Data analysis for geodesy

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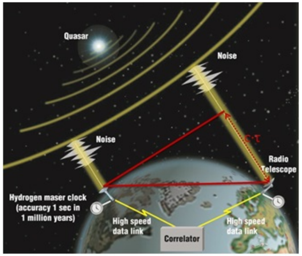
Scheduling

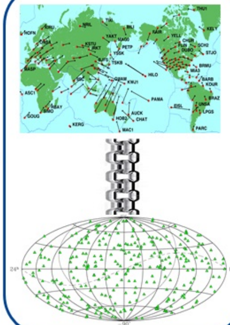
courtesy A. Iddink

Measurement & Correlator



Analysis Center

Solution



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Analyzing an Experiment

Estimation of Target Parameters

Global Solution and Combination

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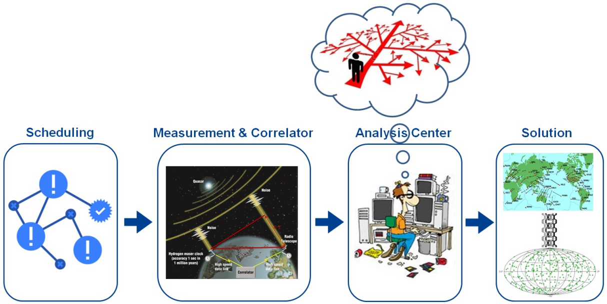
From Observations to Parameters

- given: delays with standard deviations ($\mathbf{b}$)
- we look for: station positions, ... ($\mathbf{x}$)

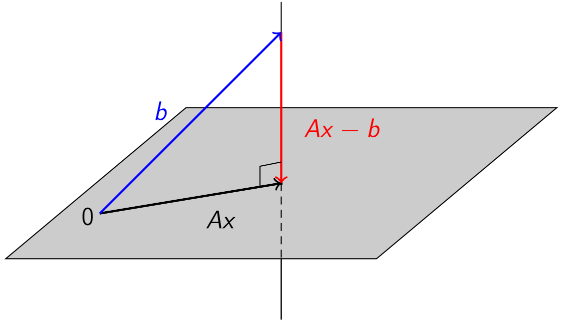


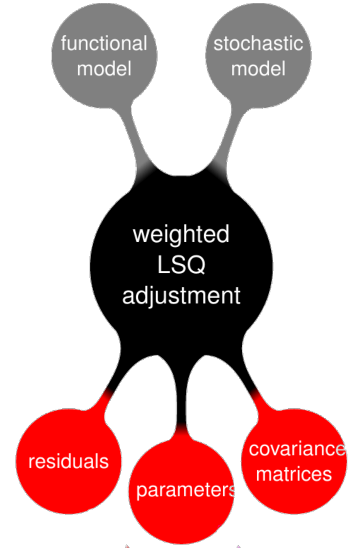
linear/linearized equation system

$$\mathbf{b} = \varphi(\mathbf{x}) = \mathbf{A} \cdot \mathbf{x}$$

- $\mathbf{A} = \frac{\partial \varphi(\mathbf{x})}{\partial \mathbf{x}}$
- a priori values necessary if equations are non-linear
- observations have to be weighted
- equation system overdetermined
- equation system singular

- Classical Least Squares
- Kalman Filter
- Least Squares Collocation
- ...



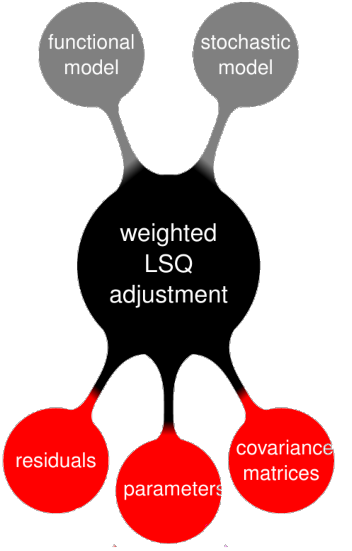


minimizing sum of squared residuals

gradient of $\|\mathbf{Ax}-\mathbf{b}\|_2^2 = \langle \mathbf{b}-\mathbf{Ax}\rangle$ vanishes $\Rightarrow 0 = \mathbf{A}^T\mathbf{r} = \mathbf{A}^T\mathbf{Ax} - \mathbf{A}^T\mathbf{b}$

$\mathbf{r} = \mathbf{Ax} - \mathbf{b}$ idea: $\sum r_i^2 \rightarrow min$

$$\Rightarrow (\mathbf{A}^T\mathbf{\Sigma}_{bb}^{-1}\mathbf{A})\mathbf{x} = \mathbf{A}^T\mathbf{\Sigma}_{bb}^{-1}\mathbf{b}$$



covariance matrices

$$\boldsymbol{\Sigma}_{xx} = \left(\mathbf{A}^T\mathbf{A}\right)^{-1}$$

$$\boldsymbol{\Sigma}_{rr} = \mathbf{I} - \mathbf{A}\boldsymbol{\Sigma}_{xx}\mathbf{A}^T$$

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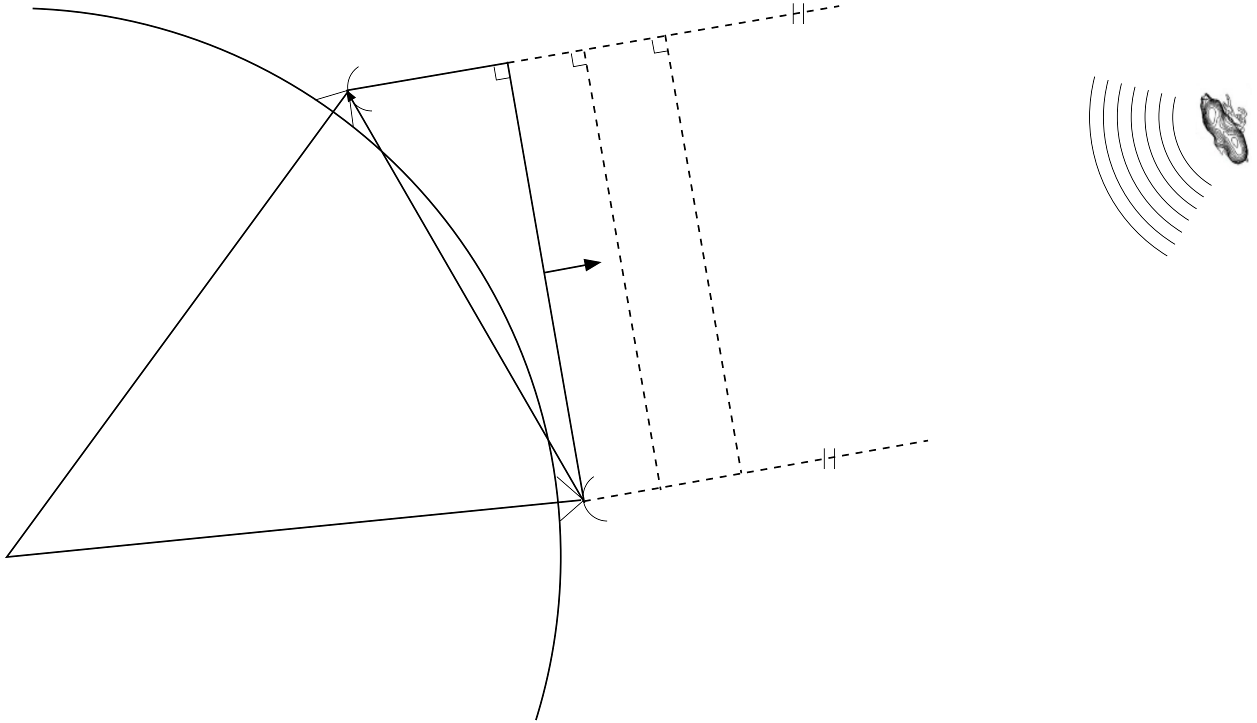
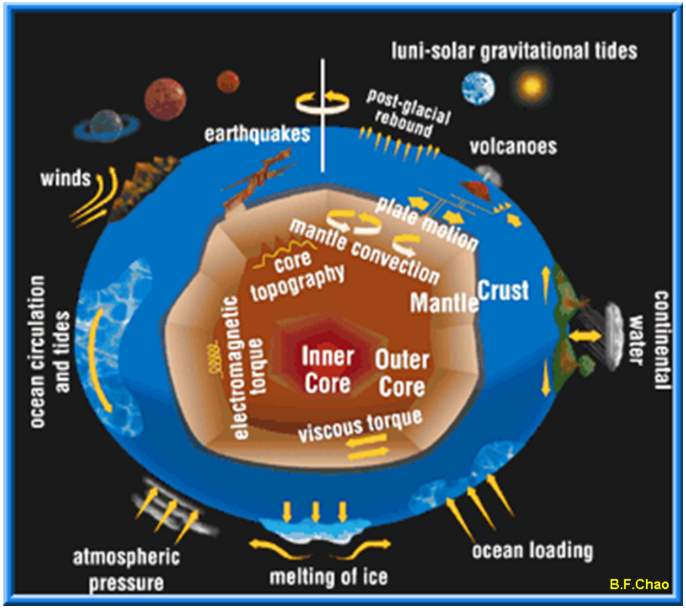
observation equation

$$\mathbf{b} = \varphi(\mathbf{x})$$



Baseline at current epoch



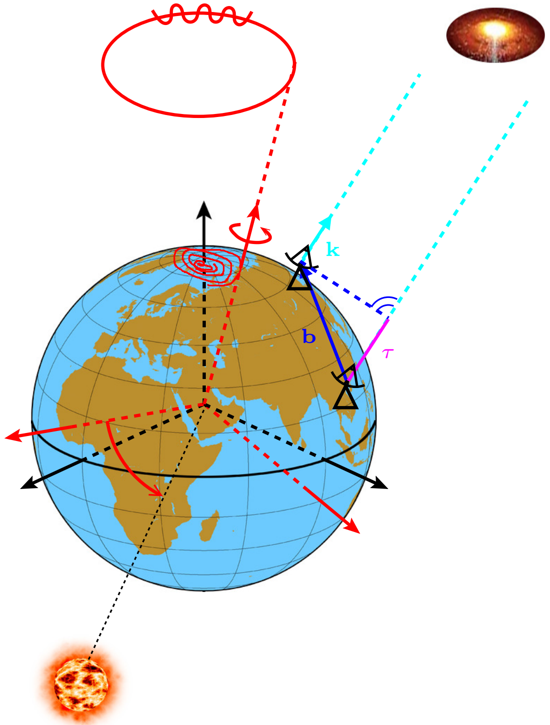
http://geodesy.agu.org

IERS Conventions

- tectonics
- solid Earth tides
- ocean loading
- pole tide loading
- deformation due to polar motion
- tidal atmospheric pressure loading

further effects

- thermal expansion
- non-linear station motions
- non-tidal atmospheric pressure loading
- hydrological loading

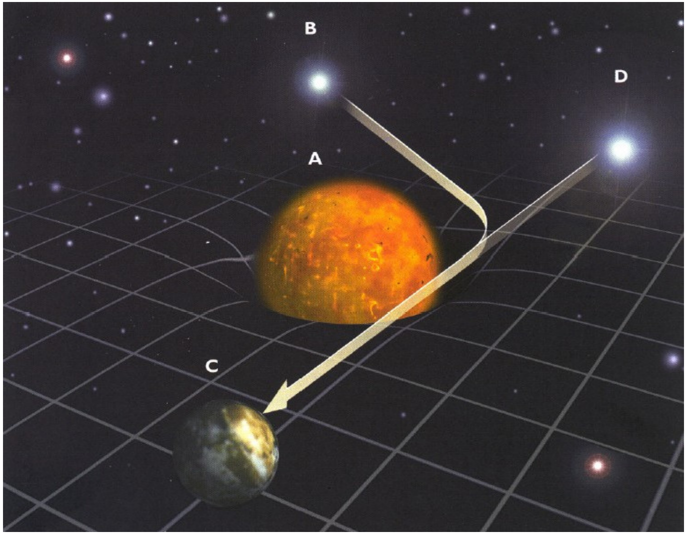


station in GCRS

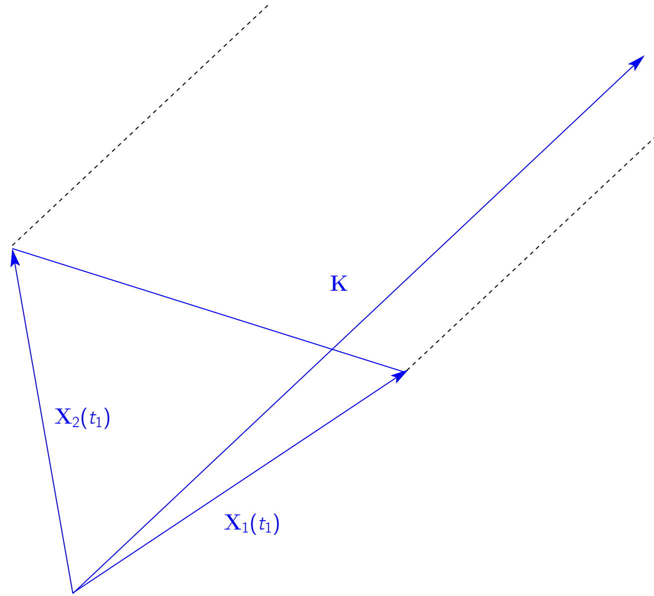
$$\mathbf{R} = \mathbf{W} \cdot \mathbf{S} \cdot \mathbf{Q}$$

$$\Rightarrow \mathbf{x}_i(t) = \mathbf{R}^T \mathbf{r}_i(t)$$

Relativistic Effects



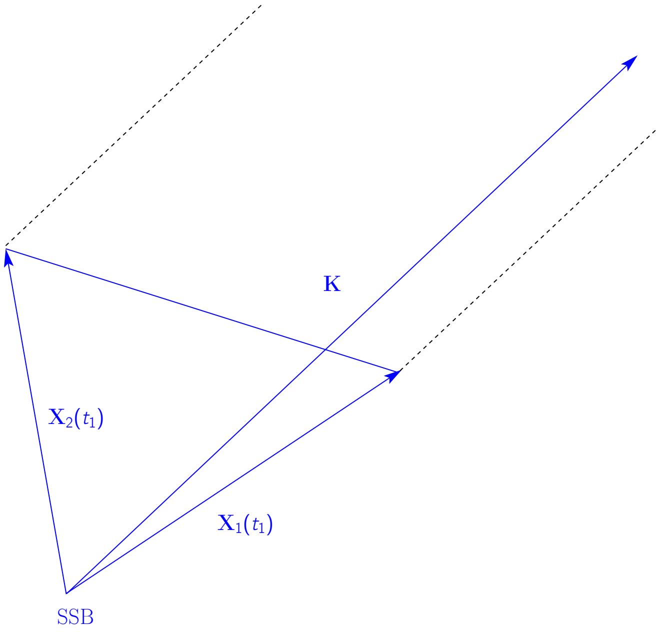
https://schwertasblog.files.wordpress.com

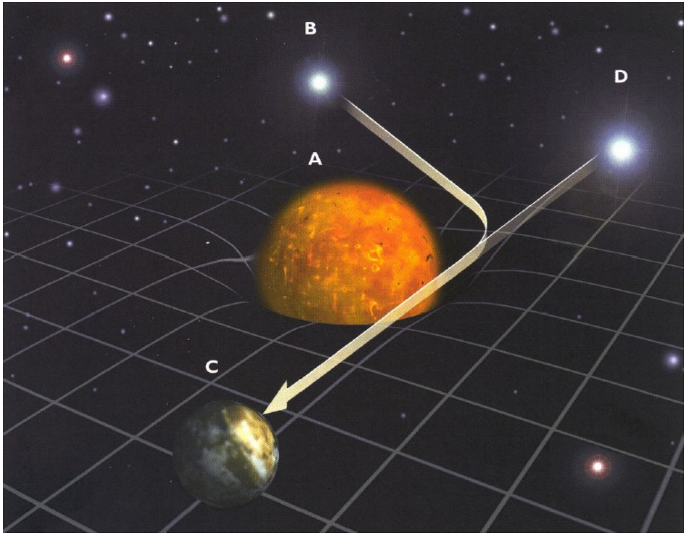


account for the motion of station 2 during the propagation time between station 1 and station 2.

$$\mathbf{X}_i(t_1) = \mathbf{X}_\oplus(t_1) + \mathbf{x}_i(t_1),$$

$$\mathbf{X}_2(t_2) = \mathbf{X}_2(t_1) - \frac{\mathbf{V}_\oplus}{c}(\hat{K} \cdot \mathbf{b})$$





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general relativistic delay, ΔT_{grav}, for the J^{th} gravitating body

$$\Delta T_{grav_J} = 2\frac{GM_J}{c^3} \ln \frac{|\mathbf{R}_{1_J}| + \mathbf{K} \cdot \mathbf{R}_{1_J}}{|\mathbf{R}_{2_J}| + \mathbf{K} \cdot \mathbf{R}_{2_J}}.$$

delay due to the Earth, $\Delta T_{grav_\oplus}$

$$\Delta T_{grav_\oplus} = 2\frac{GM_\oplus}{c^3} \ln \frac{|\mathbf{x}_1| + \mathbf{K} \cdot \mathbf{x}_1}{|\mathbf{x}_2| + \mathbf{K} \cdot \mathbf{x}_2}.$$

total gravitational delay is the sum over all gravitating bodies including the Earth

$$\Delta T_{grav} = \sum_J \Delta T_{grav_J}.$$

1. estimate the barycentric station vector for receiver 1

$$\mathbf{X}_i(t_1) = \mathbf{X}_\oplus(t_1) + \mathbf{x}_i(t_1)$$

2. estimate the vectors from the Sun, the Moon, and each planet except the Earth to receiver 1.

$$t_{1_J} = \min\left[t_1, t_1 - \frac{\hat{K}\cdot(\mathbf{X}_J(t_1) - \mathbf{X}_1(t_1))}{c}\right]$$

$$\Rightarrow \mathbf{R}_{1_J}(t_1) = \mathbf{X}_1(t_1) - \mathbf{X}_J(t_{1_J}), \qquad \mathbf{R}_{2_J} = \mathbf{X}_2(t_1) - \frac{\mathbf{V}_\oplus}{c}(\hat{K}\cdot\mathbf{b}) - \mathbf{X}_J(t_{1_J}).$$

3. estimate the differential gravitational delay for each of those bodies.

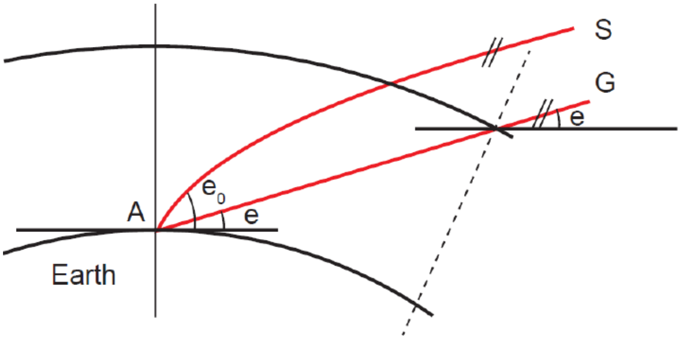
$$\Delta T_{grav_J} = 2\frac{GM_J}{c^3}\ln\frac{|\mathbf{R}_{1_J}| + \mathbf{K}\cdot\mathbf{R}_{1_J}}{|\mathbf{R}_{2_J}| + \mathbf{K}\cdot\mathbf{R}_{2_J}}.$$

4. find the differential gravitational delay due to the Earth.

$$\Delta T_{grav_\oplus} = 2\frac{GM_\oplus}{c^3}\ln\frac{|\mathbf{x}_1| + \mathbf{K}\cdot\mathbf{x}_1}{|\mathbf{x}_2| + \mathbf{K}\cdot\mathbf{x}_2}.$$

5. Sum to find the total differential gravitational delay $\Delta T_{grav} = \sum_J \Delta T_{grav_J}$
6. Compute the vacuum delay

$$t_{v_2} - t_{v_1} = \frac{\Delta T_{grav} - \frac{\hat{K}\cdot\mathbf{b}}{c}\left[1 - \frac{(1+\gamma)U}{c^2} - \frac{|\mathbf{V}_\oplus|^2}{2c^2} - \frac{\mathbf{V}_\oplus\cdot\mathbf{w}_2}{c^2}\right] - \frac{\mathbf{V}_\oplus\cdot\mathbf{b}}{c^2}(1 + \hat{K}\cdot\mathbf{V}_\oplus/2c)}{1 + \frac{\hat{K}\cdot(\mathbf{V}_\oplus+\mathbf{w}_2)}{c}}.$$



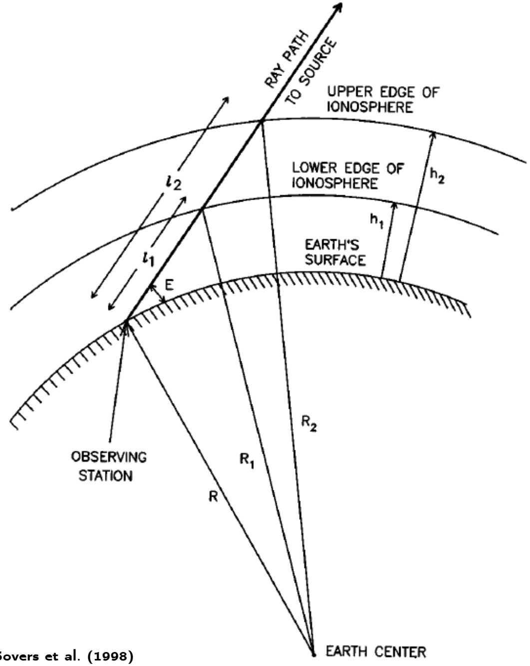
Nilsson et al. (2013)

$$\delta t_{trop} = 10^{-6} \int_S N ds$$
$$= 10^{-6} \int_S (N_h(T,p) + N_w(T,p,e)) ds$$
$$\delta t_{trop}(\epsilon) = \delta t_{at,h} \cdot m_h(\epsilon) + \delta t_{at,w} \cdot m_w(\epsilon)$$

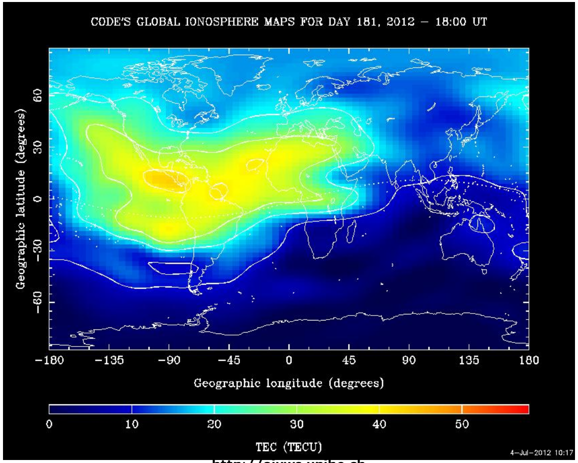
$$\delta t_{atm_i} = m_h(\epsilon) \frac{0.0022768 \cdot p}{1 - 0.00266 \cdot \cos(2\phi) - 0.28 \cdot 10^{-6} h} + m_g(\epsilon) \cdot \cot\epsilon \, (G_N \cos\alpha + G_e \sin\alpha)$$

total delay

$$t_2 - t_1 = t_{v_2} - t_{v_1} + \delta t_{atm_1} \frac{\hat{K} \cdot (\mathbf{w}_2 - \mathbf{w}_1)}{c} + (\delta t_{atm_2} - \delta t_{atm_1})$$



Sovers et al. (1998)

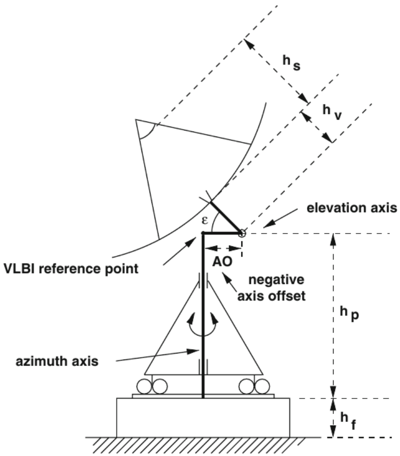


http://aiuws.unibe.ch

$$\tau_{ion,X} = (\tau_X - \tau_s)\frac{f_S^2}{f_X^2 - f_S^2}$$

$$\sigma_{\tau_{ion,X}} = \sqrt{\sigma_{\tau X}^2 + \sigma_{\tau s}^2}\,\frac{f_S^2}{f_X^2 - f_S^2}$$

Instrumental delays



Nothnagel (2009)

Contribution of axis offset

$$\Delta t_i = \frac{1}{c} AO \sqrt{1 - \left(\hat{K} \cdot \mathbf{I}\right)^2}$$

Thermal expansion

$$\Delta t_i = \frac{1}{c}[\gamma_f \cdot (T(t - \Delta t_f) - T_0) \cdot (h_f \cdot \sin\epsilon) + \gamma_a \cdot (T(t - \Delta t_a) - T_0) \cdot (h_p \cdot \sin\epsilon + AO \cdot \cos\epsilon + h_v - F_a \cdot h_s)]$$

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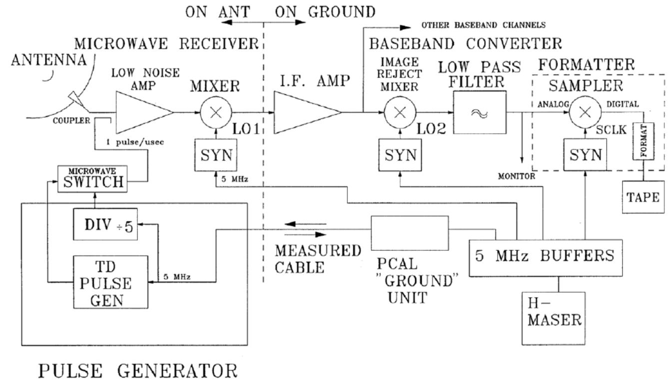
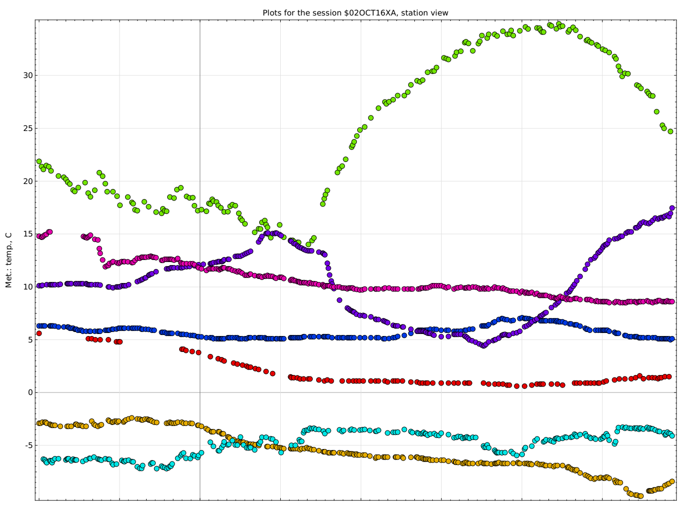
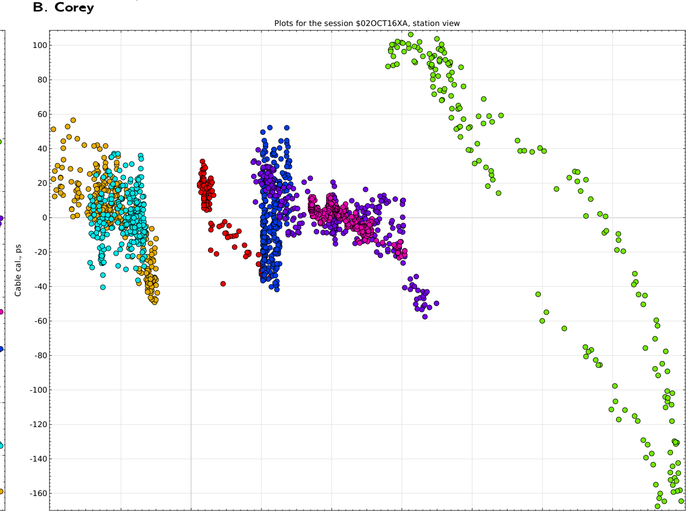
http://media.liveauctiongroup.net



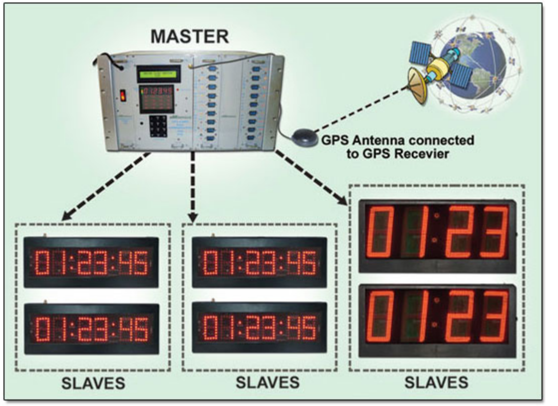
B. Corey





$$\mathbf{\Delta b} = \mathbf{b} - ((t_2 - t_1) + \tau_{ion} + \Delta\tau_{axis} + \Delta\tau_{therm} + \Delta\tau_{cable}) + \tau_{cl}$$
$$\mathbf{\Delta b} = \mathbf{A} \cdot \mathbf{\Delta x}$$

$$\tau_{cl} = \tau_{cl}^{b} - \tau_{cl}^{a}$$
$$\tau_{cl}^{a/b} = cl_0^{a/b} + cl_1^{a/b} \cdot (T_i - T_0) + cl_2^{a/b} \cdot (T_i - T_0)^2$$

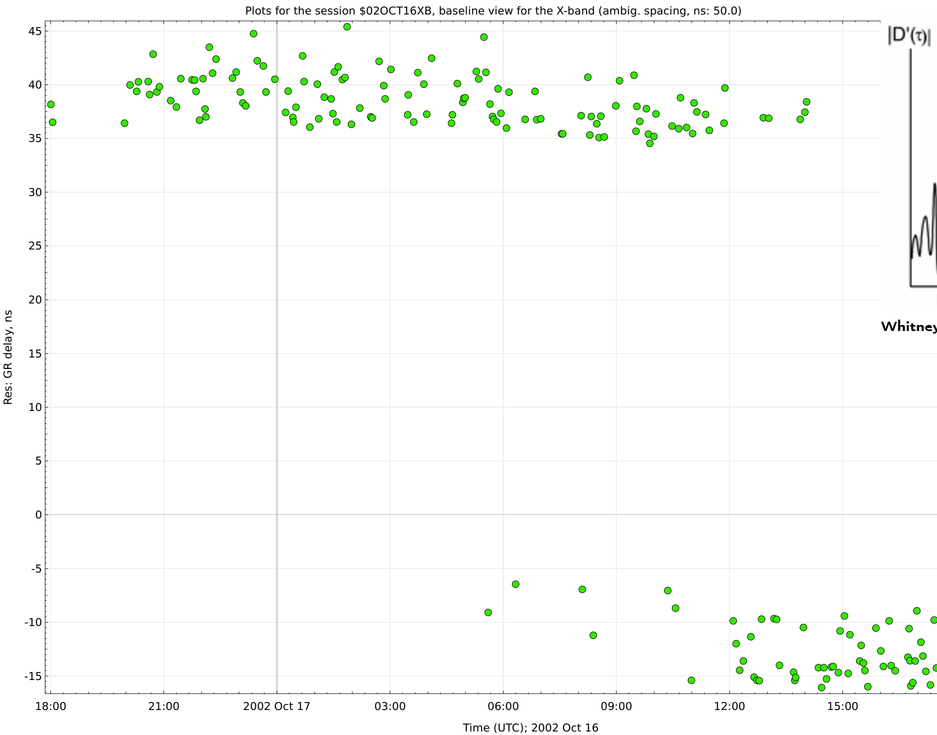


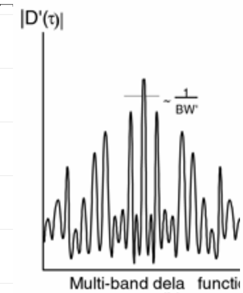
http://www.efftronics.com

rank deficiency

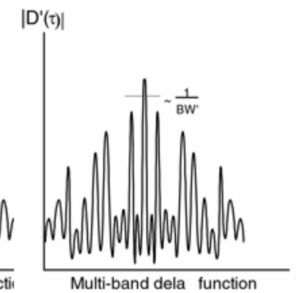
delays provide only relative information on clock behavior

$$\mathbf{x} = \begin{pmatrix} cl_0^i & cl_1^i & cl_2^i \end{pmatrix}$$





Whitney (2000)



Whitney (2000)

Ambiguity spacing can be determined from channels in bandwidth synthesis

Zenith Wet Delays & Trop. Gradients

wet part

estimated ⇒ partial derivatives

$$\tau_{at,w} = at \cdot m_w(\epsilon)$$

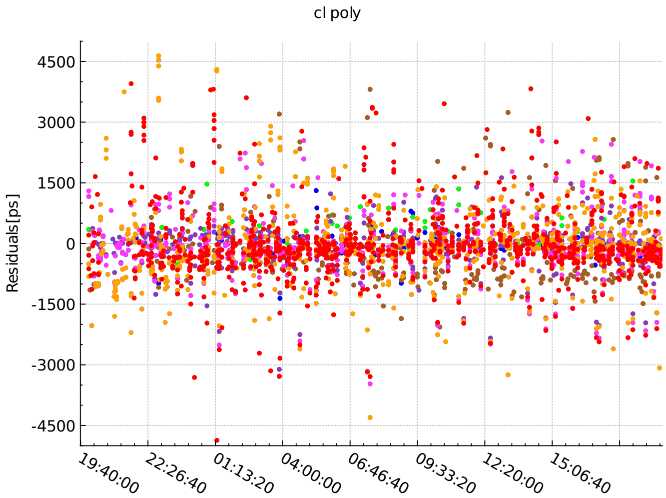
$$\frac{\partial \tau}{\partial at} = \frac{\partial \tau_{at,w}}{\partial at} = m_w(\epsilon)$$

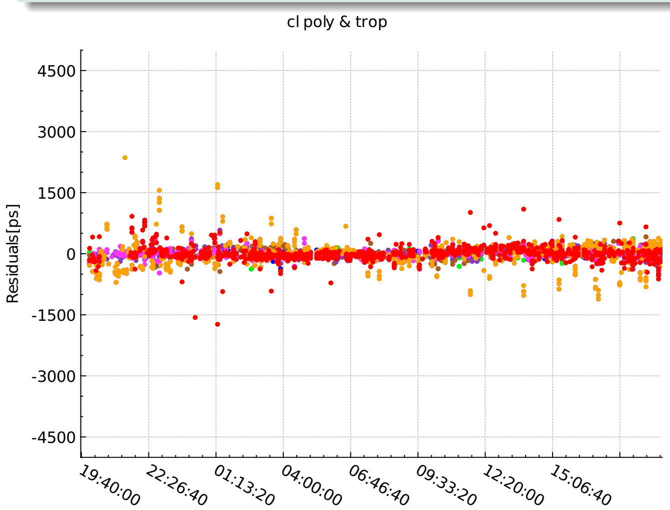
troposphere gradients

azimuthal asymmetries ⇒ estimation of troposphere gradients in north-south and east-west direction

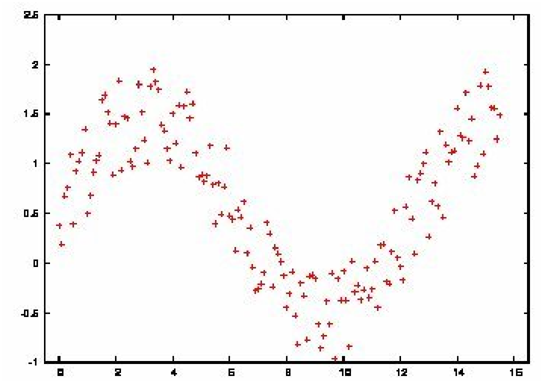
$$\frac{\partial \tau}{\partial NG} = \frac{\cos\alpha}{\tan\epsilon \cdot \sin\epsilon + 0.0032}$$

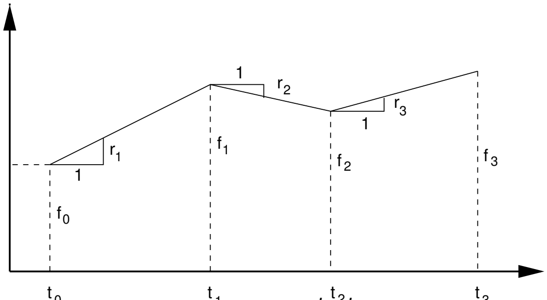
$$\frac{\partial \tau}{\partial EG} = \frac{\sin\alpha}{\tan\epsilon \cdot \sin\epsilon + 0.0032}$$





Parameterization Refinement





$$\frac{\partial f}{\partial f_{i-1}} = \begin{cases} 1 - \frac{t - t_{i-1}}{t_i - t_{i-1}} \\ 0 \end{cases}$$

<u>CPWLF:</u>

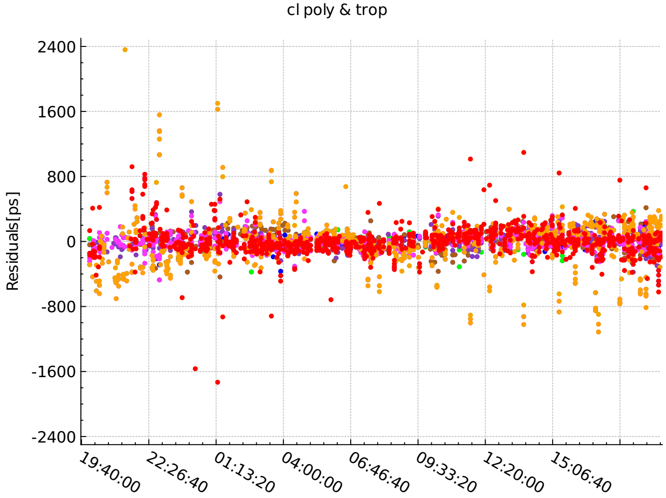
$$f(t) = f(t_0) + r_1(t_1 - t_0) + r_2(t_2 - t_1) + \ldots + r_n(t - t_{n-1})$$

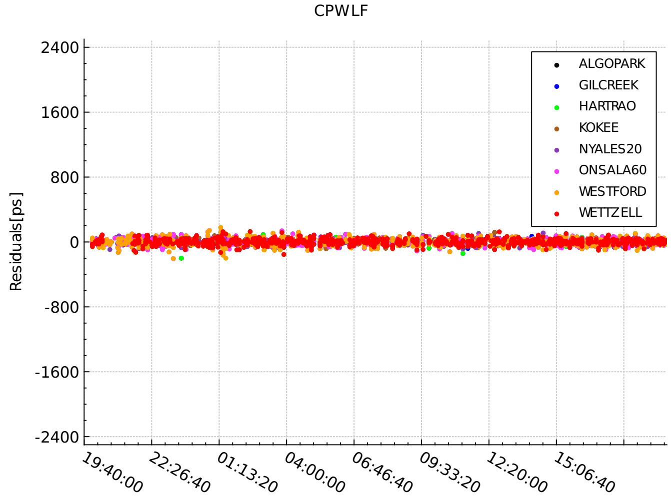
$$r_i = \frac{f(t_i) - f(t_{i-1})}{t_i - t_{i-1}}$$

$$f(t) = f(t_0) + \frac{f(t_1) - f(t_0)}{t_1 - t_0}(t_1 - t_0) + \frac{f(t_2) - f(t_1)}{t_2 - t_1}(t_2 - t_1) + \ldots$$

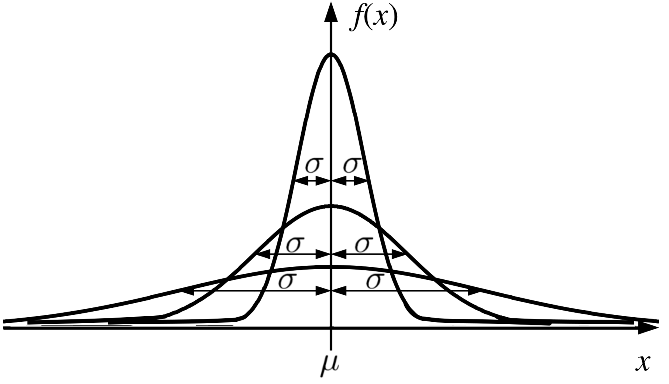
$$\frac{\partial f}{\partial f_i} = \begin{cases} \frac{t - t_{i-1}}{t_i - t_{i-1}} & \text{for } t_i < t < t_{i+1} \\ 0 & \text{all other cases} \end{cases}$$

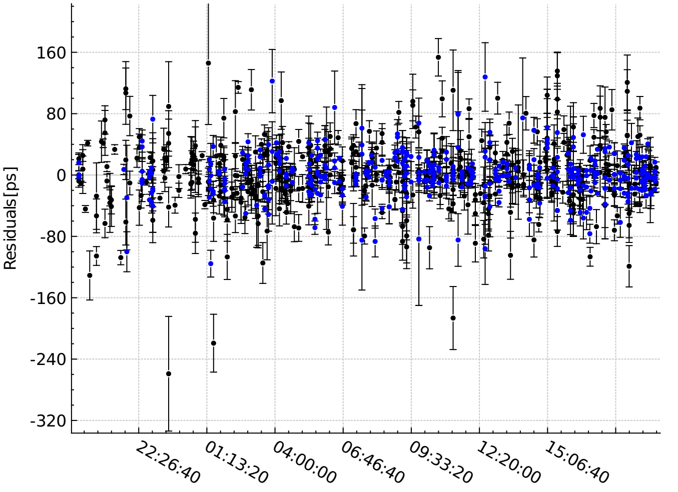
Inserting CPWLF





Removing Outliers & Reweighting





Outliers

$$\tau = \frac{r_i}{\sigma_0 \sqrt{red_i}} \sim N(0,1)$$

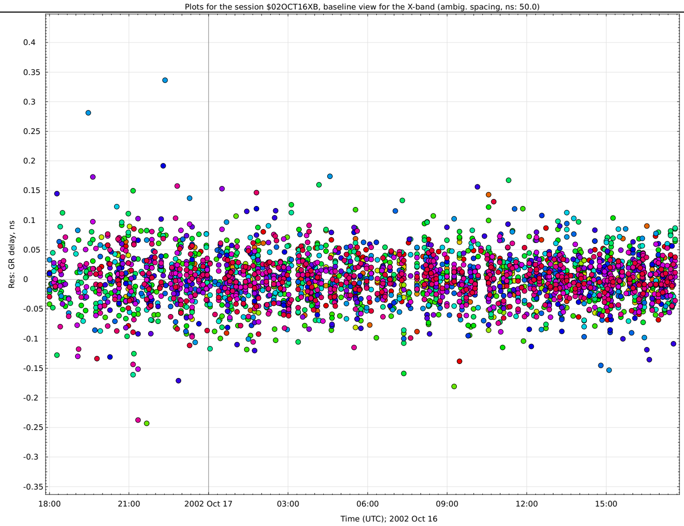
Re-weighting

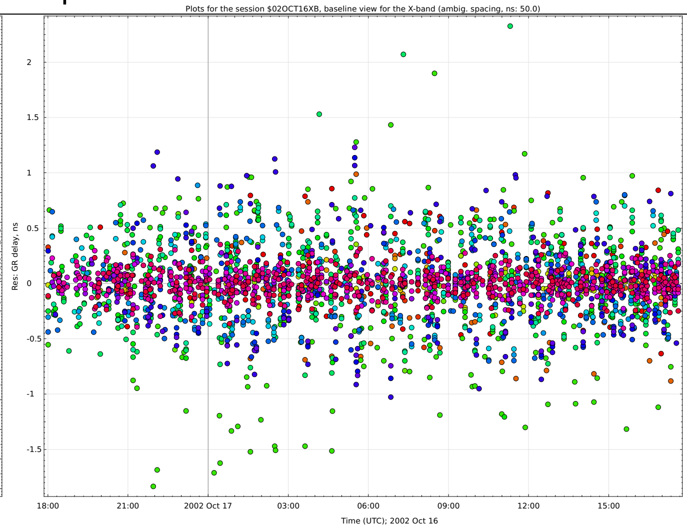
e.g. variance component estimation

$$\hat{\sigma}_0^2 = \frac{\mathbf{r}^T \mathbf{W} \mathbf{r}}{red}$$

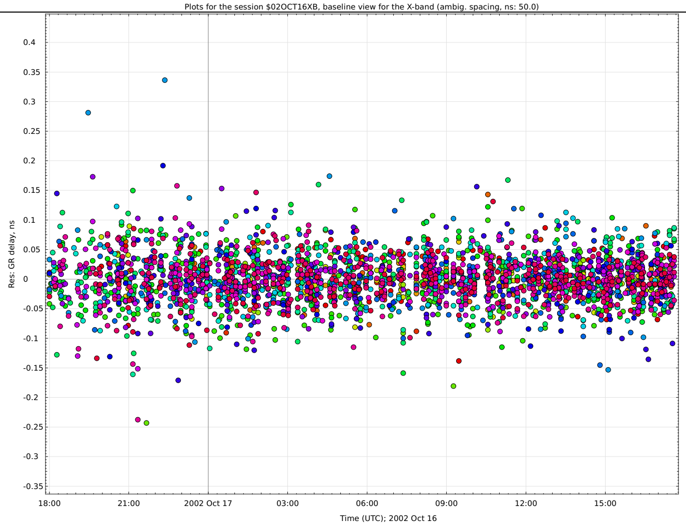
$$\hat{\sigma}_{0i}^2 = \frac{\mathbf{r}_i^T \mathbf{W}_i \mathbf{r}_i}{red_{gi}}$$

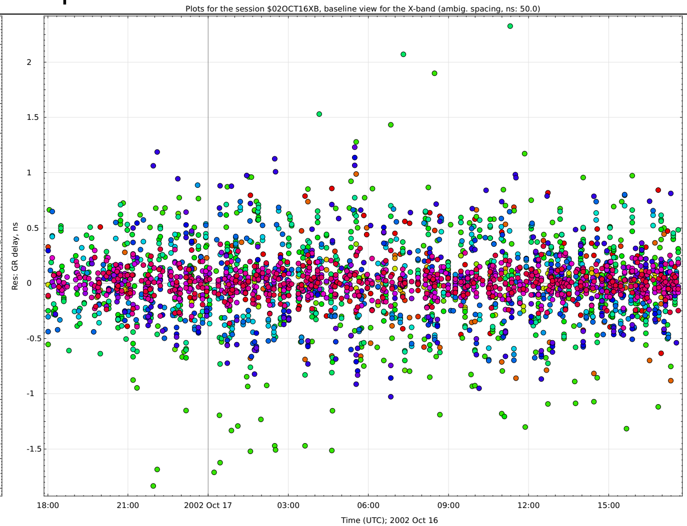
Insufficient a prioris

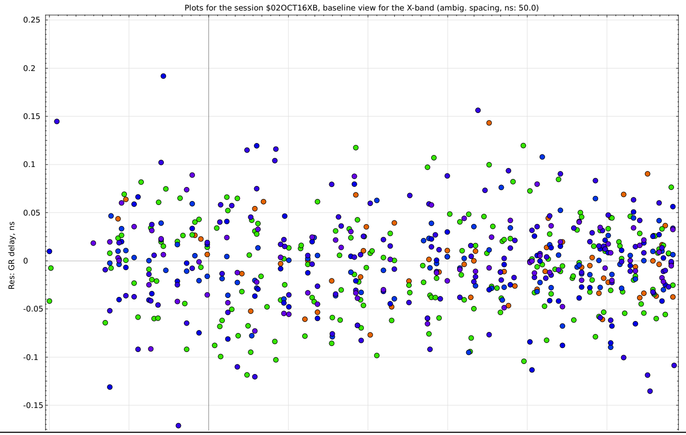


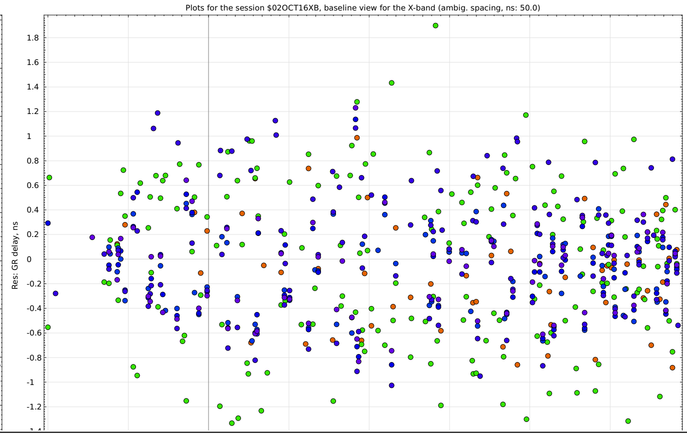


Insufficient a prioris









Initial solution

1. estimate clock polynomial
2. resolve ambiguities
3. calculate ionosphere correction
4. estimate clocks and ZWDs: CPWLF with 300 min resolution
5. find and remove outliers
6. estimate clocks and ZWDs: CPWLF with 60 min resolution and 24 h troposphere gradients
7. find and remove outliers or possibly restore earmarked observations
8. export V4 DB

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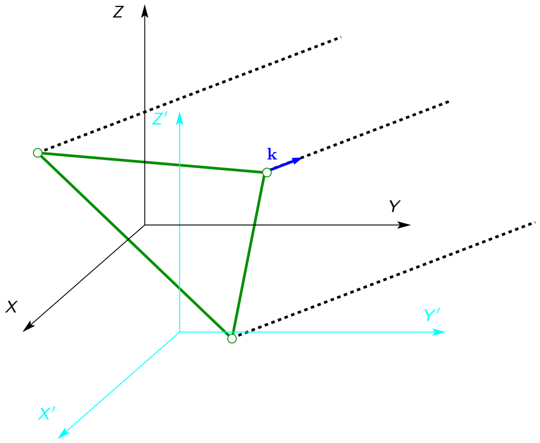
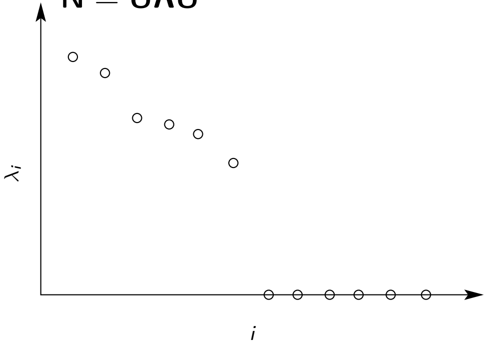
Station Positions



$$\mathbf{N} = \mathbf{A}^T \boldsymbol{\Sigma}_{yy}^{-1} \mathbf{A}$$

$$\mathbf{N} = \mathbf{U} \boldsymbol{\Lambda} \mathbf{U}^T$$

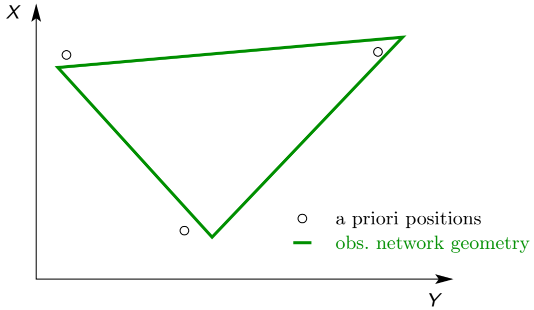


condition: optimal station estimates $\Rightarrow$ minimizing $trace(\boldsymbol{\Sigma}_{xx})$

$$\boldsymbol{\Sigma}_{xx} = \mathbf{N}^{-1}$$

$$\boldsymbol{\Sigma}_{xx} = \sum_{i=1}^{m} \frac{1}{\lambda_i} \mathbf{u}_i \cdot \mathbf{u}_i^T, \quad m : \#\lambda \neq 0$$

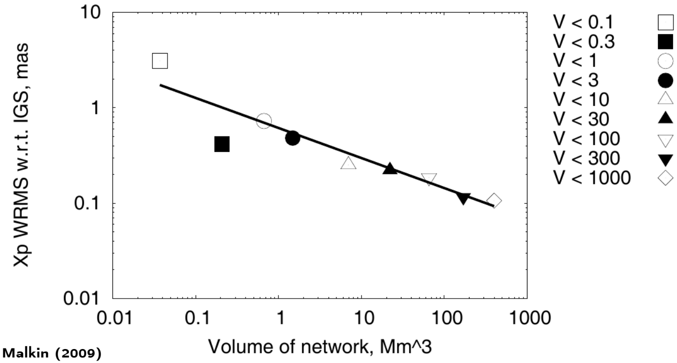
2D example



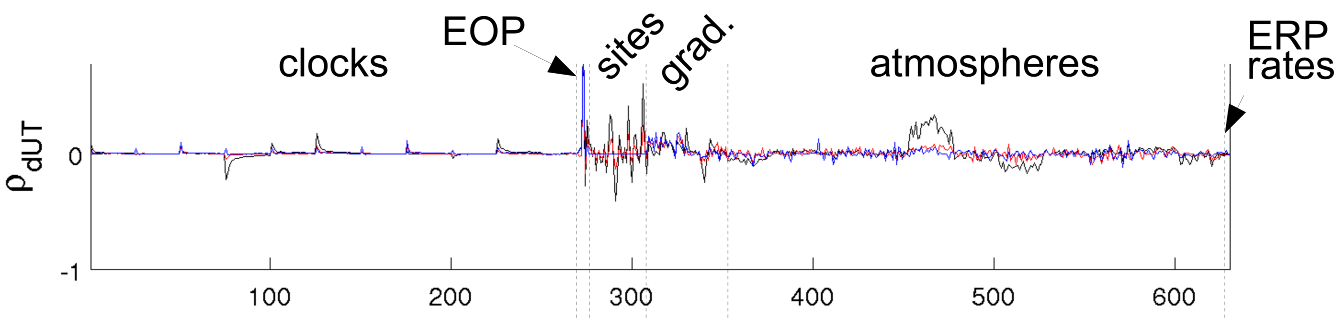
NNR/NNT condition: helmert parameter = 0

$$\mathbf{B}_i = \begin{pmatrix} 1 & 0 & 0 & 0 & -Z_i & Y_i \\ 0 & 1 & 0 & Z_i & 0 & -X_i \\ 0 & 0 & 1 & -Y_i & X_i & 0 \end{pmatrix}$$

$$\mathbf{A} = \begin{pmatrix} \dots & \frac{\partial \tau}{\partial X_1} & \frac{\partial \tau}{\partial Y_1} & \frac{\partial \tau}{\partial Z_1} & \dots & \frac{\partial \tau}{\partial X_2} & \frac{\partial \tau}{\partial Y_2} & \frac{\partial \tau}{\partial Z_2} & \dots \end{pmatrix}$$



Malkin (2009)



reliable VLBI EOPs

can only be determined when station positions are fixed

Least Squares Adjustment

Observation Equation

Analyzing an Experiment

Estimation of Target Parameters

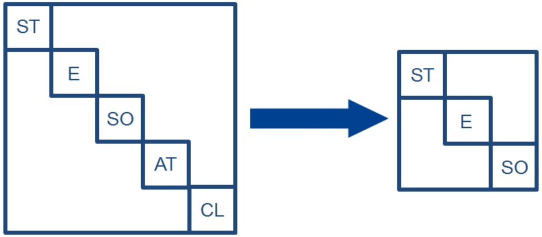
Global Solution and Combination

Global solution

- Independent solution: reduction of parameters

$$\begin{pmatrix} N_{11} & N_{12} \\ N_{21} & N_{22} \end{pmatrix} \cdot \begin{pmatrix} x_1 \\ x_2 \end{pmatrix} = \begin{pmatrix} n_1 \\ n_2 \end{pmatrix}$$

$$\Rightarrow \widetilde{N_{11}} \cdot x_1 = \widetilde{n_1}$$



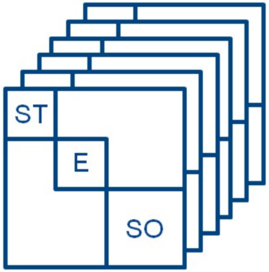
courtesy A. Iddink

$$\widetilde{N_{11}} = N_{11} - N_{12} {N_{22}}^{-1} N_{21}$$

$$\widetilde{n_1} = n_1 - N_{12} {N_{22}}^{-1} n_2$$

- stacking of datum free 24h sessions

$$\widetilde{N}_{glo} = \sum \widetilde{N}$$



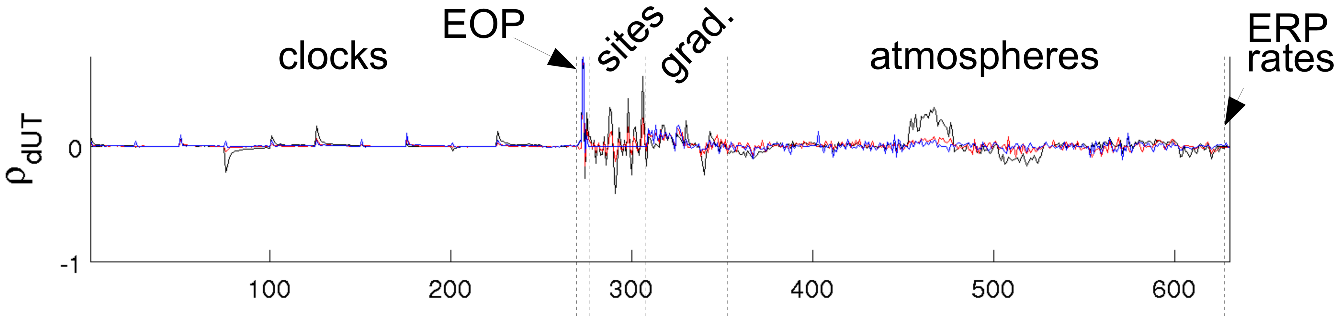
courtesy A. Iddink

- datum definition

$$\widetilde{N}_{glo} + = B^T W_B B$$

- solution for TRF, EOPs, CRF, and ...

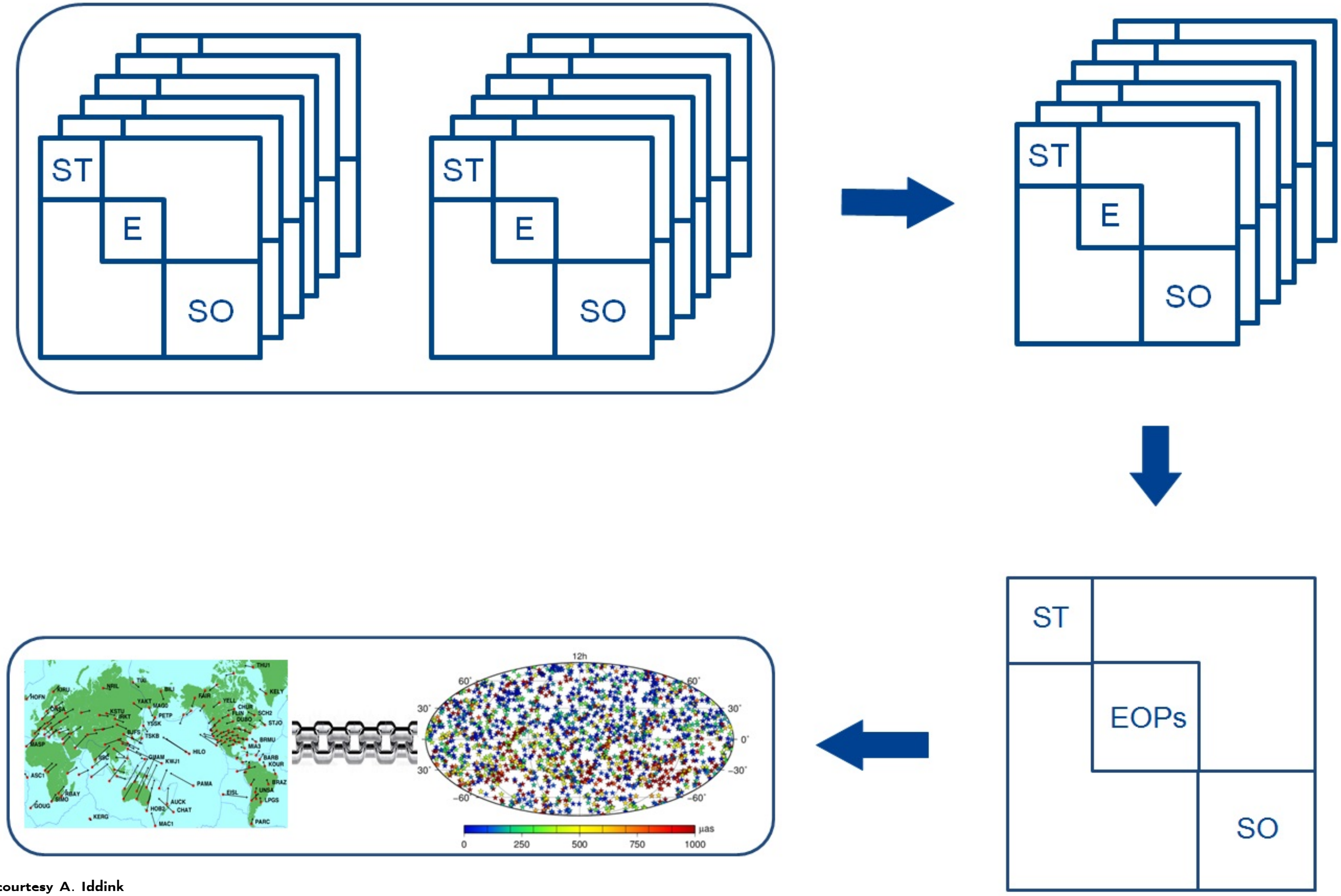
EOPs in Global solution



sufficient decorrelation

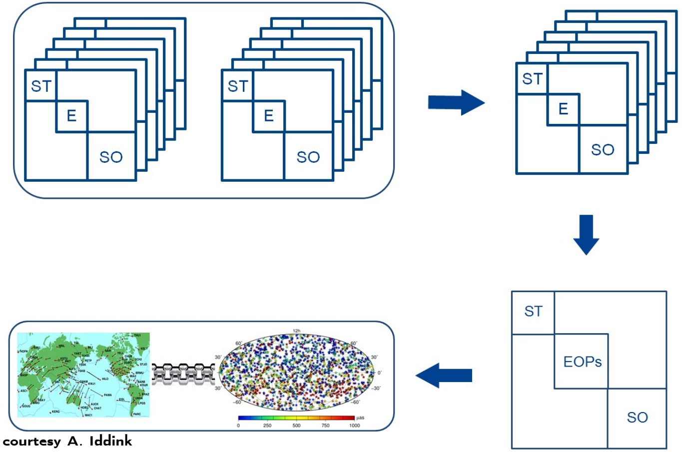
due to stacking, even for fortnightly time spans

Combination Procedure



courtesy A. Iddink

Relative Weighting of ACs



courtesy A. Iddink

$$\hat{\sigma}_{0i}^2 = \frac{\mathbf{r}_i^T \mathbf{W}_i \mathbf{r}_i}{red_{gi}}$$

$$\begin{aligned} \mathbf{r}_i^T \mathbf{W}_i \mathbf{r}_i &= (\mathbf{A}_i \hat{\mathbf{x}}_c - \mathbf{b}_i)^T W (\mathbf{A}_i \hat{\mathbf{x}}_c - \mathbf{b}_i) \\ &= \hat{\mathbf{x}}_c^T \mathbf{N}_i \hat{\mathbf{x}}_c - 2\mathbf{n}_i^T \hat{\mathbf{x}}_c + \mathbf{b}^T W \mathbf{b}_i \end{aligned}$$

$$red_i = n_{obs,i} - \frac{1}{\sigma_{0i}^2} tr(\mathbf{N}_i \mathbf{N}_c^{-1})$$