3rd IVS Training School on VLBI for Geodesy and Astrometry
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Solutions to Exercise-1: Antenna / Feed / Receiver

1. A radio telescope has a diameter, D, of 13.2 m. What is the effective area A_e if the aperture efficiency is equal to 0.7?

   The effective area is the geometric area times the aperture efficiency

   $$A_e = \eta_a \cdot A_g = 0.7 \, \frac{13.2^2 \cdot \pi}{4} = 96 \quad \mathrm{m}^2 \tag{1}$$

2. For how long time do we need to collect energy with this telescope, using a bandwidth of 4 GHz, by observing a radio source with a flux of 1 Jy in order to receive the amount of energy needed to lift a feather weighing 1 g by 1 mm?
   (Note that this is not a realistic task (why?) It is just to get an overall feeling for the quantities involved.)

   The energy needed to lift the feather

   $$W_f = m \cdot g \cdot h = 1 \cdot 10^{-3} \cdot 9.81 \cdot 1 \cdot 10^{-3} = 9.81 \cdot 10^{-6} \quad \mathrm{Ws} \tag{2}$$

   The energy collected by the telescope

   $$W_t == S_f \cdot A_e \cdot B \cdot t = 1 \cdot 10^{-26} \cdot 96 \cdot 4 \cdot 10^9 \cdot t \quad \mathrm{Ws} \tag{3}$$

   Solve for t and we find that it takes $2.6 \cdot 10^9$ s, which corresponds to 81 years.

3. What is the maximum gain for this telescope (in the direction of observation) at 3 GHz and at 12 GHz? For simplicity, let us assume that the aperture efficiency remains the same, 0.7, at these two frequencies.

   There is a simple(?) relation between the gain G and the effective area of a telescope:

   $$G = 4\,\pi \, \frac{A_e}{\lambda^2} \tag{4}$$

   which results in $1.2 \cdot 10^5$ (53 dB) at 3 GHz and $1.9 \cdot 10^6$ (63 dB) at 12 GHz.

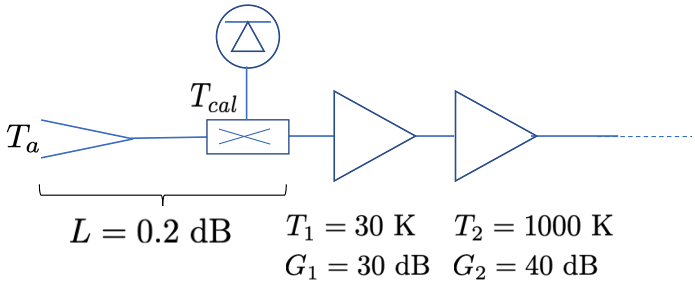
4. What is the opening angle of the main beam at 3 GHz and at 12 GHz?

   An approximate relation for the opening angle (FWHM) of the beam is

   $$\theta_{3dB} \approx 1.2 \, \frac{\lambda}{D} \tag{5}$$

   which results in 31 arcmin at 3 GHz and 8 arcmin at 12 GHz.

5. Now let us assume that the telescope is equipped with a receiver described by the following figure:



We assume that the physical temperature of lossy components in front of the first amplifier is 20 K. The noise temperatures, T_1 and T_2, indicated in the figure are referred to the input of the amplifier, and the gains of the amplifiers are denoted G_1 and G_2. What is the noise temperature of the receiver referred to the antenna input?

The receiver temperature, referred to the antenna input, consists of three terms. First the contribution from the lossy components in front of the 1st LNA, then the 1st LNA, and finally the 2nd LNA:

$$T_{rec} = T_{phys}\,(L - 1) + L \cdot T_1 + L \cdot \frac{T_2}{G_1} = 14 + 31 + 1 = 46 \quad \text{K} \tag{6}$$

6. In order to determine the total system temperature we need to add the noise entering at the antenna input, T_a. Estimate its value when we assume the ground noise pickup to be 10 K, the cosmic background radiation is 3 K, and the atmospheric attenuation at a certain moment is 0.1 dB. The effective (mean) physical temperature of the attenuating atmosphere is 250 K.

The system temperature is then the cosmic background temperature, the ground noise pickup, the contribution from the lossy atmosphere (which can be seen as the output from a lossy medium referred to the antenna input), and the receiver temperature

$$T_{sys} = T_{cos} + T_{gr} + T_{atm}\left(1 - \frac{1}{L_{atm}}\right) + T_{rec} = 3 + 10 + 250\left(1 - \frac{1}{1.023}\right) + 46 = 65 \quad \text{K} \tag{7}$$

7. Let us now assume that the calibration temperature injected into the receiver via the cross coupler is unknown. In order to determine its effective value we point the telescope towards the moon. The moon's effective temperature is assumed to be 250 K. When pointing at the moon with the noise diode off ($T_{cal} = 0$) the receiver output increase by 1.6 dB compared to the output when pointing away from the moon and the noise diode is activated. What is T_{cal} with the noise diode activated?

   We can form a ratio between these to measurements for the equivalent temperatures at the antenna input (1.6 dB corresponds to a factor equal to 3.98)

$$\frac{T_{sys+moon,caloff}}{T_{sys,nomoon,calon}} = \frac{T_{sys} + T_{moon}/L_{atm}}{T_{sys} + T_{cal}} = \frac{65 + 250/1.023}{65 + T_{cal}} = 3.98 \tag{8}$$

   We can now solve for $T_{cal} = 13$ *K.*

8. What is the system equivalent flux density (SEFD in Jy) for the system? (Note that in real life we expect a frequency dependence for the SEFD, but here we have a constant antenna efficiency and system noise temperature across the frequency band.)

   The equation for SEFD is

$$SEFD = \frac{2\,k \cdot T_{sys}}{A_e} \cdot 1 \cdot 10^{26} = 1868 \quad \text{Jy} \tag{9}$$

   in practice the uncertainty of both T_{sys} *and* A_e *may be significant. An alternative formulation for the SEFD is*

$$SEFD = \frac{S_f}{\left(\dfrac{P_{on-source}}{P_{off-source}} - 1\right)} \tag{10}$$

Final remark: discuss if this is an expected procedure to learn about the performance of the system, i.e. which parameters are known first and which parameters are inferred from these.