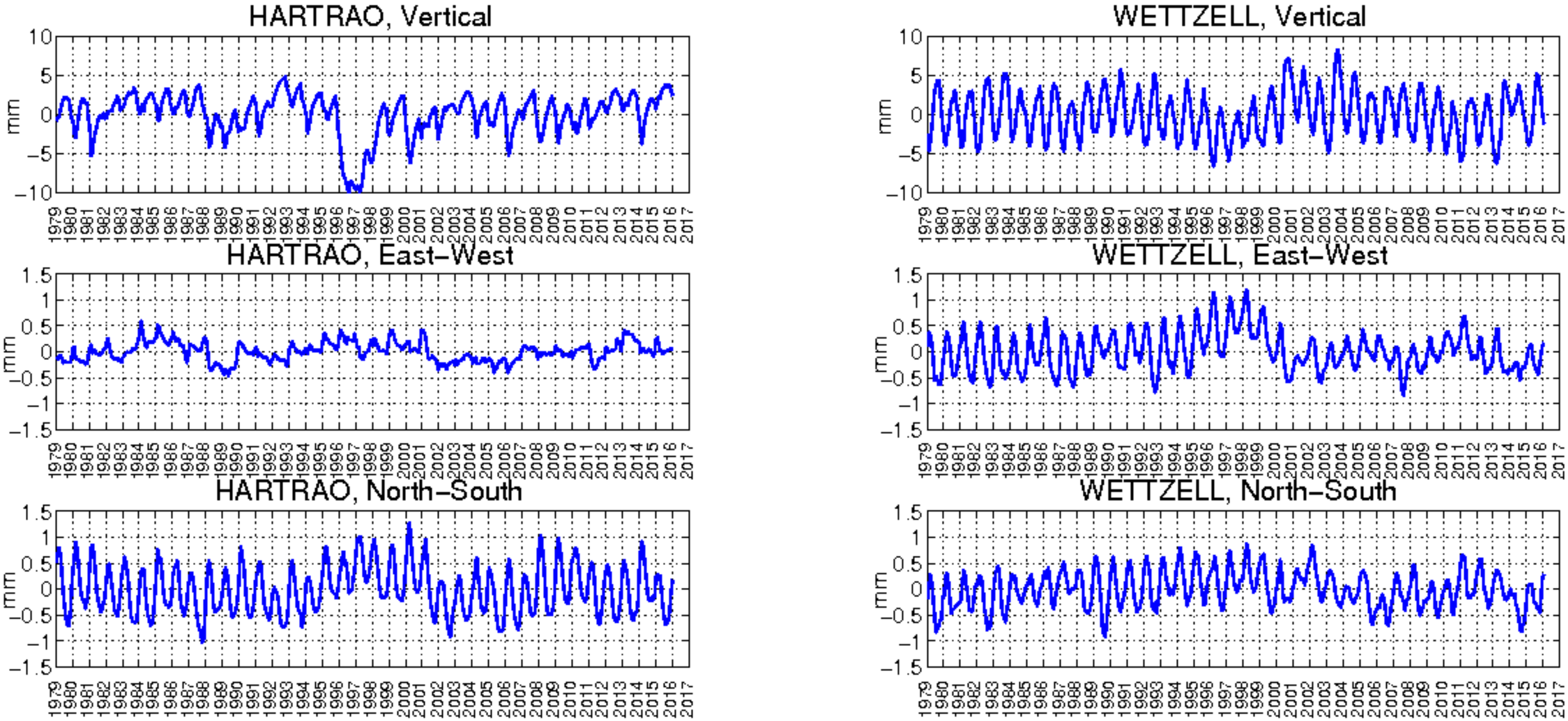
- Need to know admittance factor $\alpha_{pressure}$
- Determined usually from VLBI data fitting
- Admittance factor station/site dependent!
- Linear model, i.e. total dependence on the accuracy of the pressure information (numerical weather model)

Hydrology loading

- Compute the displacement due to water loading on the surface (crust)
- Requires good hydrologic model
- Provided as time-series for selected stations
- Services available

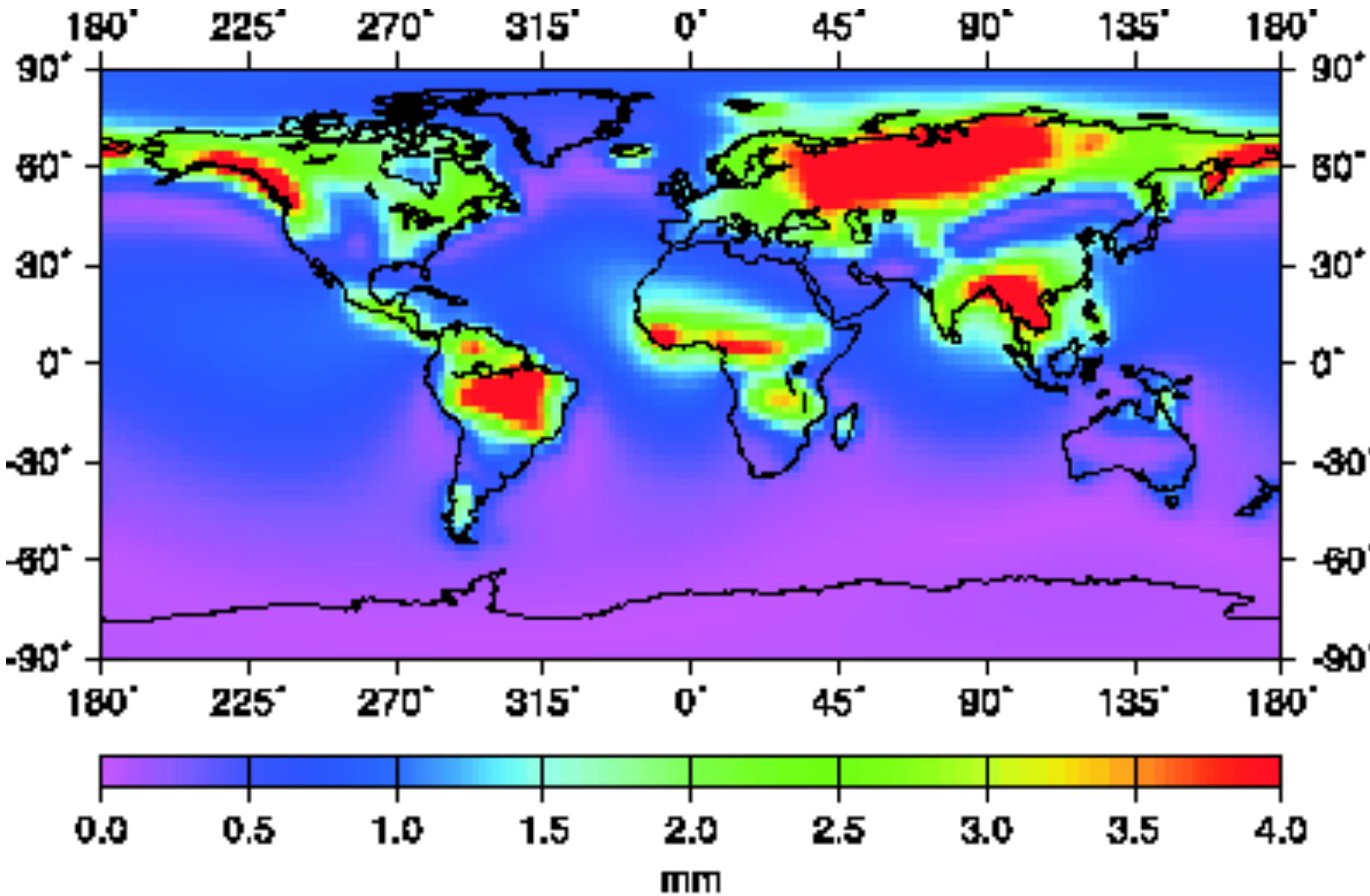
  e.g. http://lacerta.gsfc.nasa.gov/hydlo/

Hydrology loading (examples)



(Source http://lacerta.gsfc.nasa.gov/hydlo/Pics/hartrao_wettzell.png)

Hydrology loading – where ?

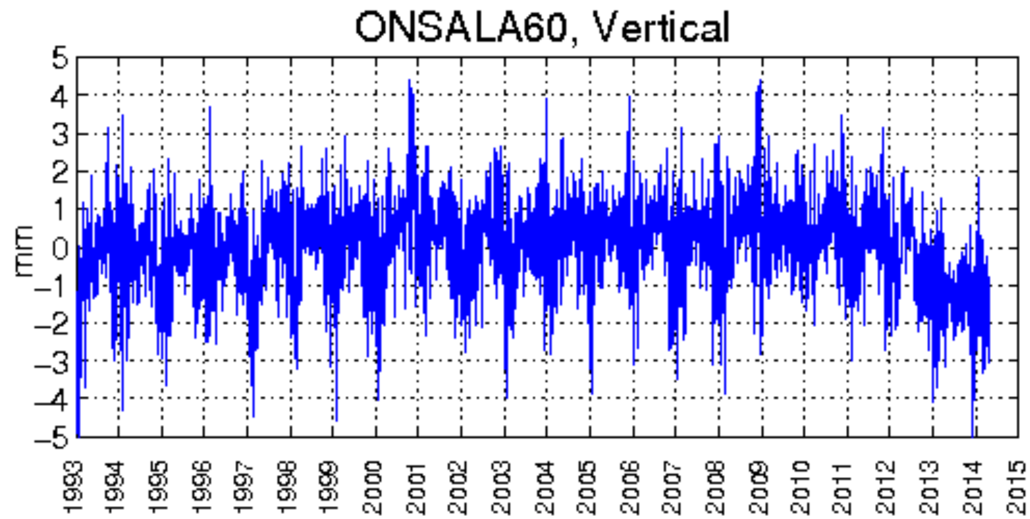


(Source: http://ivs.nict.go.jp/mirror/publications/gm2004/macmillan1/macmillan1.html)

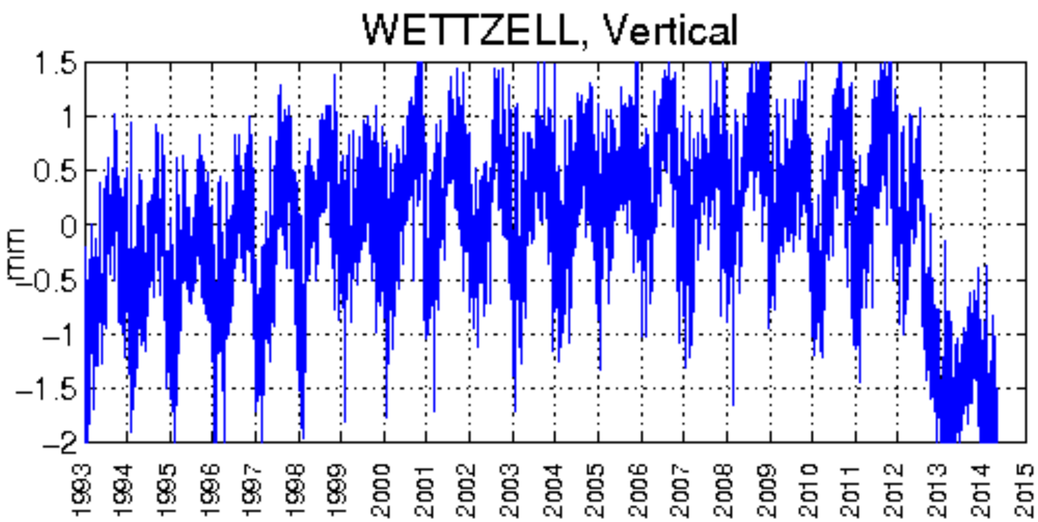
Non-tidal ocean loading

- usually very seasonal
- most significant near the coast
- Computed from ocean bottom pressure data / models, e.g. ECCO (JPL, 12h resolution)
- Provided as
  - Station dependent corrections (time-series)
  - Grid (NetCDF)
- Download http://lacerta.gsfc.nasa.gov/oclo/

Non-tidal ocean loading



ONSALA –
close to the sea



WETTZELL –
far away from the coast

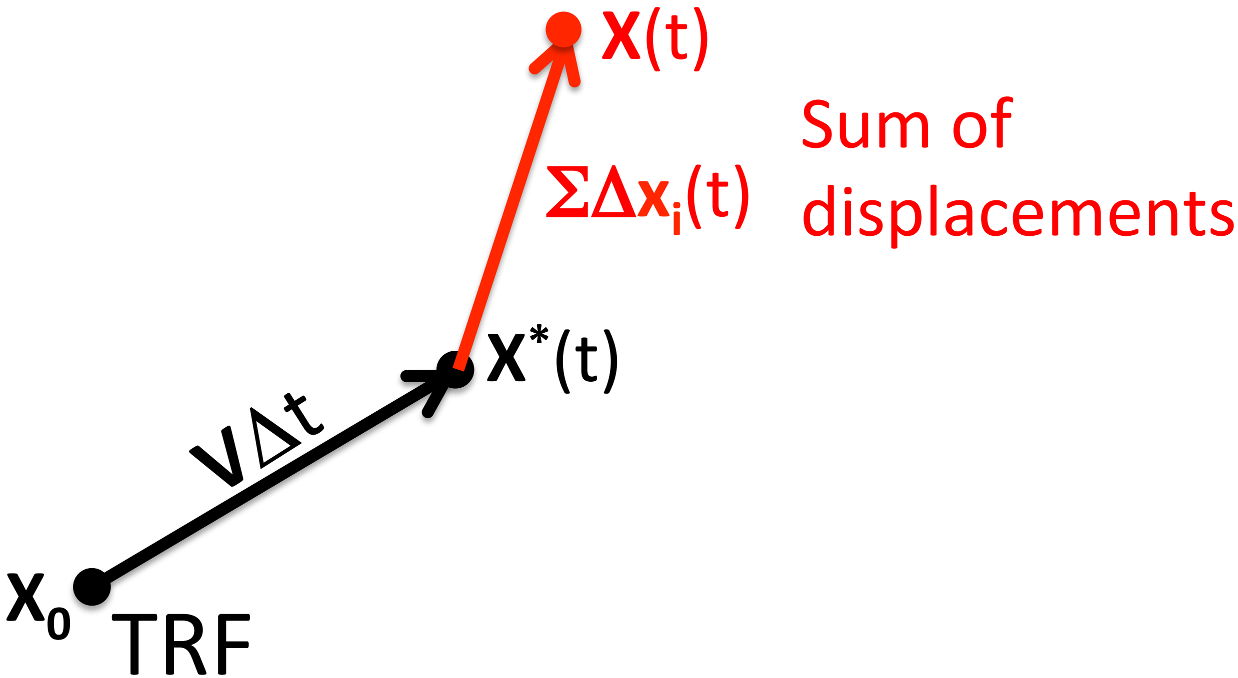
(source: http://lacerta.gsfc.nasa.gov/oclo/)

Total displacement

=

Σ of all effects discussed here

Remember



We have now

- 3D coordinate of station #1 at obs. Epoch
- 3D coordinate of station #2 at obs. epoch

We need to

- Transform both stations from TRF to CRF (BCRS) where we can model the theoretical (relativistic) delay
- This requires Earth Orientation Parameters (EOPs) and transformations which relate between TRF and CRF and vice versa.

Transformation – TRF-CRF

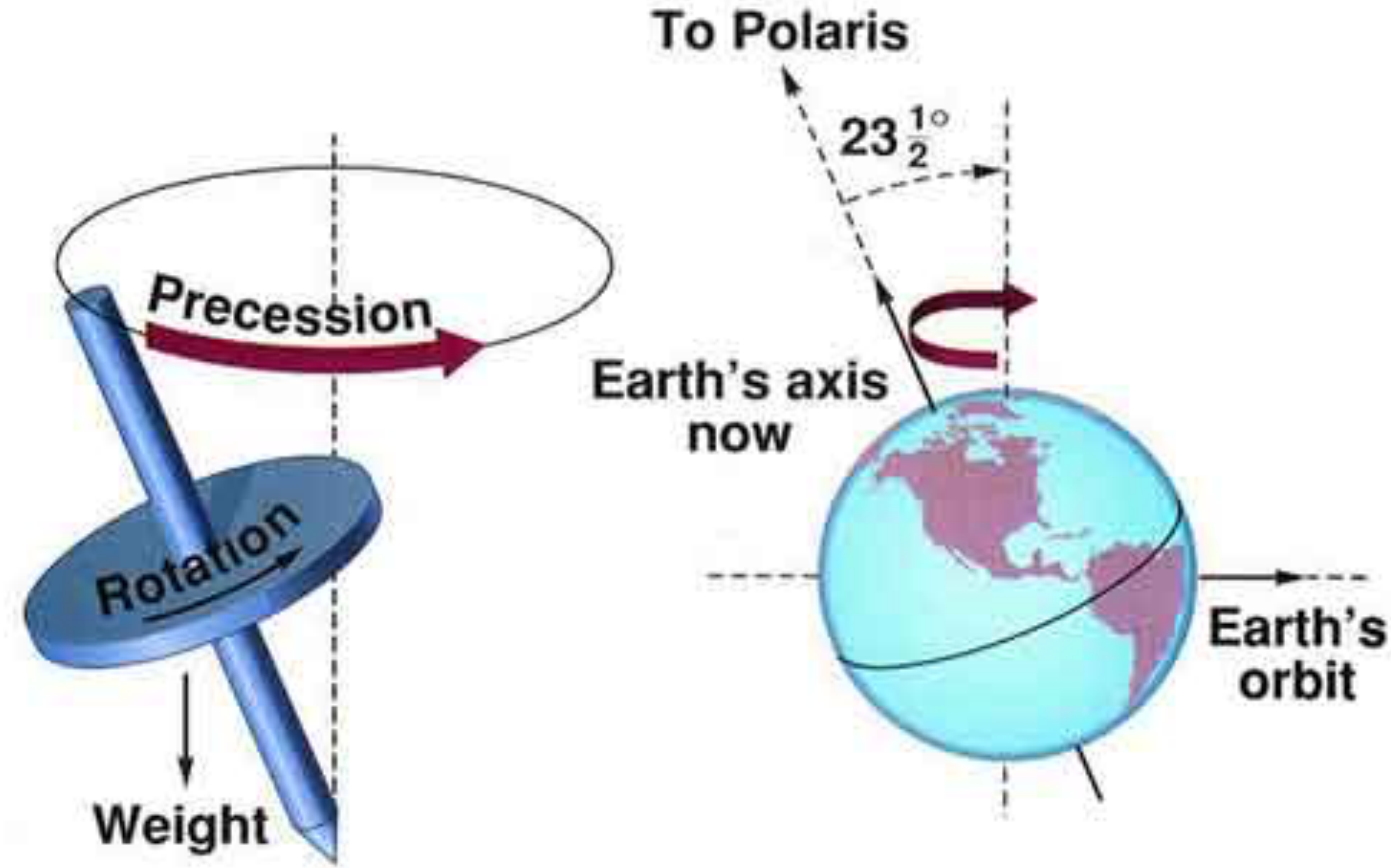
Compute rotation matrix Q that translates between celestial and terrestrial from

$$Q = P \cdot N \cdot U \cdot X \cdot Y$$

Precession Nutation Spin (UT1) Polar Motion

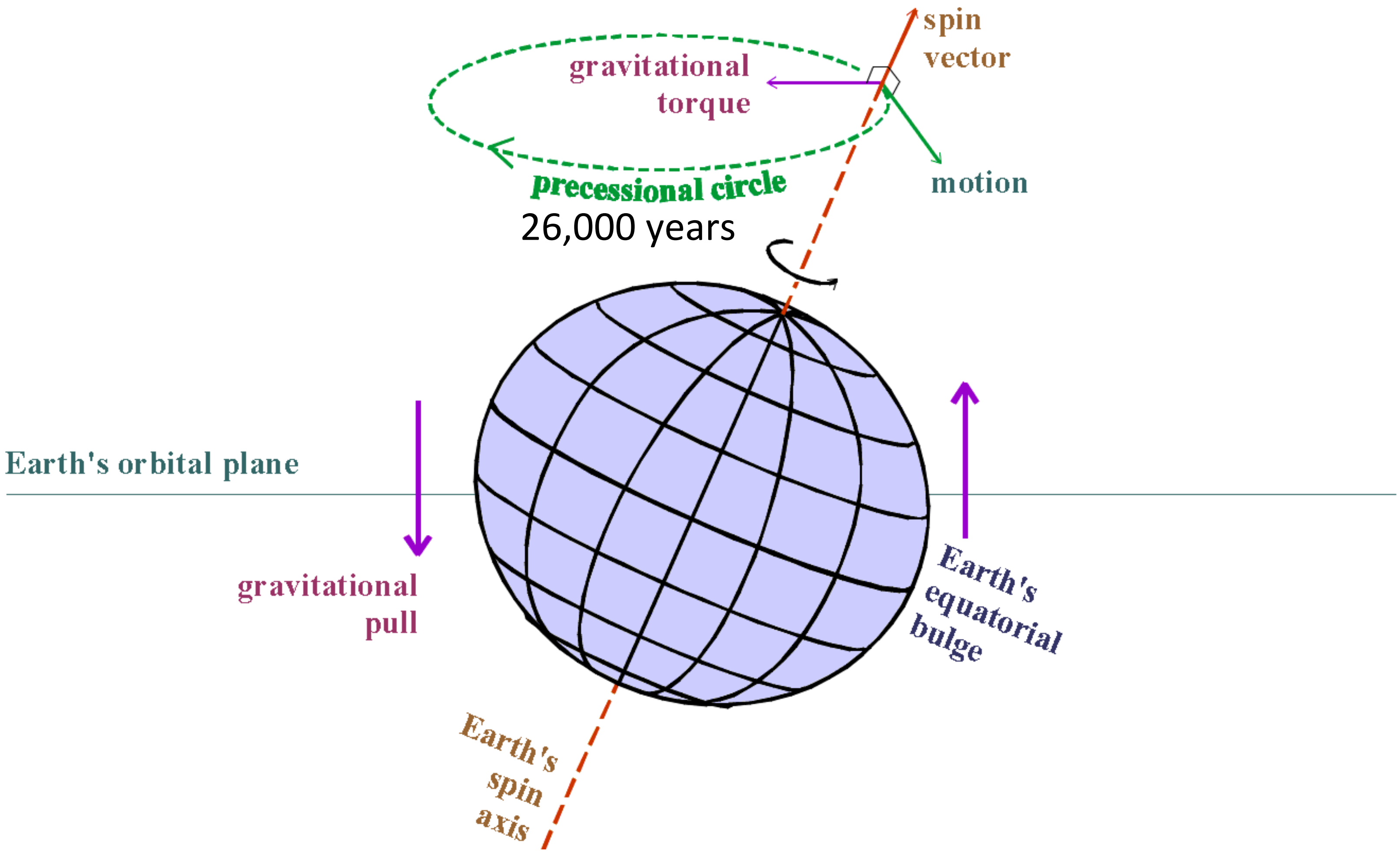
$$\vec{r}_c = Q \cdot \vec{r}$$

Precession



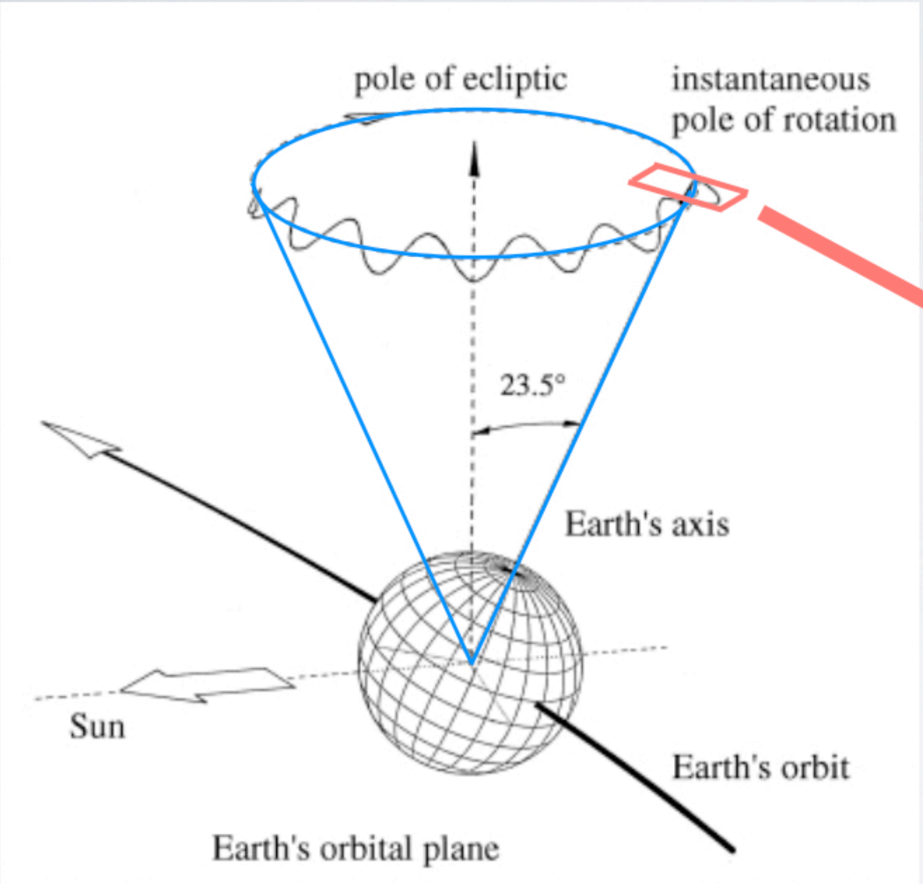
(Source: http://oceanworld.tamu.edu/students/iceage/images/precession_1.jpg)

Precession in detail

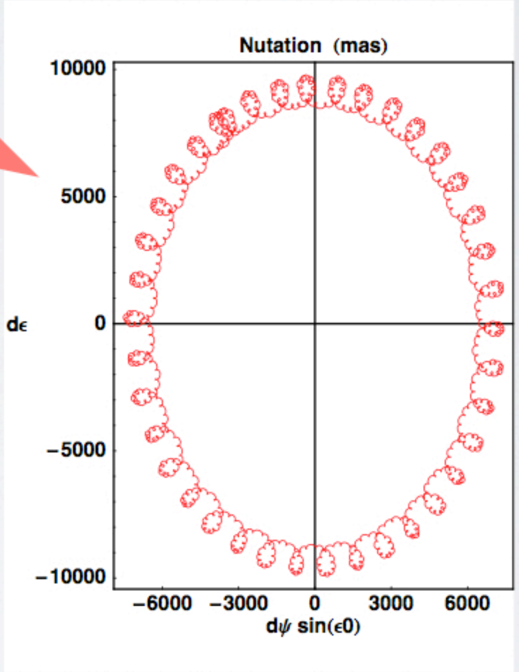


(Source http://star-www.st-and.ac.uk/~fv/webnotes/PRECESS1.GIF)

Precession / Nutation



Main cause: gravitational torque from the Moon, the Sun and the planets

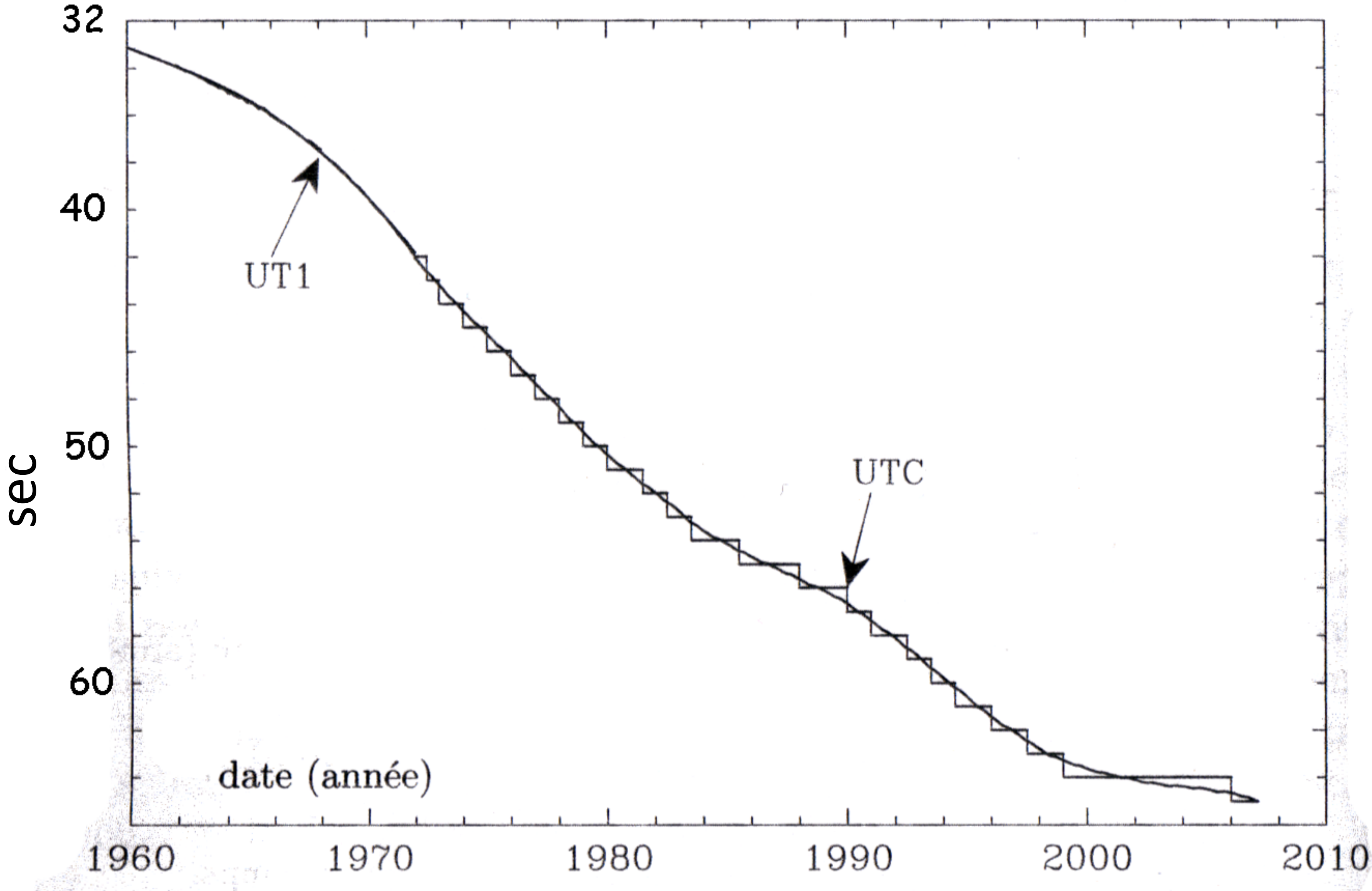


(source: https://www.ualberta.ca/~dumberry/nutation.jpg)

Universal Time (UT1)

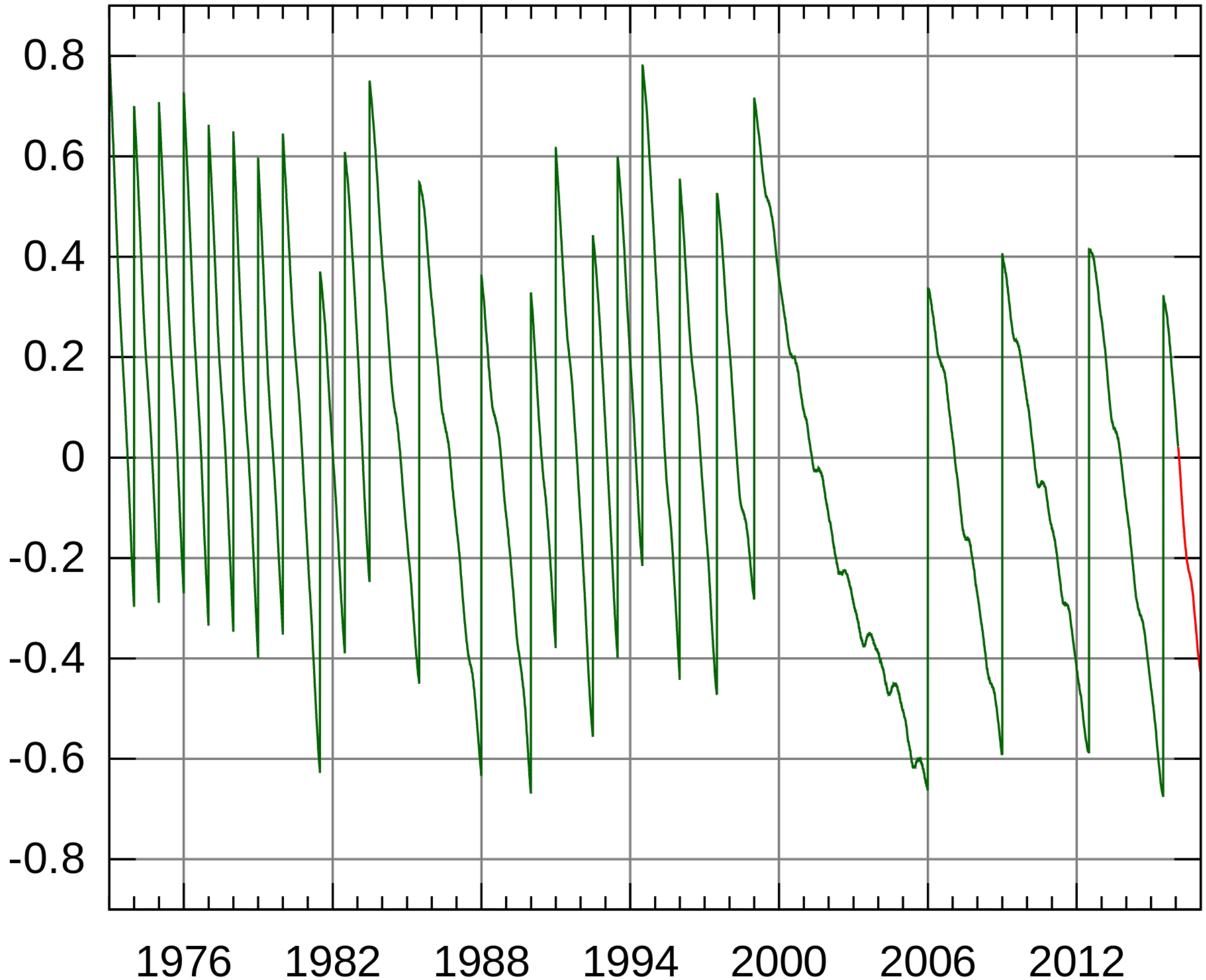
- UT1 is related to the Greenwich mean sidereal time (GMST) by a conventional relationship
- gives access to the direction of the ITRF zero-meridian in the ICRS.
- Is expressed as the difference
  - UT1-TAI or UT1-UTC

leap seconds !!



(source: https://www.imcce.fr/langues/en/grandpublic/systeme/promenade-en/images/gifa/a64.png)

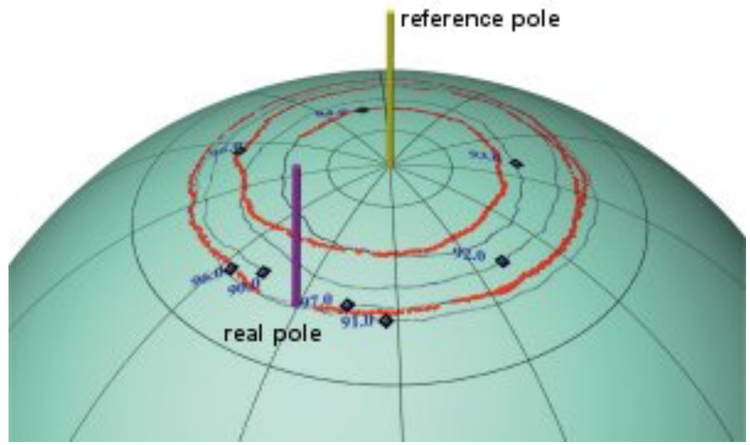
UT1 – UTC (in seconds)



(source: http://wikipedia.org)

Polar Motion

- motion of the rotation axis of the earth relative to the crust

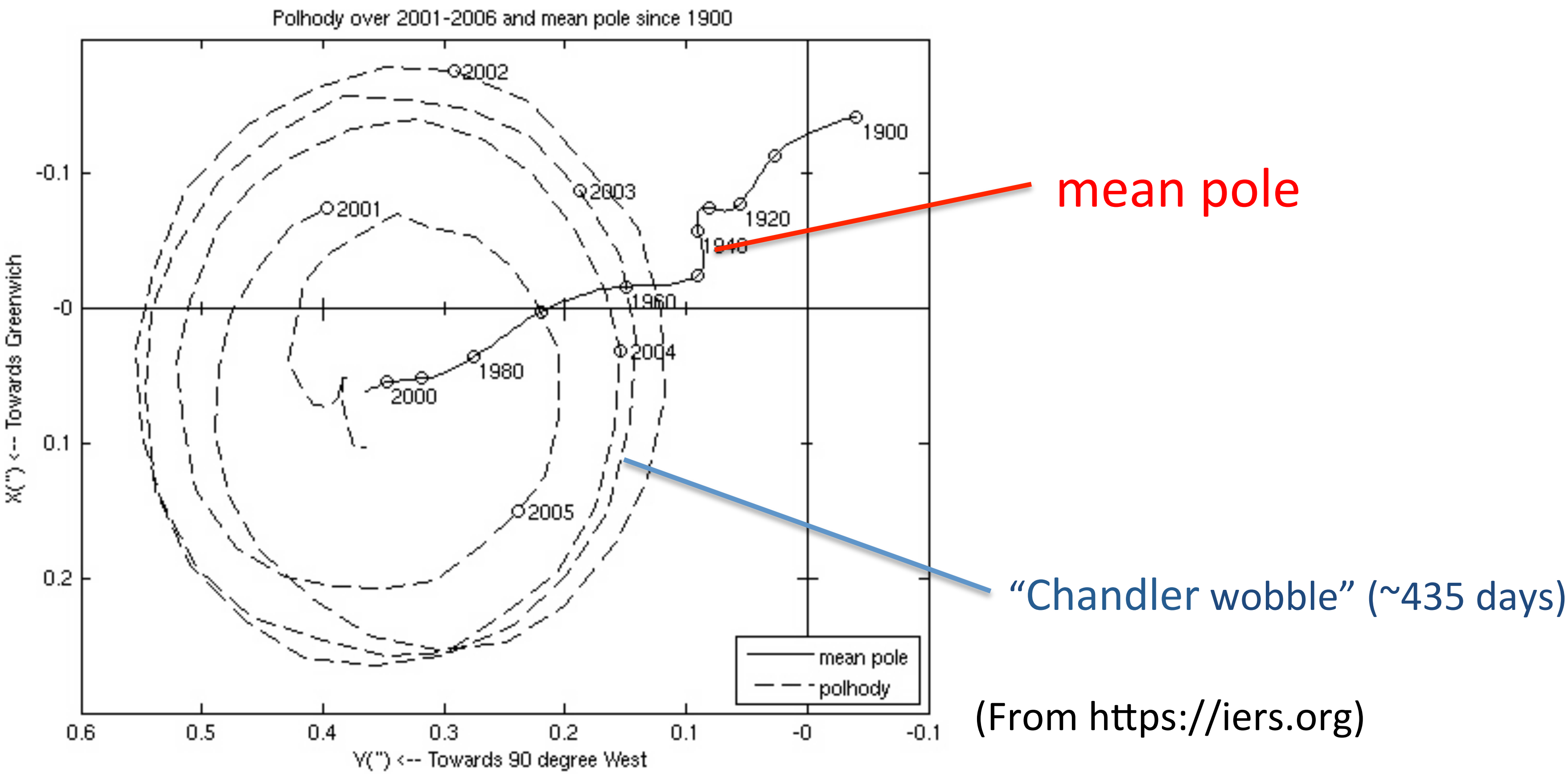


(source: http://www.aviso.altimetry.fr/uploads/pics/200112_pole_3d_uk.jpg)

- 3 major components
  - Long-term drift ("mean pole")
  - free oscillation with period about 435 days (Chandler wobble)
  - an annual oscillation forced by the seasonal displacement of air and water masses

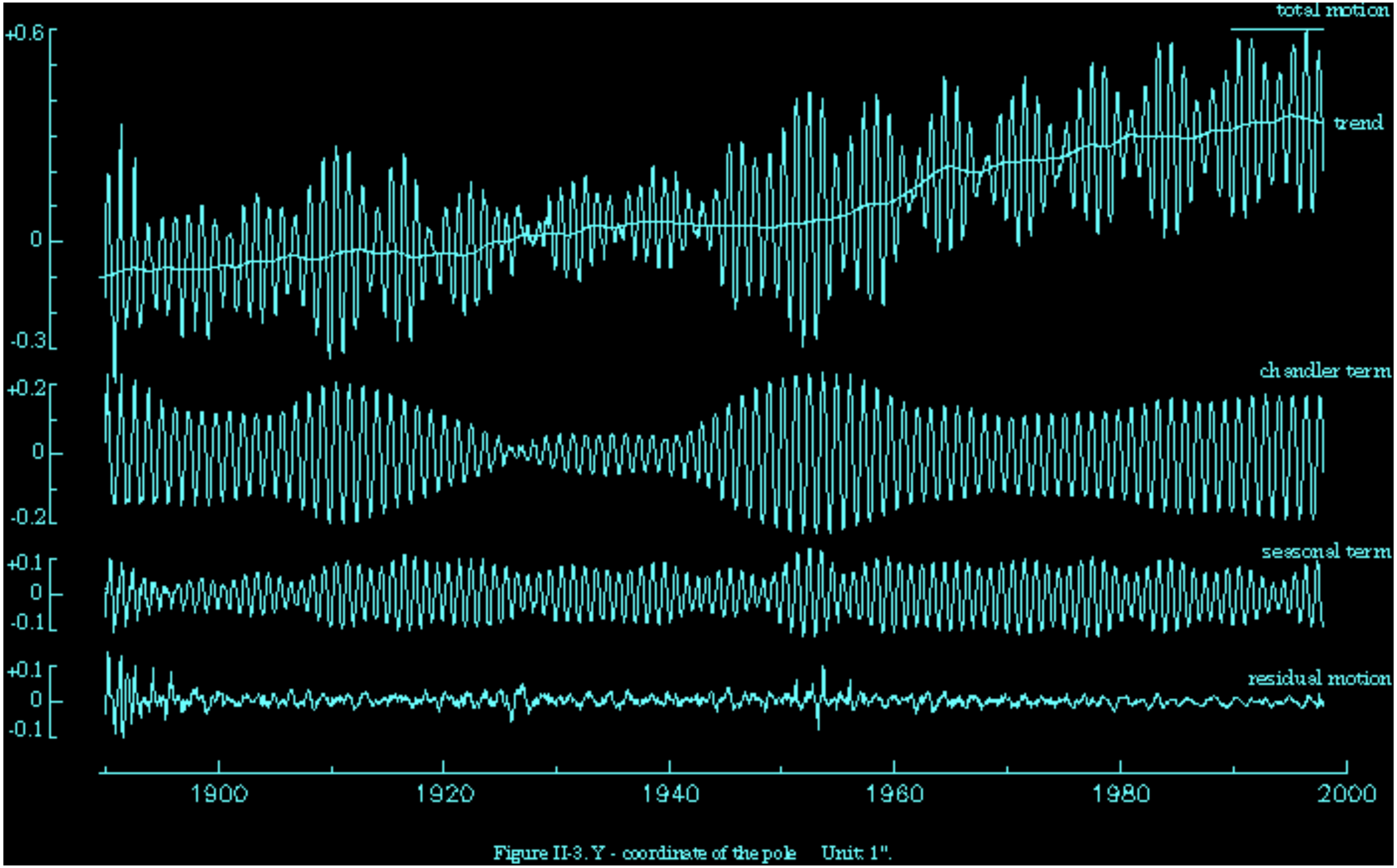
Polar Motion

Do you remember this sketch when we talked about pole tides?



(From https://iers.org)

Polar Motion (just the Y-component)



Earth orientation parameters (EOPs)

- EOPs describe the irregularities of the earth's rotation
- EOPs are the parameters which provide the rotation of the ITRS to the ICRS as a function of time
- EOPs include:
  - UT1 + the excess revolution time is called length of day (LOD).
  - Coordinates of the pole (x_p,y_p).
  - Celestial pole offsets

Disclaimer

- Many geophysical effects contribute to EOP variations and are available as models (tidal terms, elasticity, …)
- Not discussed here!

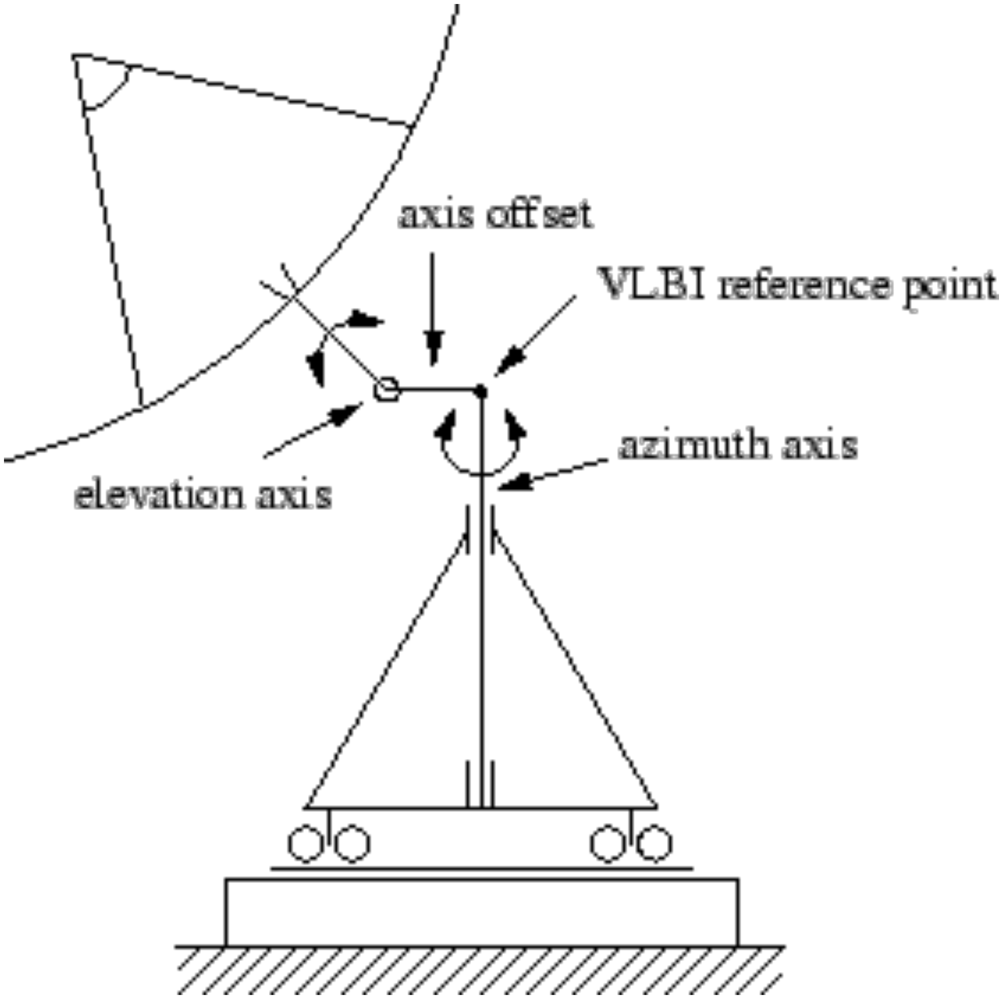
Are we done?

- We have both stations in the CRF (BCRS) ✔
- We can calculate the theoretical delay ✔

BUT

- Two more "geophysical" (rather mechanical) models that need to be applied as corrections
  - Antenna axis offsets
  - Antenna thermal deformations

Axis offset models – why?



VLBI reference point is defined as the <u>intersection of the telescope rotation axis</u> – we need to make sure that we consider axis offsets !

(source: http://ivs.nict.go.jp/mirror/publications/ar2004/ivs-ct/img3.gif)

Thermal deformation

- VLBI telescopes are mechanical structures with different materials (steel, aluminum, concrete, …)
- Environmental temperature changes
- expansion/contraction of structures, i.e. the antenna "deforms"
- Need to correct this by appropriate models

J Geod (2009) 83:787–792
DOI 10.1007/s00190-008-0284-z

SHORT NOTE

Conventions on thermal expansion modelling of radio telescopes for geodetic and astrometric VLBI

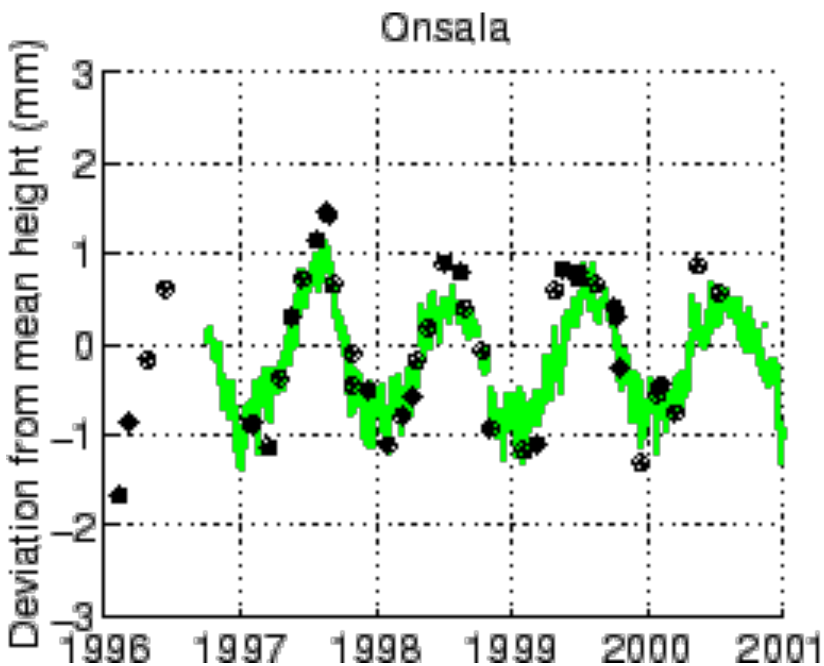
Axel Nothnagel

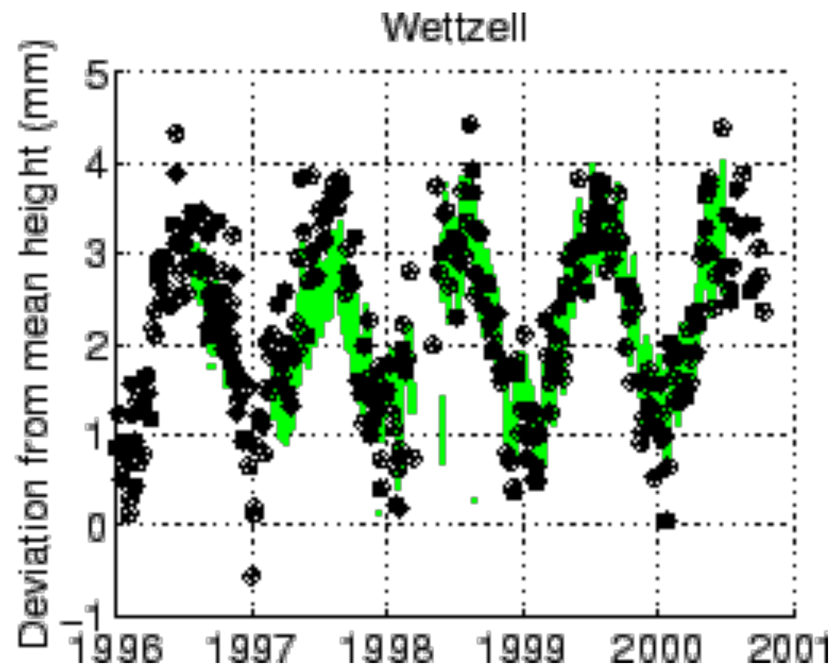
Thermal Deformation (examples)

Vertical height at Onsala and Wettzell

solid line - measured by the invar rod measuring systems;

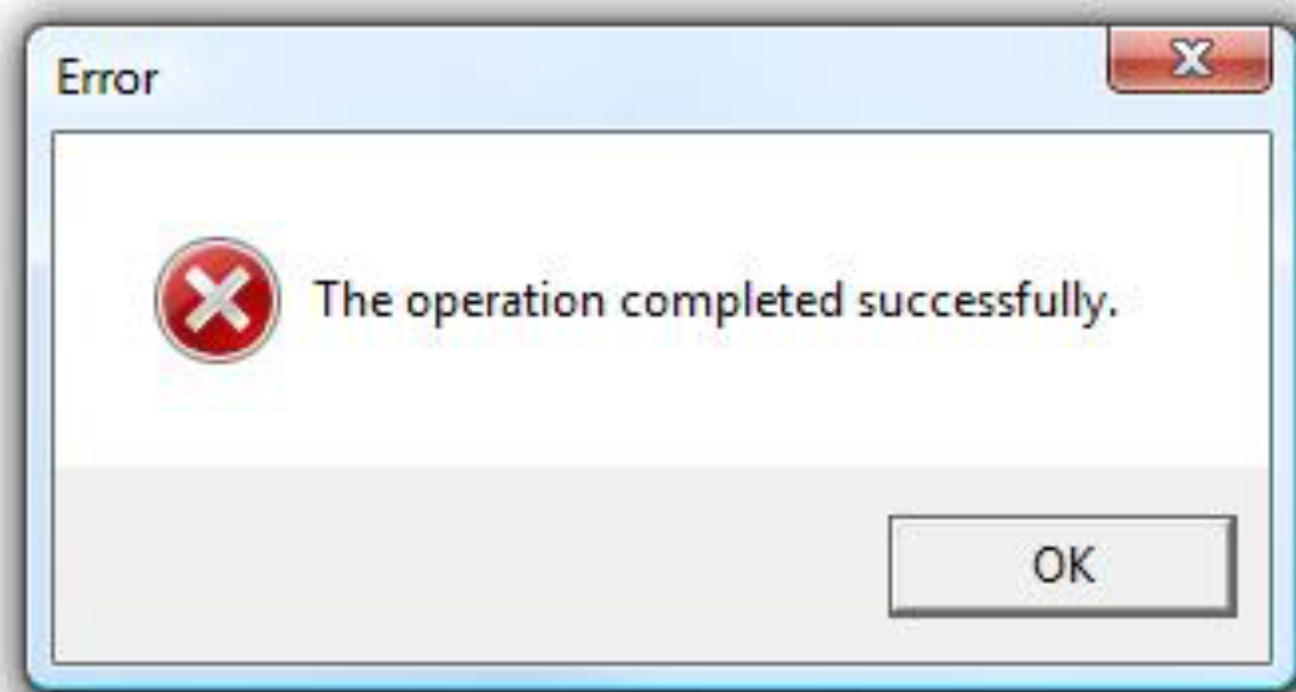
black points - a simple model based on daily mean temperature from the VLBI data base, thermal expansion coefficient, and the telescope dimensions.





(source: http://ivscc.gsfc.nasa.gov/publications/ar2000/acoso/)

We are done !



Recap: main tasks of geophysical modelling

1. Compute a-priori station positions
   a. Get coordinates in TRF at observation epoch
   b. Add (model) "deformations" at observation epoch
2. Transform TRF -> CRF
   1. Precession/Nutation
   2. Polar Motion
   3. UT1
3. Correct for delays due to antenna "imperfectness"
   1. Antenna axis offset
   2. Thermal deformation

Which models are recommended?

- See latest IERS Conventions (2010, i.e. IERS Technical Note No. 36)
  - http://www.iers.org/IERS/EN/Publications/TechnicalNotes/tn36.html
  - On that HP you will also find software tools which implement the models described in the conventions (and discussed here)
  - Contains all the references to the models

Thanks for listening!

Questions?

Now or mail me

(thomas.hobiger@chalmers.se)