3rd IVS training school on VLBI
March 2019, Gran Canaria

L09 – How do we determine group delays? Fringe-fitting with Fourfit

Rüdiger Haas

Material taken from:

- 2nd VLBI school, Lecture 8 by Roger Cappallo
  http://www.oso.chalmers.se/evga/vlbi_school_2016/
- 1st VLBI school, Lecture 8 by Alessandra Bertarini
  http://www.oso.chalmers.se/evga/vlbi_school_2013/

Post-correlation Analysis & Fringe-fitting

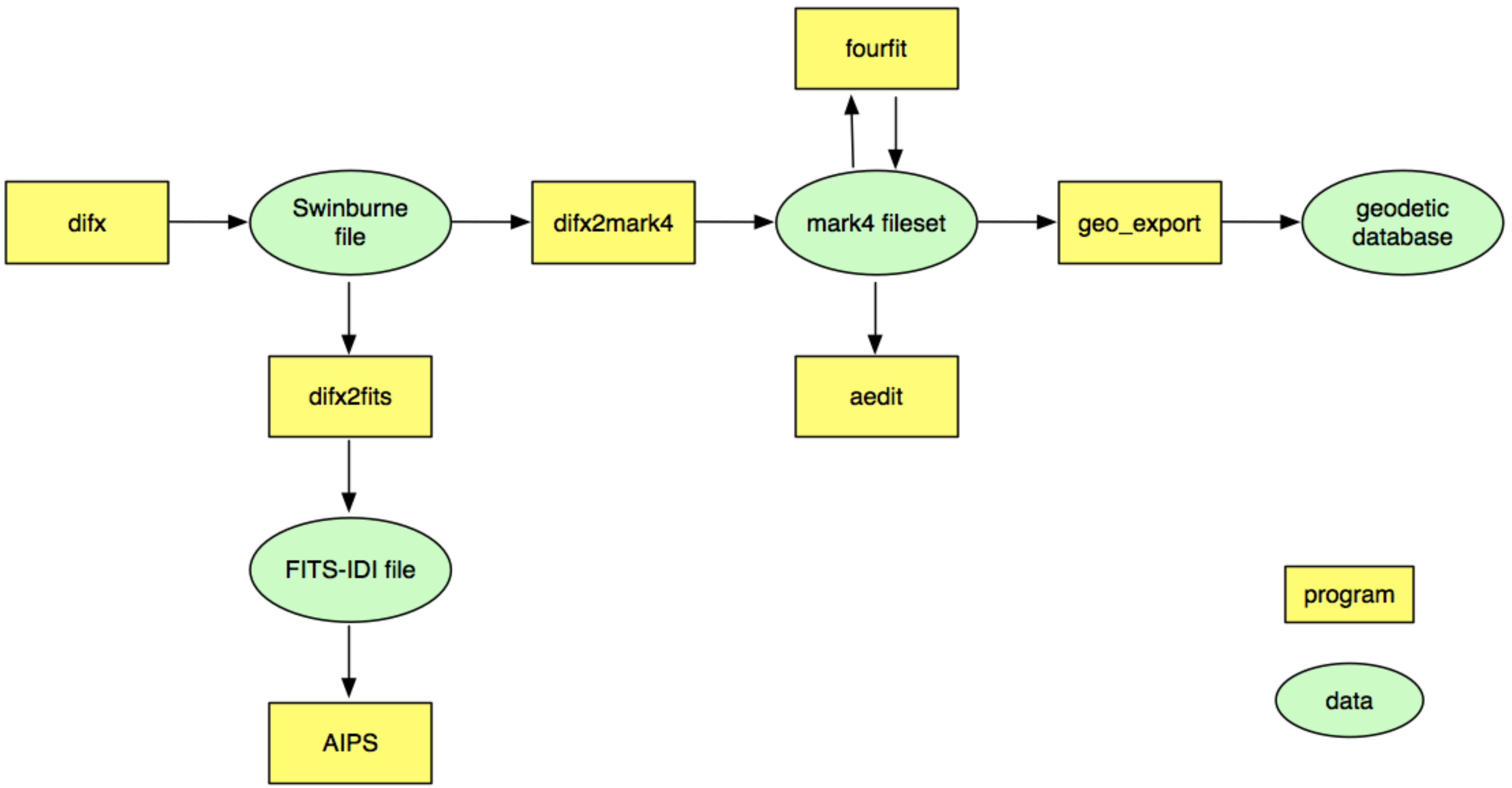
2nd IVS VLBI School – Hartebeesthoek, SA

Roger Cappallo
MIT Haystack Observatory
2016.3.11

overview

- fringe-fitting
  - theory
  - practical example within fourfit
- data quality analysis
  - key to successful operations
- ~~data export to geodetic databases~~

typical processing dataflow



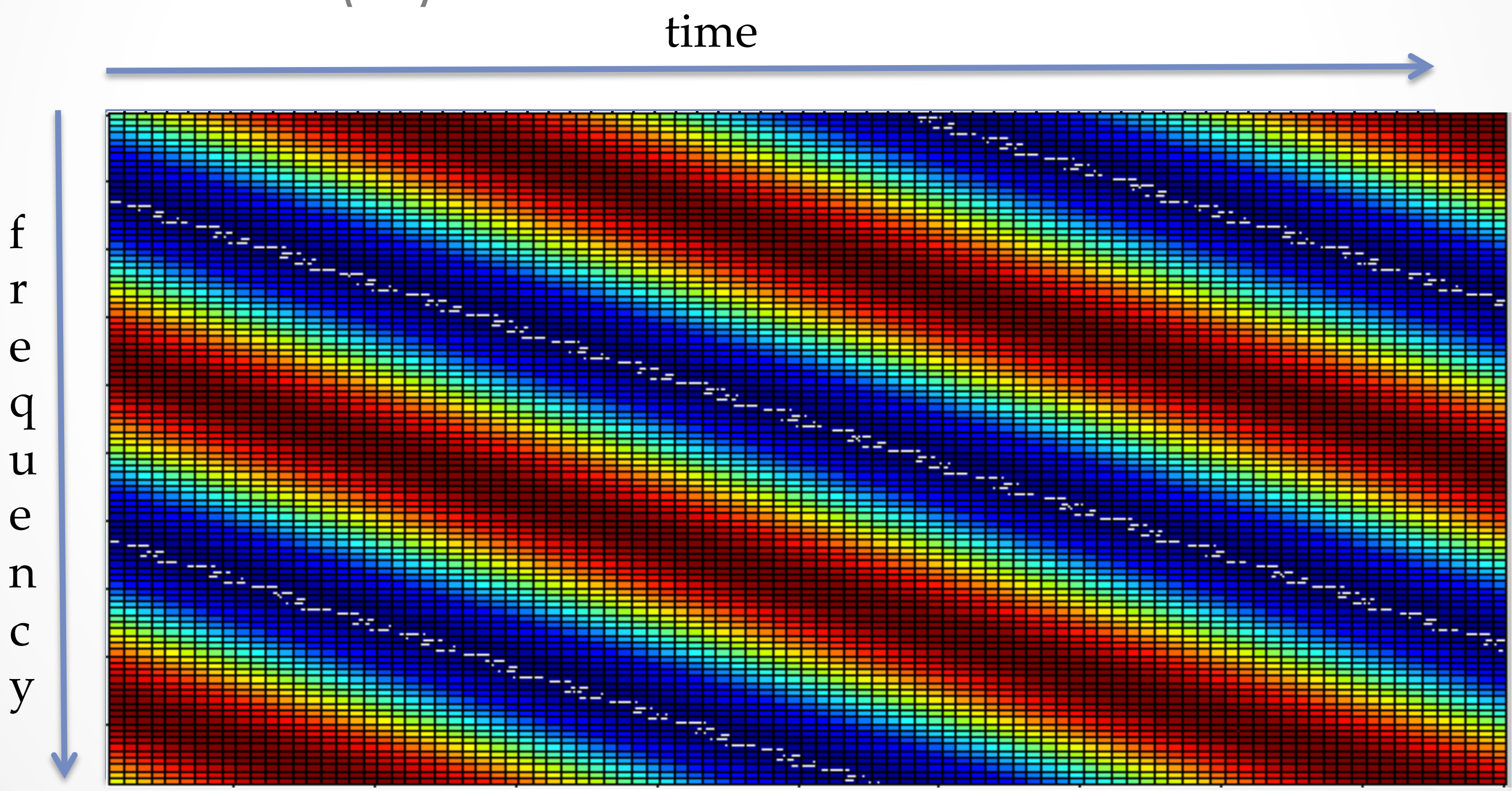
why is fringe-fitting even necessary?

- Correlator model is good, but not perfect
- Typically, antenna models and locations are now very good, but...
- Source positions are imperfect, and can vary with time and frequency
- Atmosphere and ionosphere are time-variable and unpredictable
- GPS clock information has significant errors at the VLBI level of accuracy

Fringe-fitting removes remaining non-random signatures by incremental changes to the correlation parameters

central concept of fringe-fitting

- correlator produces a 2-D complex array of visibilities $\mathcal{V}$ (f,t)



typical patterns in visibilities

- mean amplitude
- quasi-linear drift of phase with time
- quasi-linear drift of phase with frequency

(all trends have noise added to them, often dominant)

extracted parameters

- principally for astronomy:
  - ρ amplitude
  - Φ phase

  } visibility → FT → image

- principally for geodesy:
  - τ_g group delay: variation of phase with frequency
  - $\dot{\tau}_g$ delay-rate: rate of change of τ_g, derived from the variation of phase with time

- nuisance (at least for us)
  - o ΔTEC: differential Total Electron Content (of ionosphere)

snr example

- signal-to-noise ratio for a scan is given by:

  $$snr_{scan} = snr_{sample} \times \sqrt{(\#samples)}$$

- where:
  - $snr_{sample} = \rho$
  - #samples = 2 B T

- VGOS example 4 GS/s * 30s = 1.2e11 samples, so snr increases by a factor of ~ 10^5
- this turns a 0.001 correlation into a scan snr of 300
- for typical scans a minimum snr of 6 or 7 is required for detection

Fourier Transform

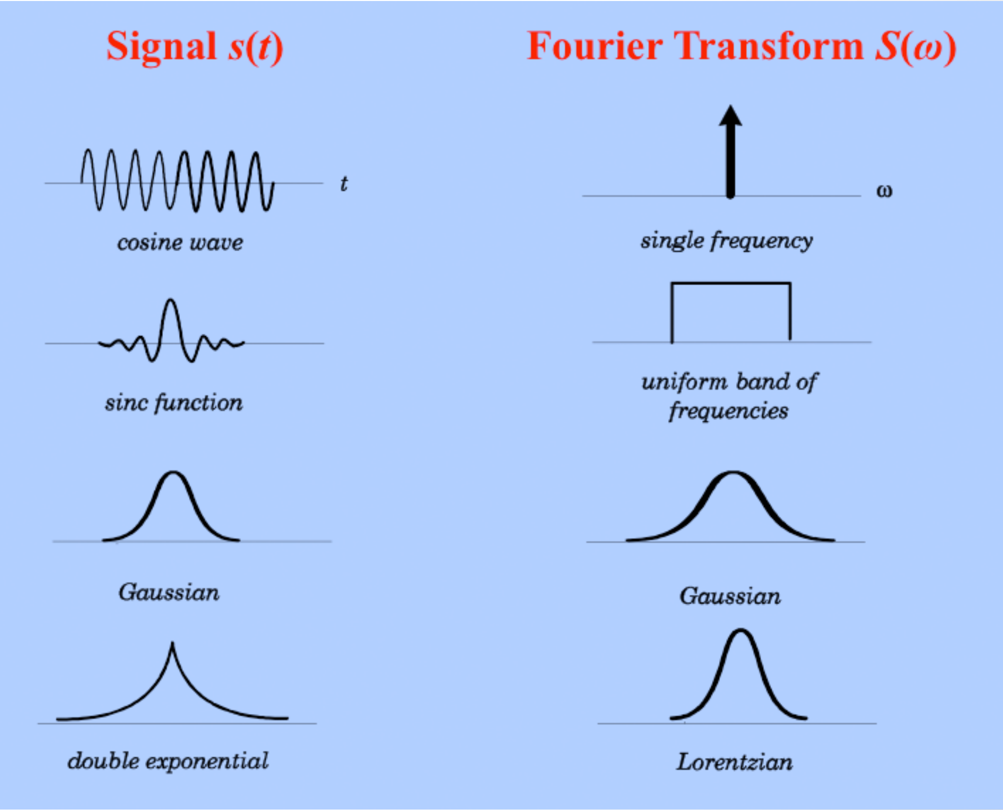
- generates alternative representation of a function in its conjugate domain (e.g. time ~ frequency)

$$F(\omega) = \int_{-\infty}^{\infty} f(t) e^{-i\omega t} dt$$

$$f(t) = \frac{1}{2\pi} \int_{-\infty}^{\infty} F(\omega) e^{i\omega t} d\omega$$

- dft
- fft

$$X_k = \sum_{n=0}^{N-1} x_n e^{-i 2\pi k \frac{n}{N}} \qquad k = 0, \ldots, N-1$$



fringe-fitting in *fourfit*

- 2 steps
  - coarse grid search
  - refinement of parameter estimates

- grid search done via FFT's:
  - over frequency to find delay
  - over time to find fringe/delay rate
  - over "lag" to find single-band delay

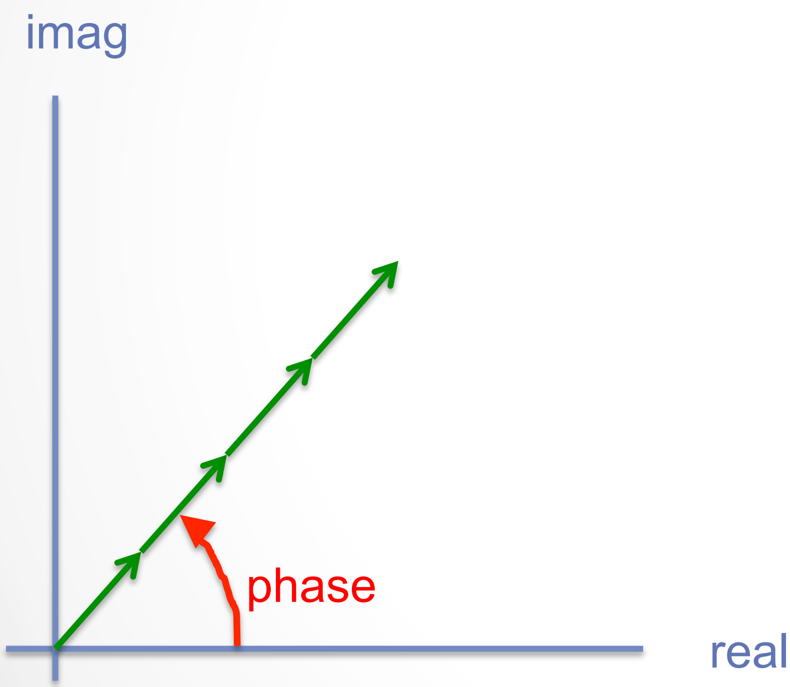
- refinement
  - counter-rotate data and coherently sum:

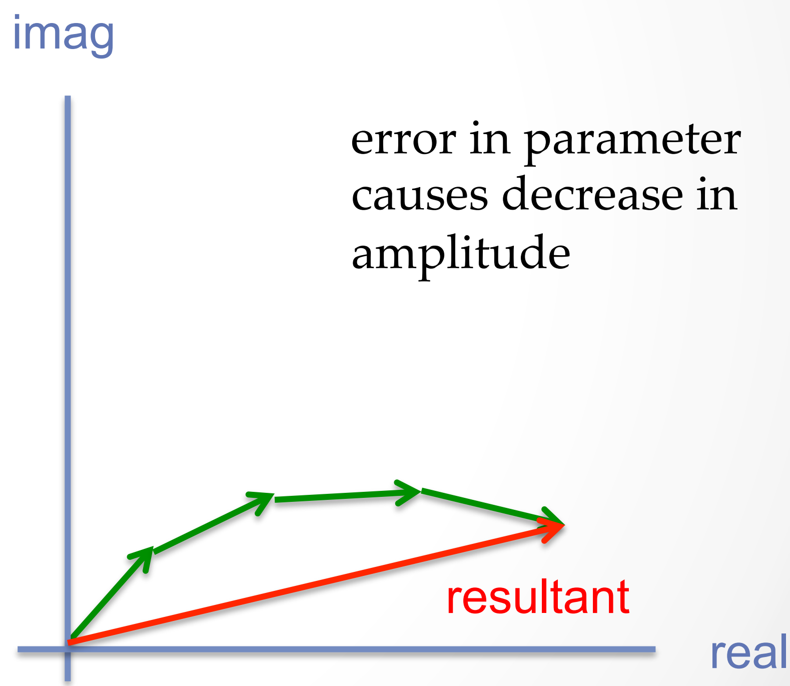
$$\mathbf{g}(\tau, \dot{\tau}) = \sum_f \sum_t \boldsymbol{V}(f, t) e^{-2\pi i (f \dot{\tau} t + f \tau + \delta\phi)}$$

  - interpolate from closely-spaced grid-points

coherent addition of visibilities

(idealized noiseless case)





$$\mathbf{g}(\tau, \dot{\tau}) = \sum_f \sum_t \boldsymbol{V}(f, t) e^{-2\pi i (f \dot{\tau} t + f \tau + \delta\phi)}$$

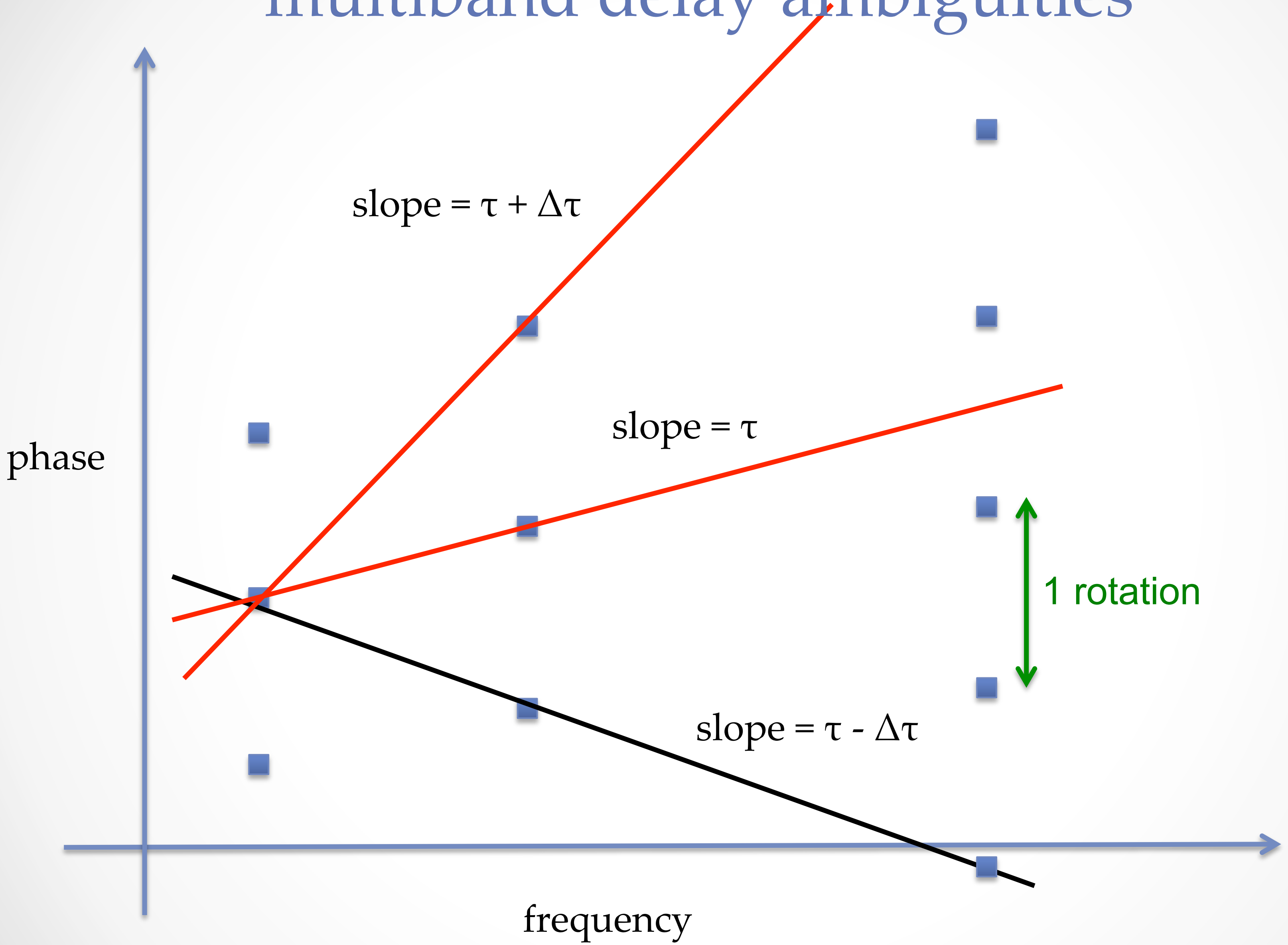
singleband delay

- slope of phase vs. frequency within one channel (e.g. 32 MHz, for VGOS)
- determined on a baseline by estimating slope of phase vs. freq. of a radio source
- instrumental contribution for a single antenna can be found by using the phases of phase-cal tones embedded in the channel

multiband delay

- determined by “collapsing” each channel’s data down to a single phase per channel, and then finding the slope of those phases across their frequency range
- by spacing channels apart, a wider range of frequencies is covered, leading to a more accurate slope
- technique is called “bandwidth synthesis”
- ambiguities are unavoidable...

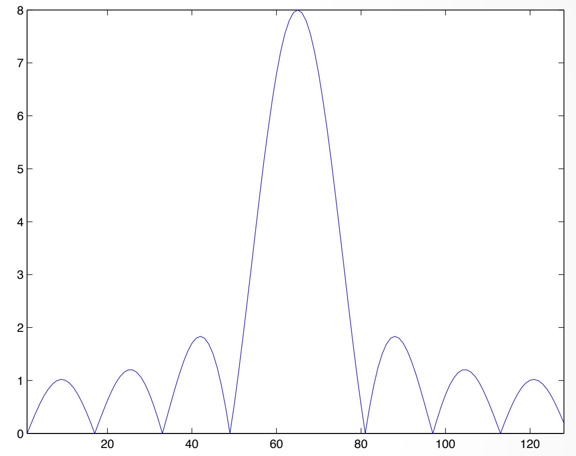
multiband delay ambiguities



delay resolution function

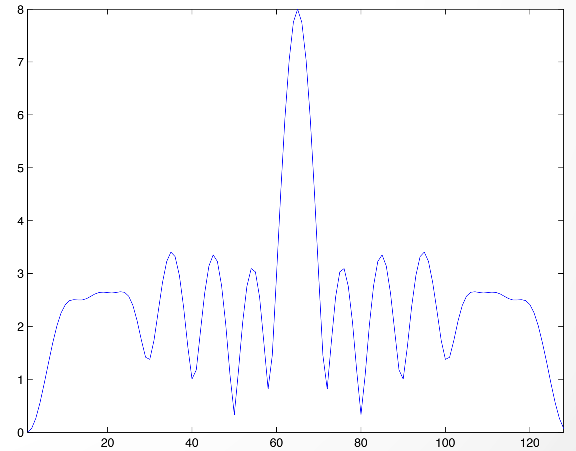
FFT of cross-power spectrum

freqs = [1 2 3 4 5 6 7 8]



freqs = [1 2 5 8 10 13 14 15]

has narrower peak, higher sidelobes



(cf. Arsac arrays & Golomb rulers)

multiband delay vs. singleband delay

- different due to things that affect single channels or groups of channels (e.g. cable lengths, filter delays)

- by correcting channels with pcal-derived delays there is hope to go to (just) multiband delay

- ambiguity spacing in delay is inverse of greatest common frequency difference
  - VGOS mbd: 32 MHz spacing ➔ 31.25 ns ambiguity
  - VGOS sbd: 128 spectral pts/channel ➔ 1/8 MHz spacing ➔ 8 μs ambiguity

delay-rate vs. fringe-rate

$\frac{d\tau}{dt}$ delay-rate (group-delay rate) is rate of change of delay, and is dimensionless

$\frac{d\phi}{dt}$ fringe-rate is rate of change of fringe phase, typically in Hz or mHz. It is the differential Doppler-shift

related by $$\frac{d\tau}{dt} = \frac{1}{f} \times \frac{d\phi}{dt}$$

overcoming instrumental shortcomings

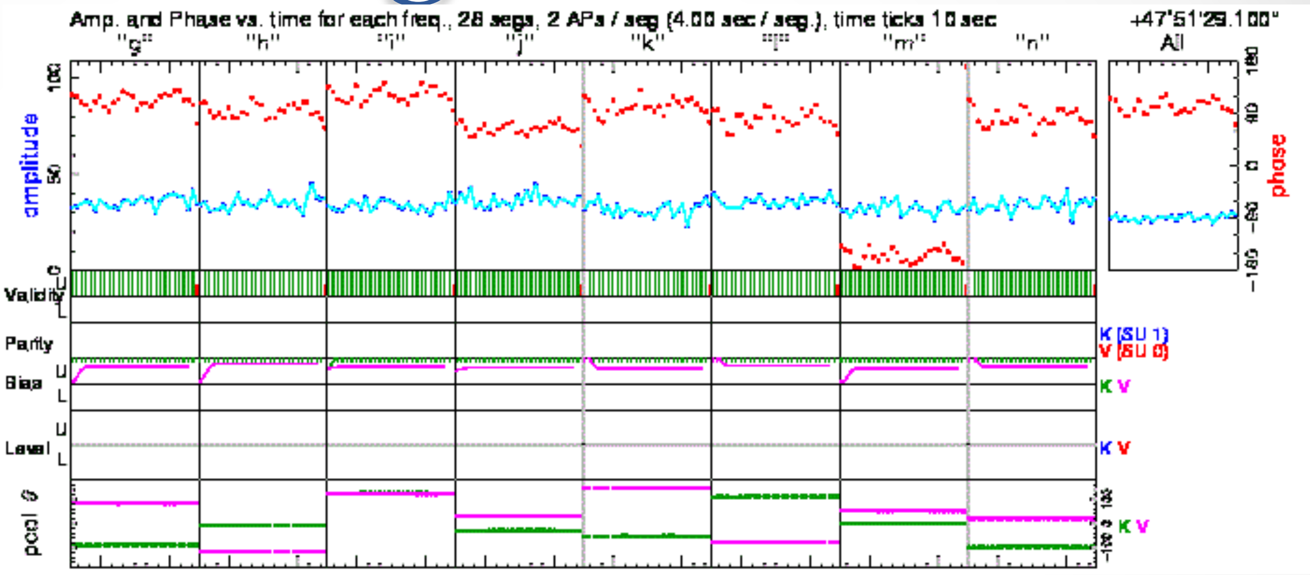
- GPS clock errors
  - search over range of delays
- RFI
  - delete channels or times as necessary
- data defects (e.g. off-source)
  - delay start or stop of fit
- phase & delay (mis)calibration
  - use pcal tones and/or manually adjust delays and pcal phases

phase calibration

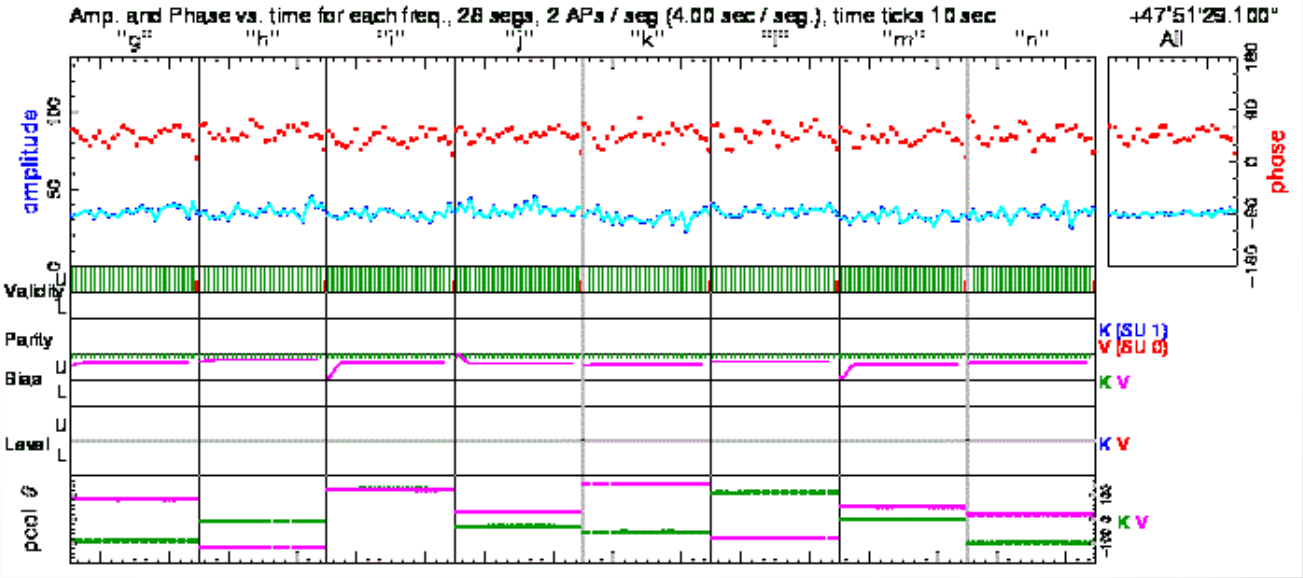
- legacy “normal”
  - 1 tone per channel
  - deprecated!
- multitone
  - many tones per channel
  - capability to correct channel delay
- manual
  - typically set to constant values for whole experiment
  - line up phases with strong calibrator source
  - slight tweaks just change the clock estimate

phase-cal aligns channels

before:

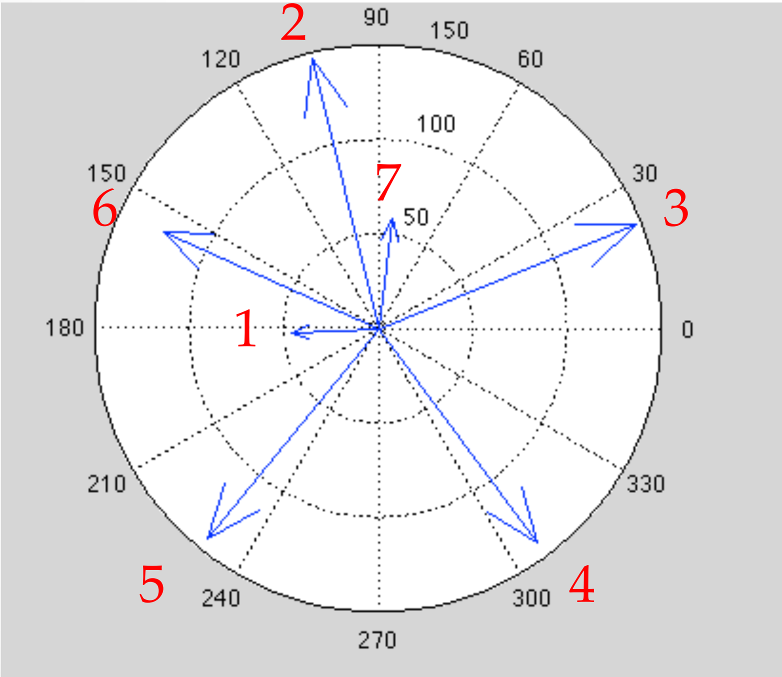


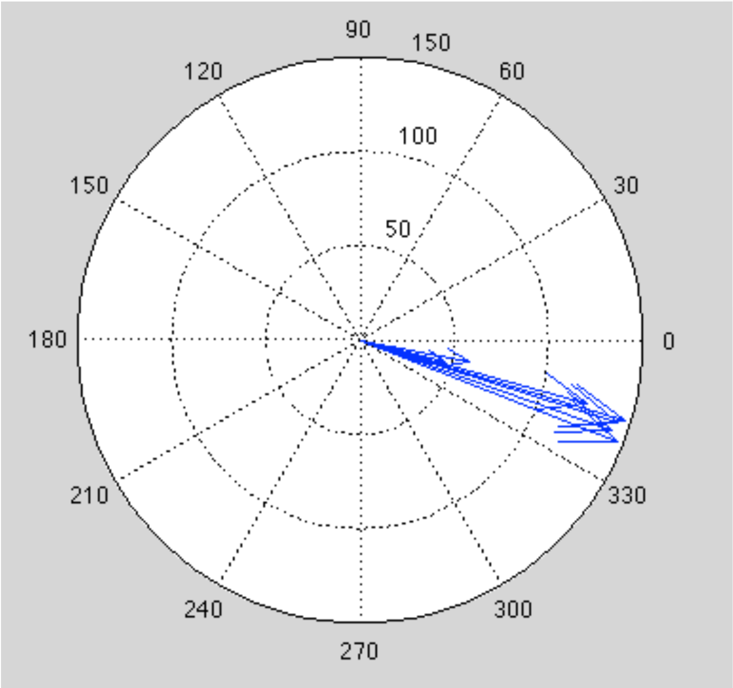
after:



channel-based delay correction using multitone pcal

$$\boldsymbol{V^*}(f_k) = \boldsymbol{V}(f_k)e^{-2\pi i(f_k \delta\tau)}$$



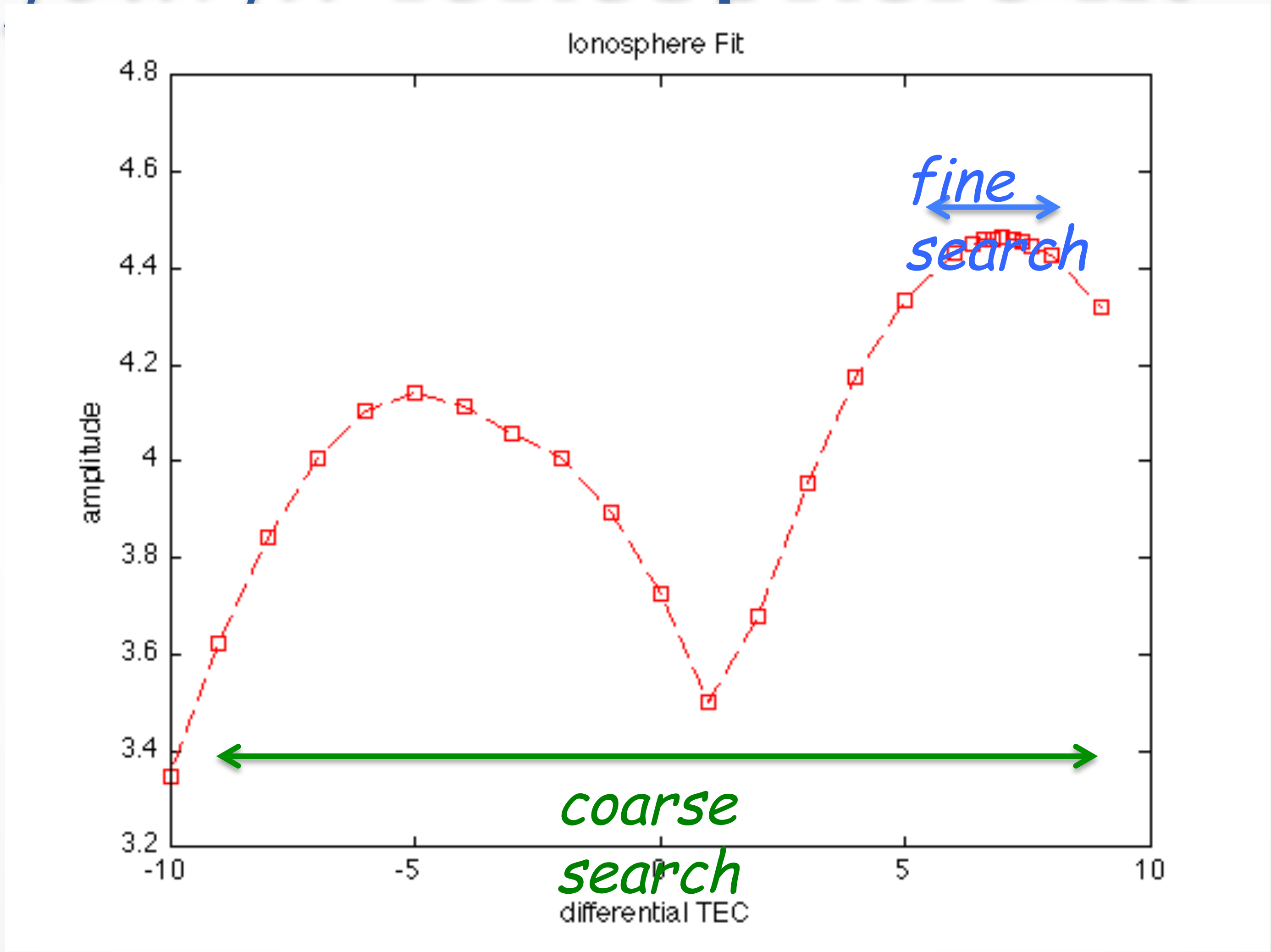


ionosphere

- phase of each freq channel affected by differential path integral of charges (Total Electron Content)
- 1 TEC unit = 10^{16} electrons / m^2
- $\Delta \phi = c \times \Delta TEC / f$

- differential TEC can be fit and/or specified *a priori*
  - all-sky models from GPS available, but not yet used
  - fit made difficult by nonlinearity
  - search for peak of coherent sum of all bands
- ionosphere and group delay estimate are strongly correlated ~92% for VGOS

fourfit ionosphere fit



combining linear polarizations in fourfit

- Maximize sensitivity in τ_g by combining all 4 Stokes polarization products
- Form an approximation to Stokes I:
  - from the 4 correlation products form

$$I \cong (HxH + VxV)\cos\Delta + (HxV - VxH)\sin\Delta$$

Δ = differential parallactic angle

  - correct to first order in the D terms
- Also have mixed combinations to legacy stations

e.g. {RxV, RxH, LxV, LxH}

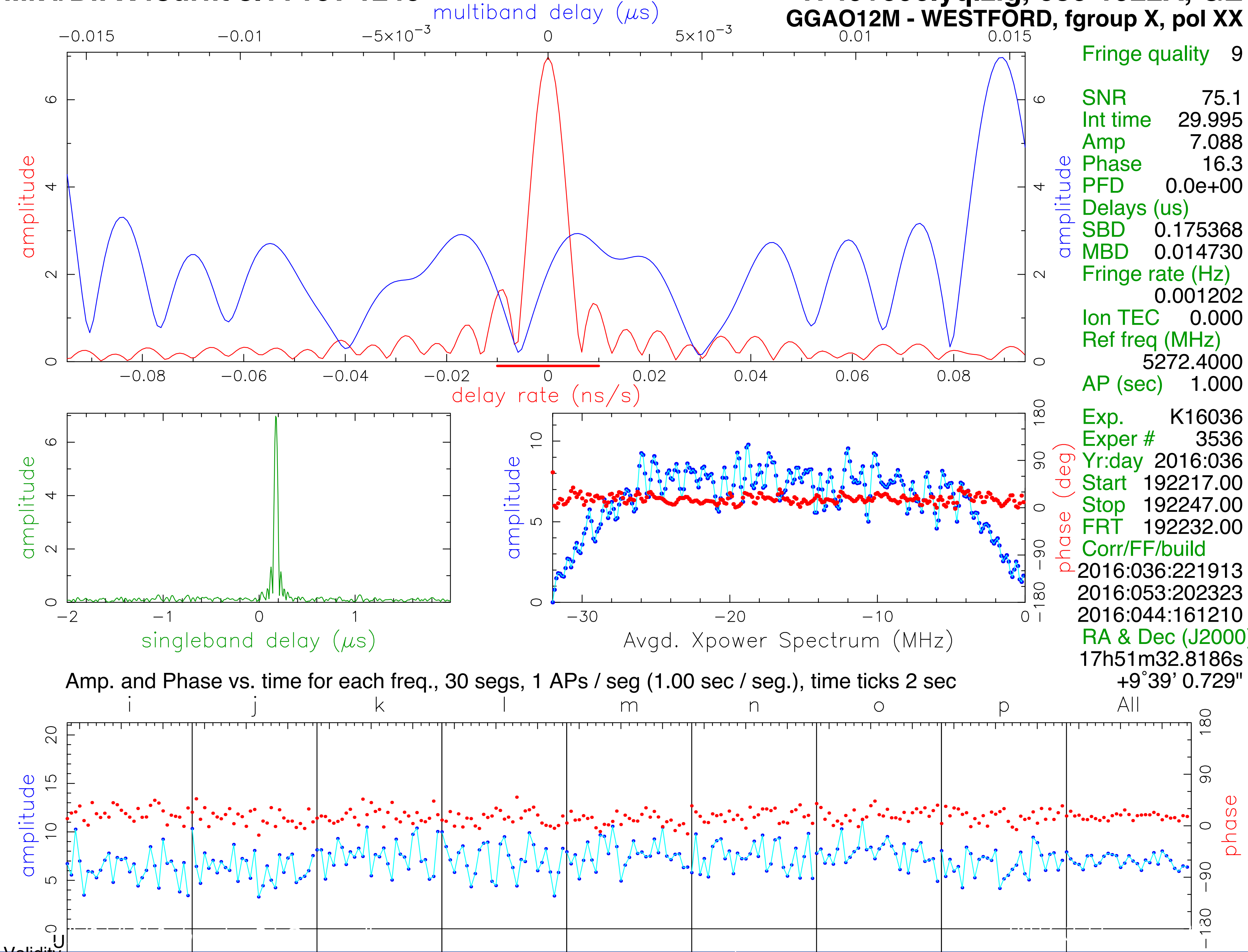
fourfit output

- "mk4" data file
  - used for fourfit input as well
  - set of files tied by a common suffix
    - type 0: root file, contains vex statements for scan
    - type 1: correlator output (visibilities), 1 per baseline
    - type 2: fourfit output, per baseline & by ff fit
    - type 3: station files, 1 per station
- fringe plot
  - single page w/ graphical and printed summary

fringe-plot example

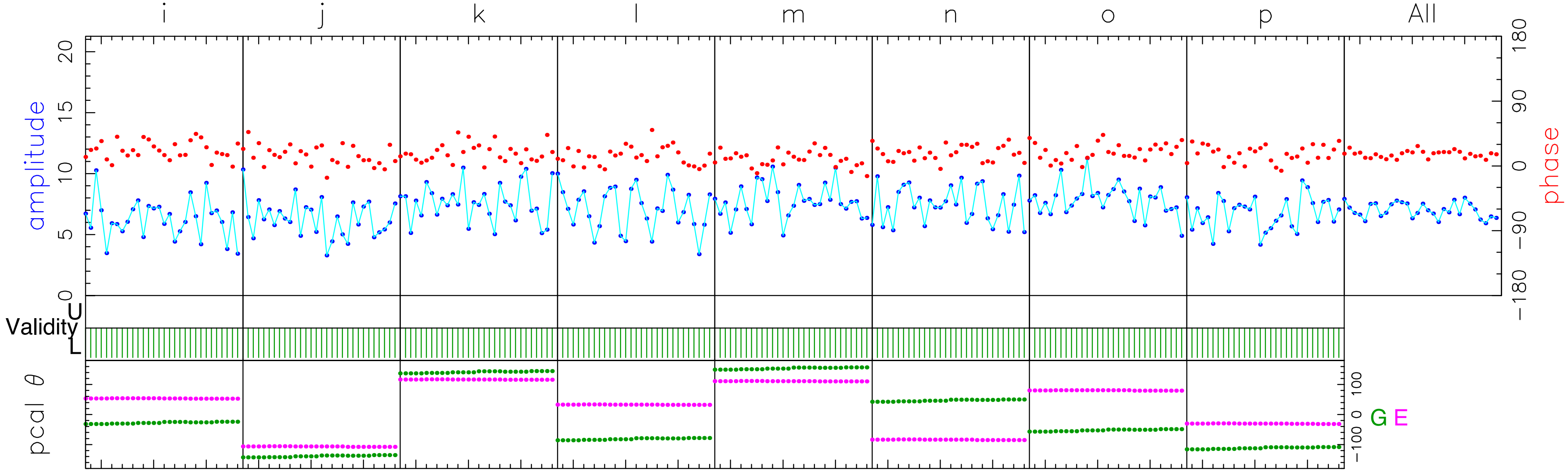
- concise summary, but crowded
- plots
  - multiband delay
  - singleband delay
  - delay-rate
  - cross-power spectrum
  - phase & amp by channel as function of time
  - pcal amp & phase(t)
  - data fractions
- text
  - residual fit parameters
  - total values
  - metadata
  - statistics

Mk4/DiFX fourfit 3.11 rev 1240 **1749+096.yqlzig, 036-1922A, GE**



17h51m32.8186s
+9°39' 0.729"

Amp. and Phase vs. time for each freq., 30 segs, 1 APs / seg (1.00 sec / seg.), time ticks 2 sec



	5272.40	5304.40	5336.40	5464.40	5560.40	5624.40	5688.40	5720.40	Freq (MHz)	All
	21.7	15.0	17.2	13.8	9.1	18.3	19.5	16.2	Phase	16.3
	6.1	6.2	7.4	7.0	7.6	7.3	7.7	6.6	Ampl.	7.0
	279.5	279.4	279.5	279.5	279.4	279.5	279.4	279.3	Sbd box	279.4
U/L	0/30	0/30	0/30	0/30	0/30	0/30	0/30	0/30	APs used	
G	166.8	164.3	162.8	160.3	159.6	159.9	160.4	162.0	PC X delays (ns)	
E	140.1	137.3	135.4	133.2	132.2	133.1	133.4	134.4	PC X delays (ns)	
G:E	-28:53	-139:-107	141:116	-82:33	154:111	47:-84	-53:80	-112:-31	PC phase	
G:E	0:0	0:0	0:0	0:0	0:0	0:0	0:0	0:0	Manl PC	
G	90	93	87	88	74	90	97	84	PC amp	
E	70	75	77	79	64	69	67	57		
G	XAB00LX	XAB01LX	XAB02LX	XAB03LX	XAB04LX	XAB05LX	XAB06LX	XAB07LX	Chan ids Tracks	
E	XAB00LX	XAB01LX	XAB02LX	XAB03LX	XAB04LX	XAB05LX	XAB06LX	XAB07LX	Chan ids Tracks	

Group delay (usec)(model)	1.45049874487E+03	Apriori delay (usec)	1.45048401506E+03	Resid mbdelay (usec)	1.47298E-02	+/-	1.3E-05
Sband delay (usec)	1.45065938306E+03	Apriori clock (usec)	5.6711163E+01	Resid sbdelay (usec)	1.75368E-01	+/-	2.3E-04
Phase delay (usec)	1.45048402363E+03	Apriori clockrate (us/s)	1.2900000E-06	Resid phdelay (usec)	8.57320E-06	+/-	8.0E-07
Delay rate (us/s)	-5.03668732117E-02	Apriori rate (us/s)	-5.03671011913E-02	Resid rate (us/s)	2.27980E-07	+/-	4.6E-08
Total phase (deg)	347.8	Apriori accel (us/s/s)	-8.30345929750E-06	Resid phase (deg)	16.3	+/-	1.5

	RMS	Theor.		
			Amplitude	7.088 +/- 0.094
ph/seg (deg)	4.8	4.2	Search (64X64)	6.941
amp/seg (%)	8.5	7.3	Interp.	0.000
ph/frq (deg)	4.2	2.2	Inc. seg. avg.	6.983
amp/frq (%)	8.4	3.8	Inc. frq. avg.	6.986

Pcal mode: MULTITONE, MULTITONE PC period (AP's) 5, 5
Pcal rate: 0.000E+00, 0.000E+00 (us/s)
Bits/sample: 2x2 SampCntNorm: disabled
Sample rate(MSamp/s): 64
Data rate(Mb/s): 1024 nlags: 256 t_cohere infinite

sb window (us)	-1.000	1.000
mb window (us)	-0.016	0.016
dr window (ns/s)	-0.010	0.010
ion window (TEC)	0.00	0.00

G: az 271.9 el 13.0 pa 52.0 E: az 274.7 el 9.2 pa 48.1 u,v (fr/asec) 17.766 -32.192 simultaneous interpolator

Control file: default Input file: /data-sc02/rjc/trivgos/3536/036-1922/GE..yqlzig Output file: Suppressed by test mode

fourfit control files

- text files with simple syntax
- there are ~95 keywords known to *fourfit*
- syntactic elements
  - if, and, or, not, <, >, ?
- data selectors
  - station, baseline, source, scan, f_group
- filtering
  - freqs, start, stop, etc.
- corrections
  - pc_mode, pc_phases, ionosphere, ref_freq, lsb_offset, etc.
- search control
  - sb_win, mb_win, dr_win, ion_win, etc.

example control file

```
ref_freq  8213.15                     * global commands come first

start -10
if scan 288-210210
   sb_win .37 .37

if scan > 289-132510                  * don’t use any scans after 1325
   skip true

if station L and f_group X
   freqs a+ b c d- e f g h

if station L and f_group S
  pc_mode manual
  pc_phases ijkmn 4.5 -78 39 +12 0
if station A
   pc_mode multitone
   pc_period 30
   pc_tonemask abcdefgh 0 0 8 0 4 0 5 0
   pc_phases_l abcdefgh 12 13 11 12 24 -6 38 110
   pc_phases_r abcdefgh 11 29 14 11 64 -2 44 132
   samplers 2 abcd efgh
   pc_delay_l 30.2  pc_delay_r -5.9
   ionosphere 18.0
if station V or baseline KT and source 3C279
   sb_win -0.5 0.5    mb_win 0.02 0.02  dr_win -1.0E-6 0.5E-6
```

fourfit quality codes

Aedit: Quality Code

QC = 0 Fringes not detected (PFD > 1e-4).

= 1-9 Fringes detected, no error condition. Higher number => better quality.

= B Interpolation error in fourfit.

= D No data in one or more freq. channels.

= E Max fringe amplitude at the edge of SBD, MBD or DR window.

= F "Fork" problem in processing.

= G Fringe amp. in one or more channels is < 0.5 mean amp. (for SNR > 20).

= H Low pcal-amplitude.

= N No valid correlator data.

A. Bertrarini

56

DiFX Correlation & Post-Correlation Analysis

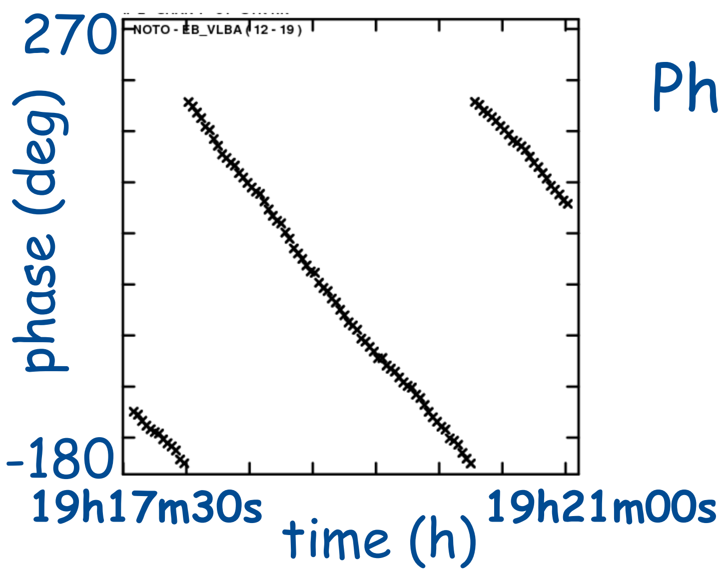
Alessandra Bertarini

IGG University of Bonn & MPIfR Bonn

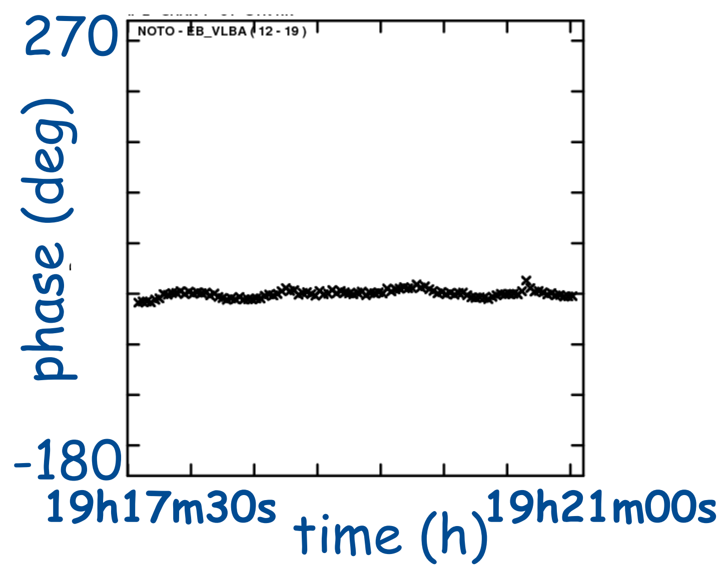
Max-Planck-Institut für Radioastronomie

Post Correlation Software

Due to errors in the model, the correlator phases still show a slope vs time:



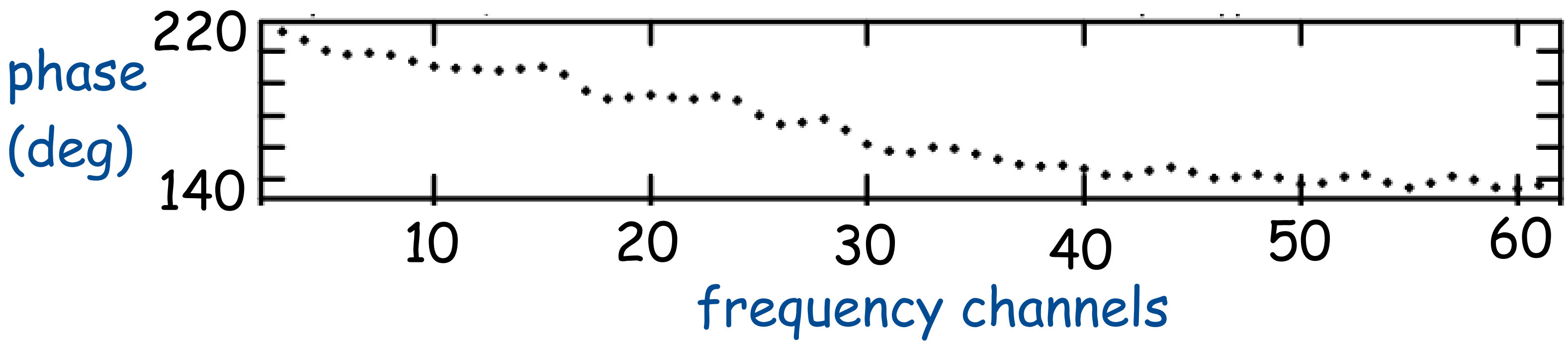
Phase slope vs time is "fringe rate"



Fringe Fit refines the model removing the fringe rate

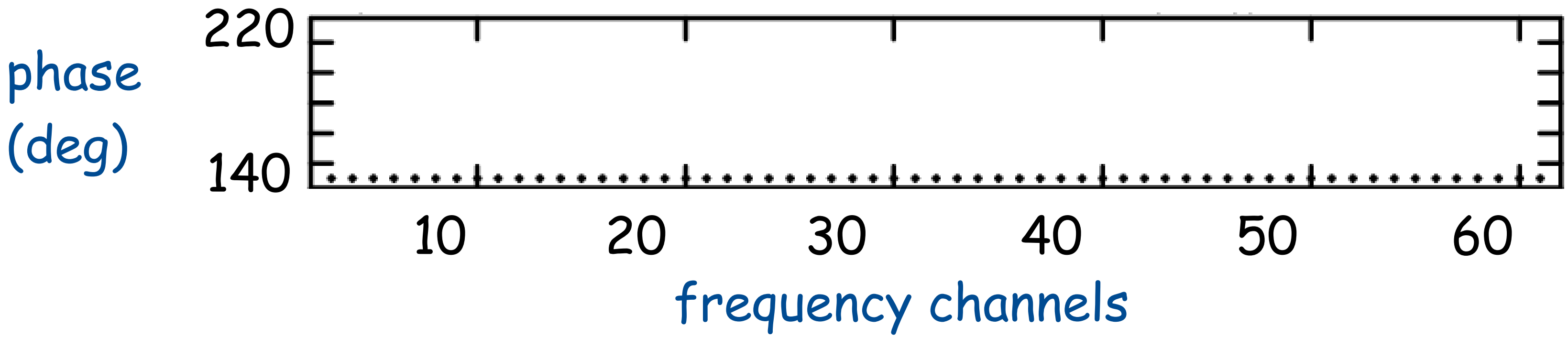
Post Correlation Software

Due to errors in the model, the correlator phases still show a slope vs frequency:

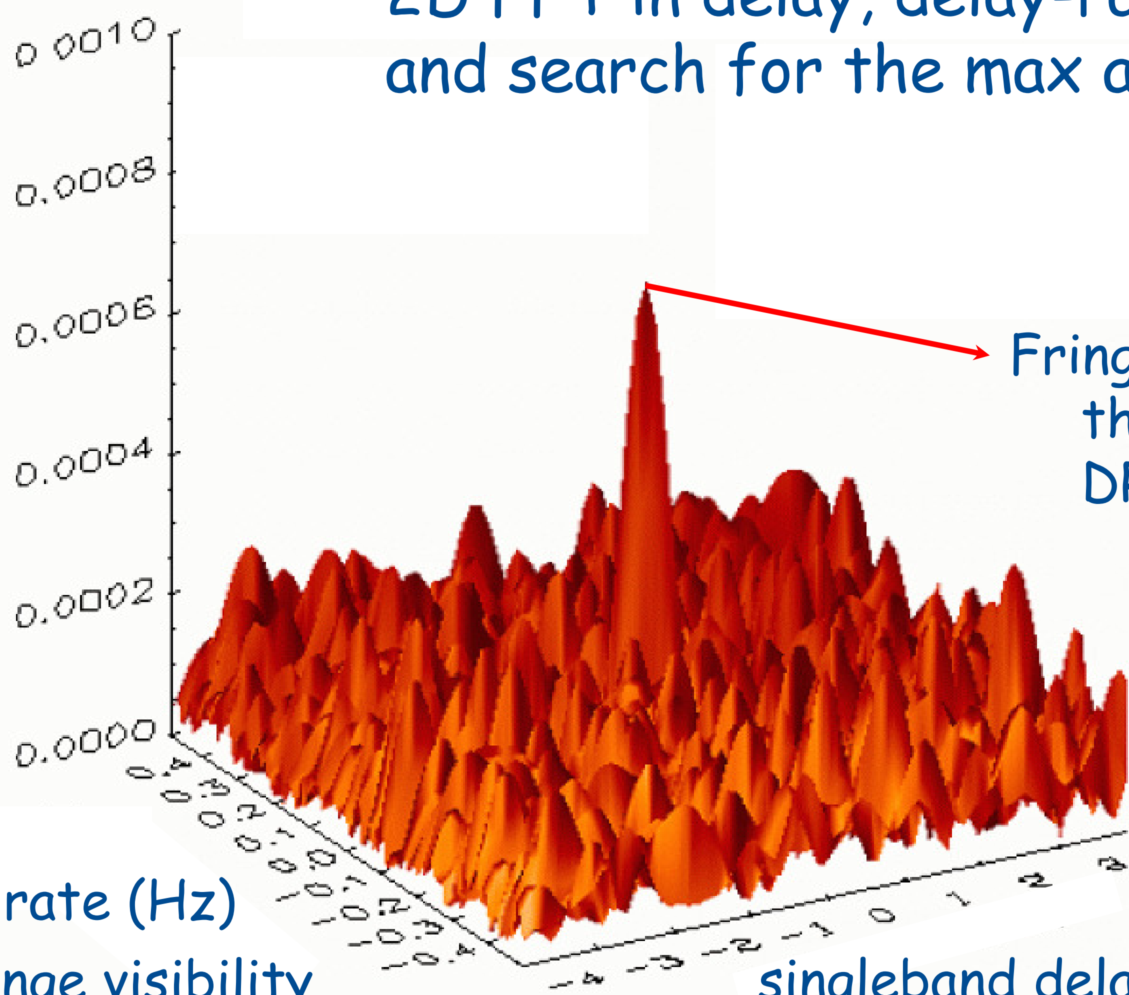


Phase slope in frequency is delay.

Fringe Fit corrects the delay pivoting around a reference frequency



2D FFT in delay, delay-rate space
and search for the max amplitude



Corr. amplitude

Fringes are located at this position in SBD, DR space

Fringe rate (Hz)
(FT fringe visibility with respect to time)

singleband delay (μs)

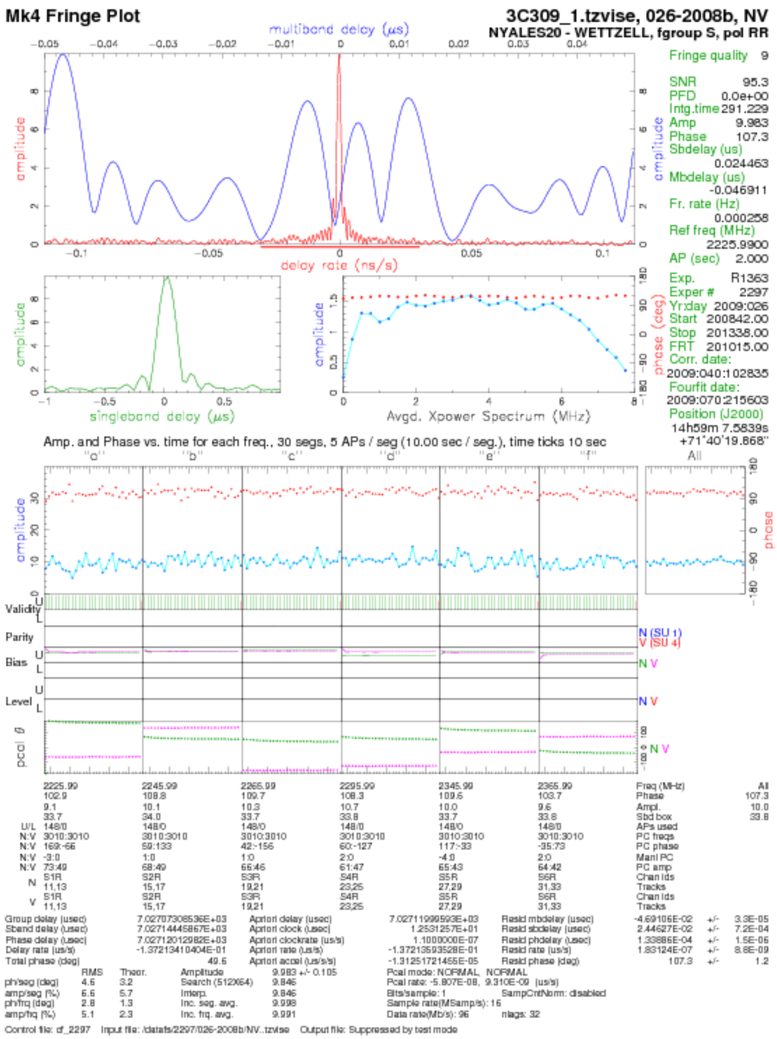
Image courtesy of K. Kingham

Fringe Fit: *Fourfit* Overview

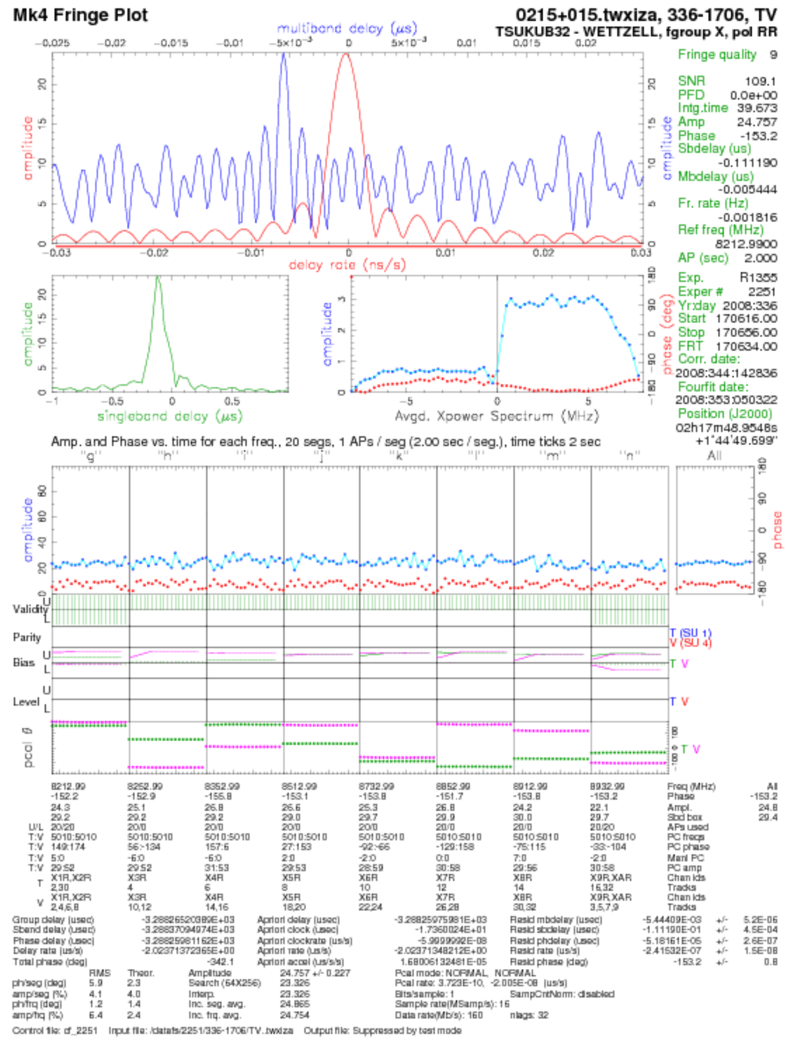
igg

universitätbonn

S-Band:



X-Band:



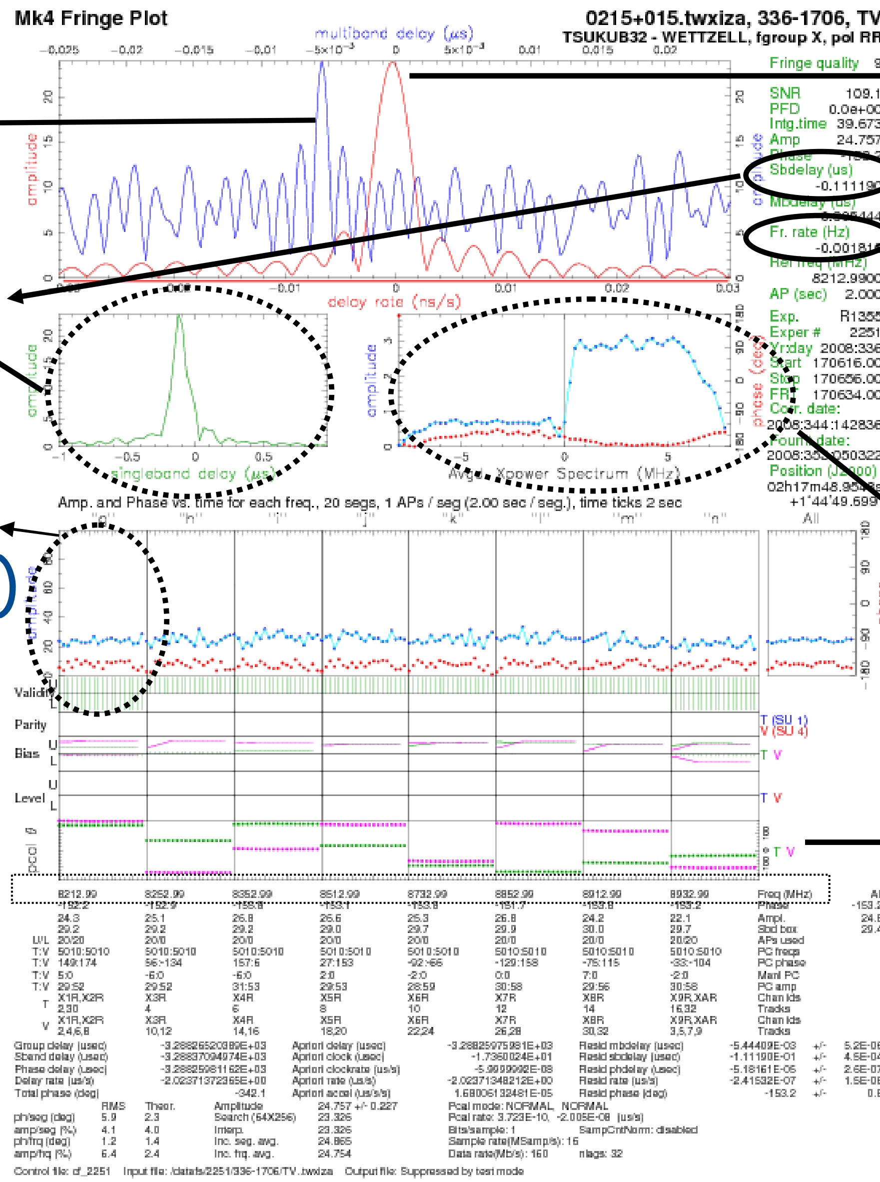
Fourfit

Multiband delay (µs)

Single band delay (µs)

Phase (red) & amp (blue) vs time for every BBC

Sky freq.



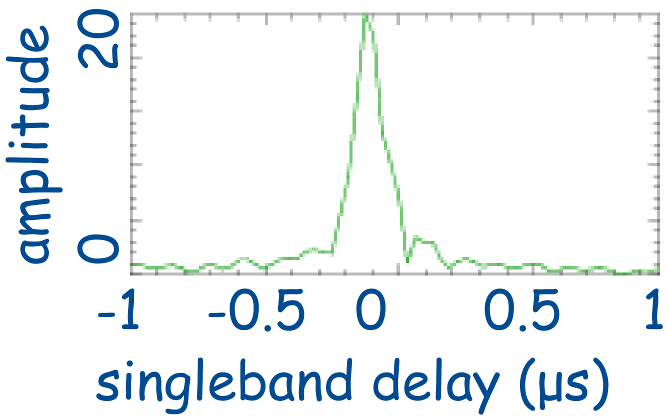
Delay rate.

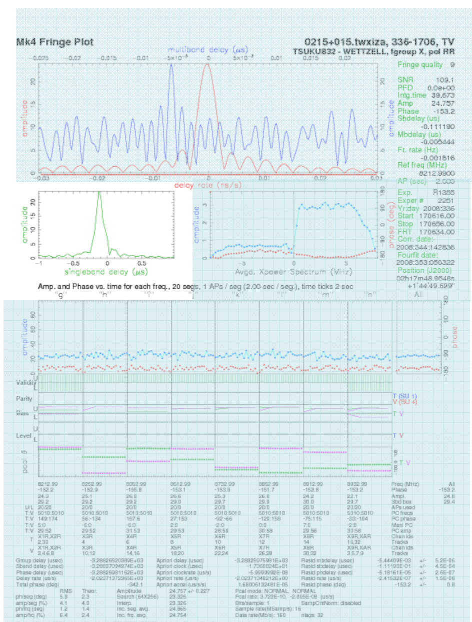
Fringe rate (Hz) = Delay Rate • Sky freq.

FT of lag spectrum

Pcal phases

Fourfit: Lag Spectrum





Lag spectrum: output of the correlator integrated over the scan duration.

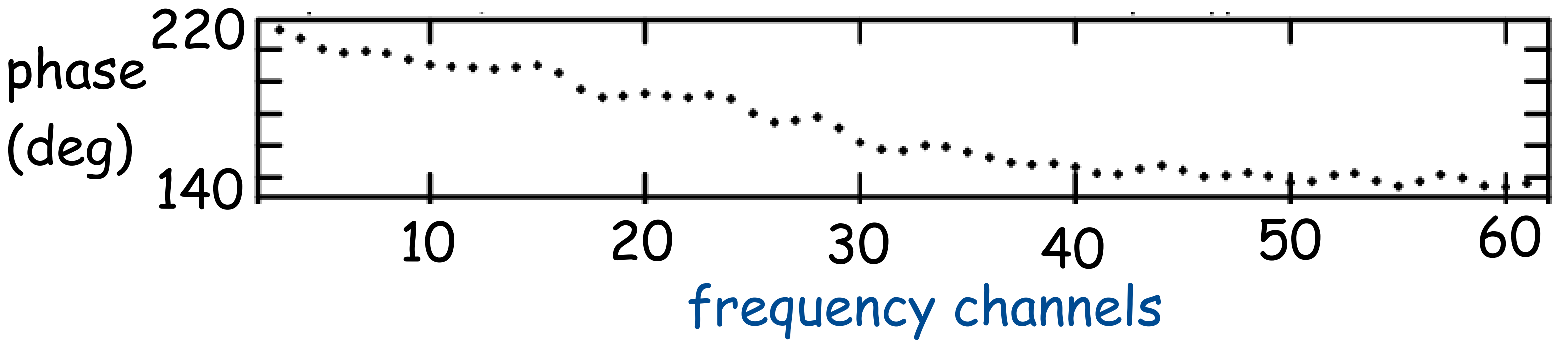
Lag spectrum shown is lag spectra of all BBC stacked.

8 MHz/BBC => 16 Msample/s => sample period = 1 / 16 Msample/s = 0.0625 µs = > 0.0625 µs * 32 lags = 2 µs SBD window width.

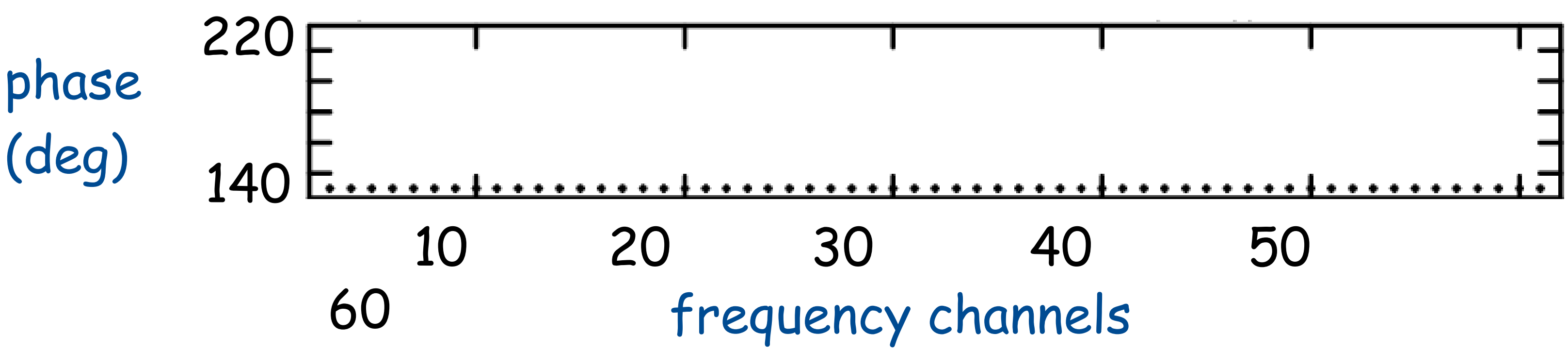
Indicates residual correlator model errors, part of which can be absorbed in the clock offset.

Fringe Fit: Visibility Phase vs Frequency

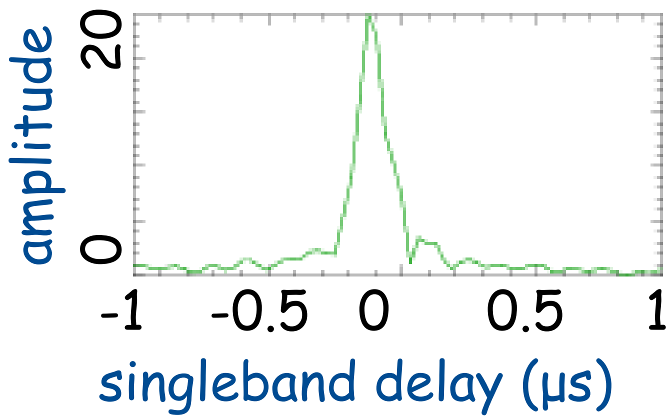
Raw:



Fringe fitted:

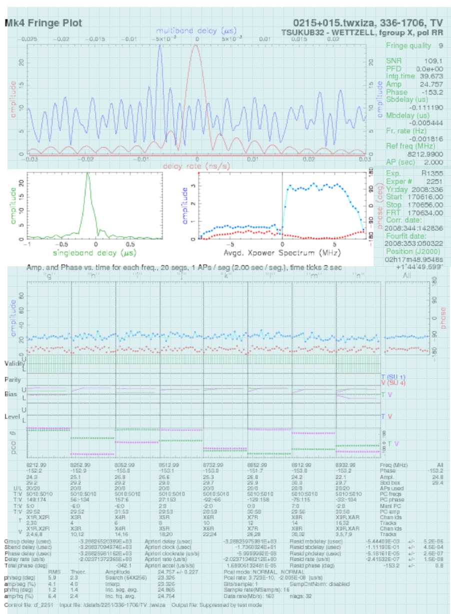


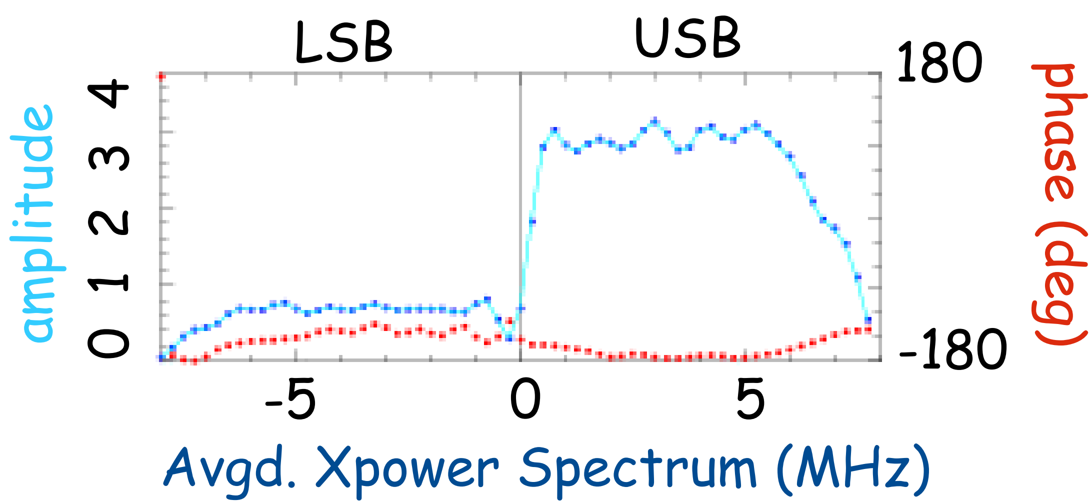
Fringe Fit: from Lag to Frequency



Fourier Transform

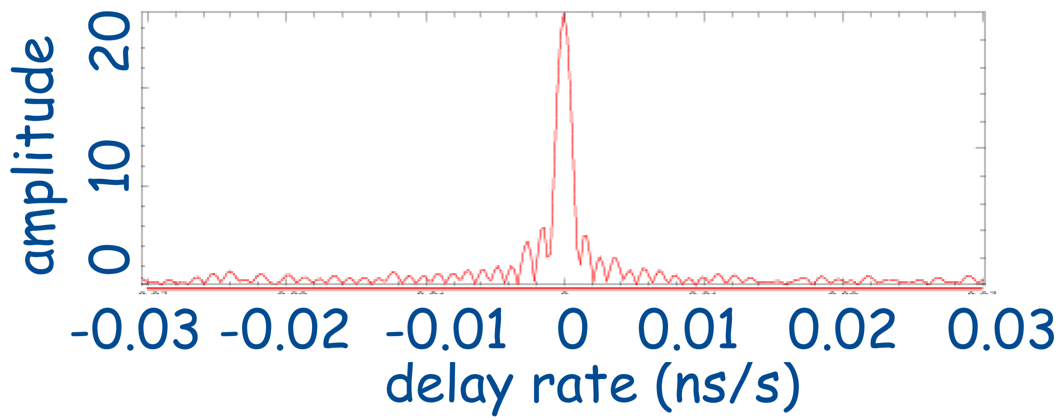
$$V(u,v,\tau) = \int V(u,v,\nu) e^{2\pi i \tau \nu} d\nu$$

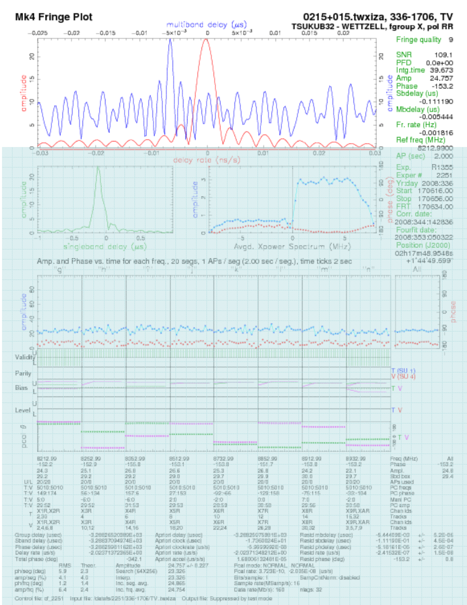




The data are already fringe fitted.

Fourfit: Delay Rate (DR)





Fringe rate (FR) is the Fourier transform of fringe visibility with respect to time.

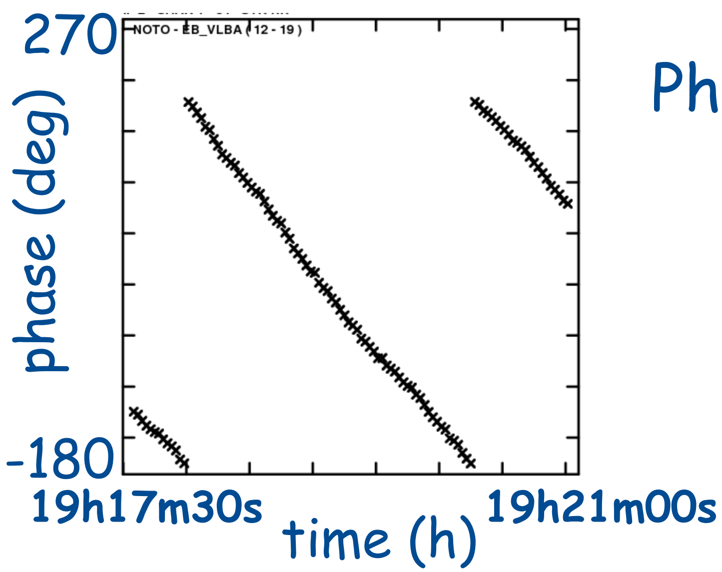
DR = FR / Observing frequency.

DR window = [1 /(2 * AP)] / Obs. Freq.

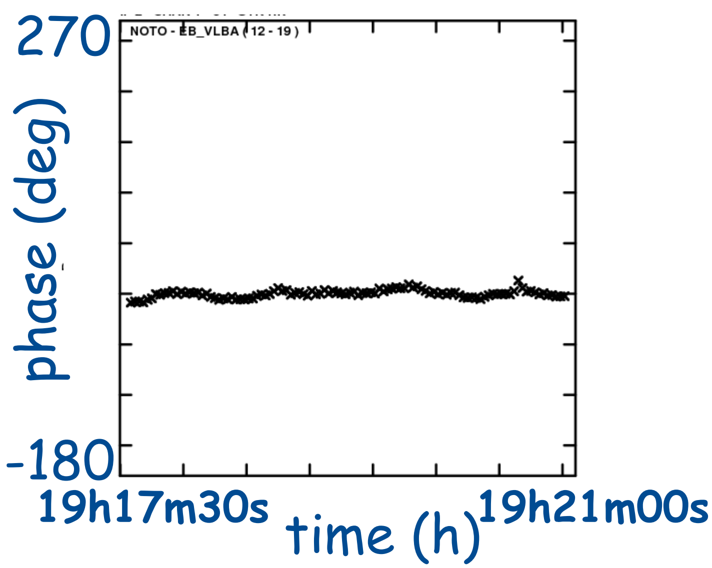
DR tells how fast the fringes move away from the phase centre due to correlator model error. It can be absorbed in the clock rate.

Post Correlation Software

Due to errors in the model, the correlator phases still show a slope vs time:



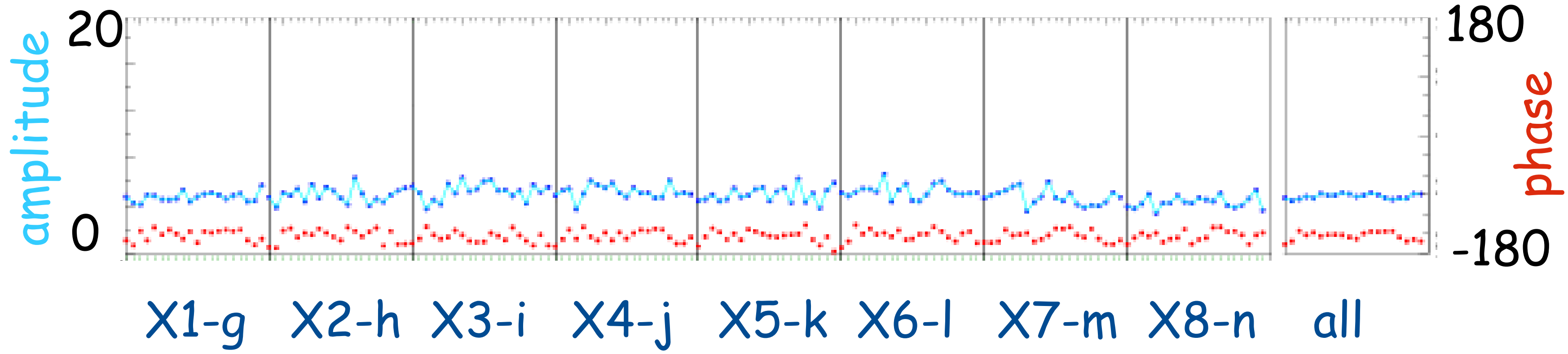
Phase slope vs time is "fringe rate"

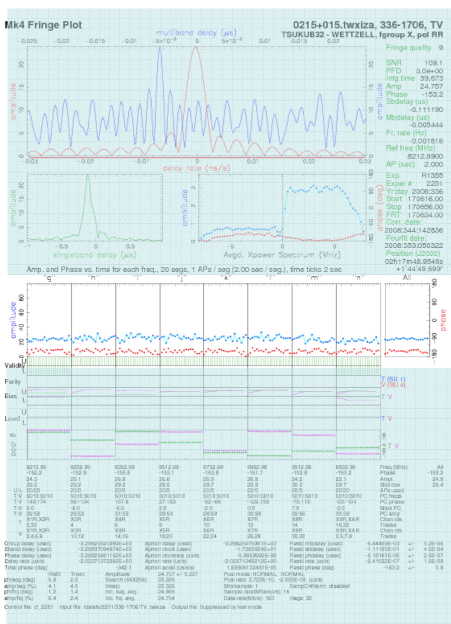


Fringe Fit refines the model removing the fringe rate

Fringe Fit: Amp. & Phase vs Time

Amp. & Phase vs time for each frequency



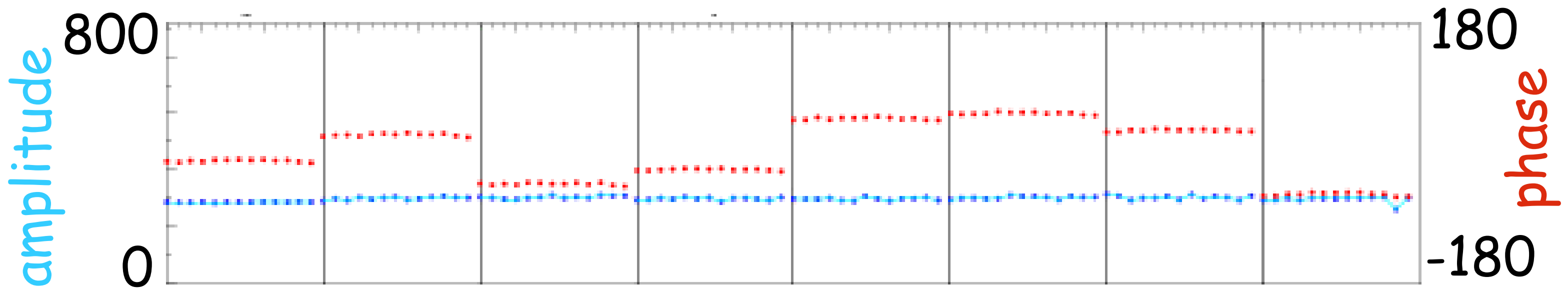


- Every dot represents the phase (red) and amplitude (blue) of the visibility for every segment (~ AP).
- Data are already fringe fitted and pcal has been applied.
- Every BBC/VC channel is represented.

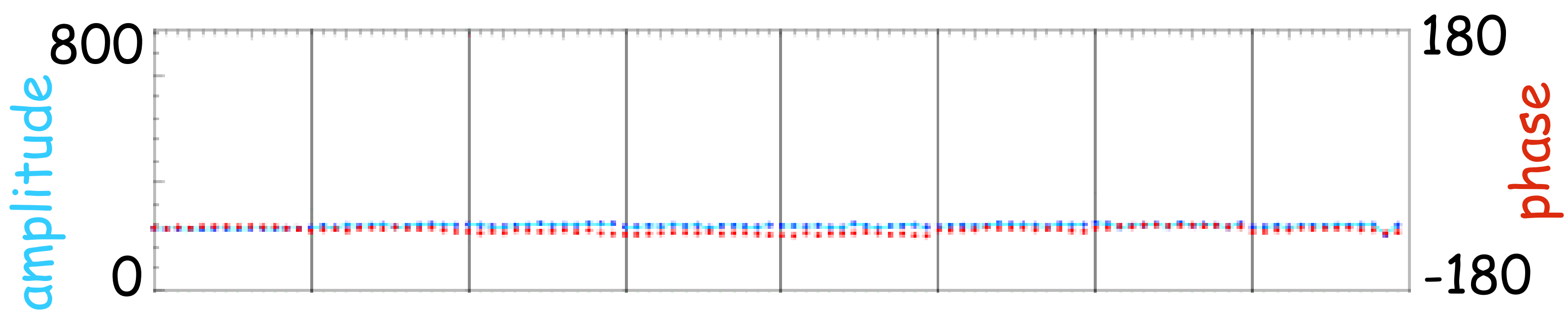
Fringe Fit: Phase Cal Phase

Corrects phase offset of each BBC/VC.

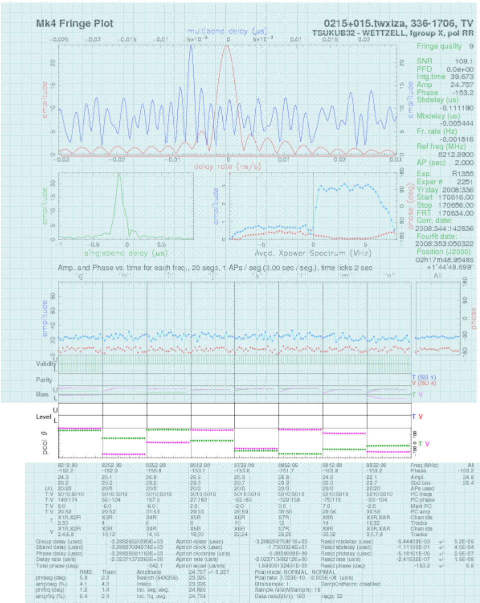
Phase offsets within the BBCs/VCs still present.



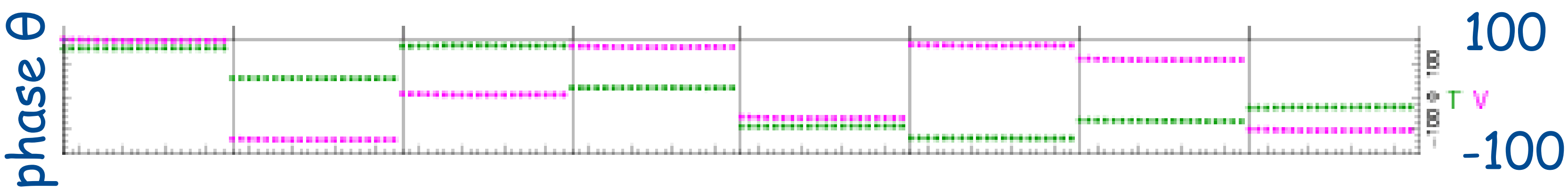
Phase cal phase flattens the phases across the band.



Fringe Fit:Phase Cal Phase Plot



Phase cal phase are plotted whilst only the value of the mean coherent pcal amplitude (PC amp.) is written for each channel.



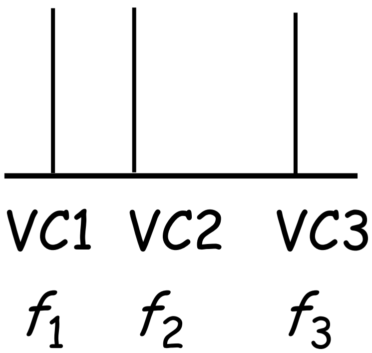
Reference Station ··············

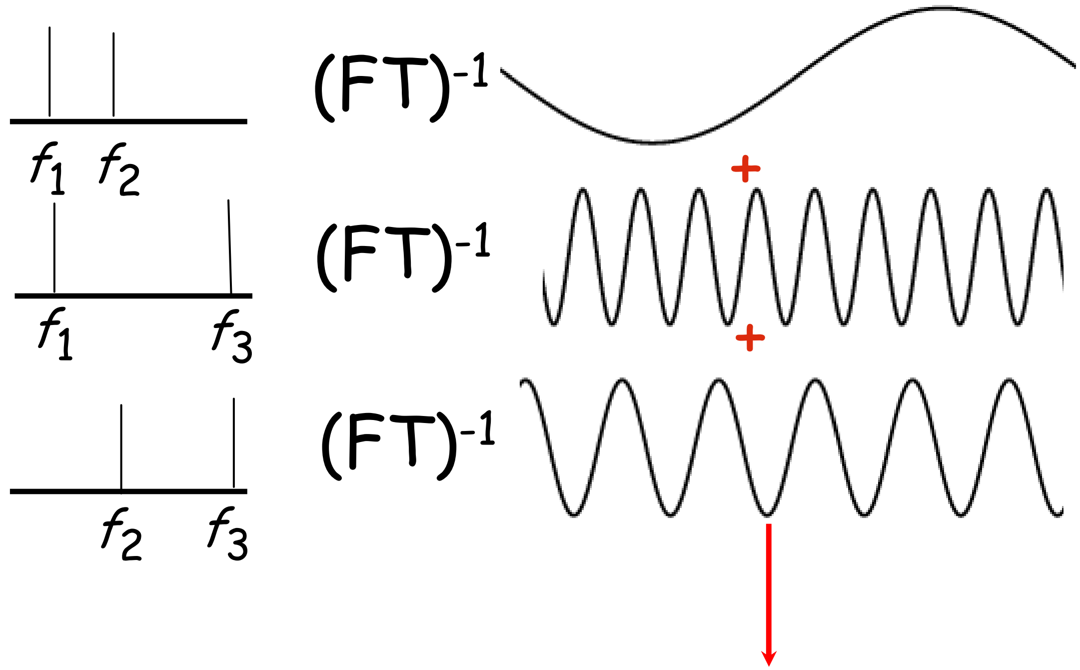
Remote Station ··············

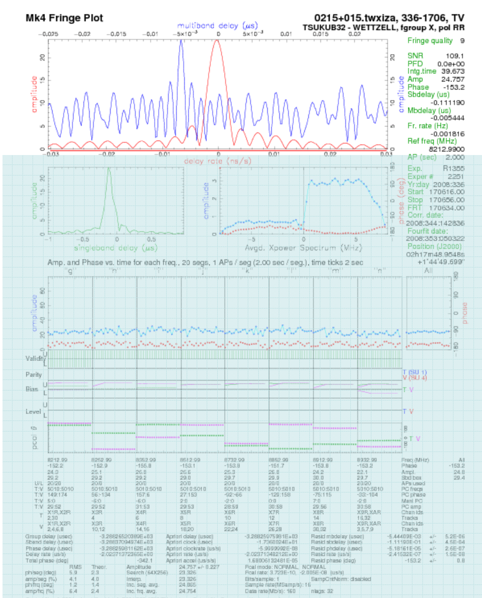
igg

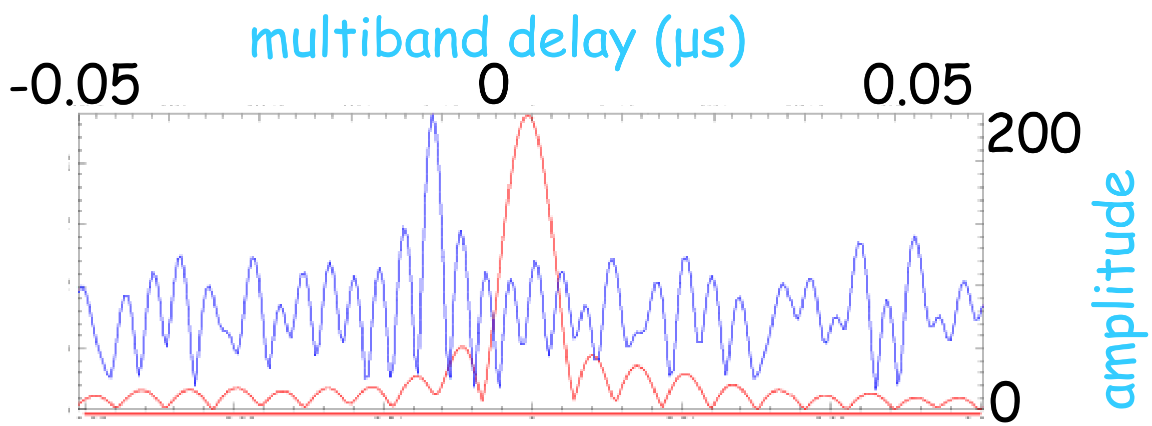
Fringe Fit: Multiband Delay (MBD)

universitätbonn



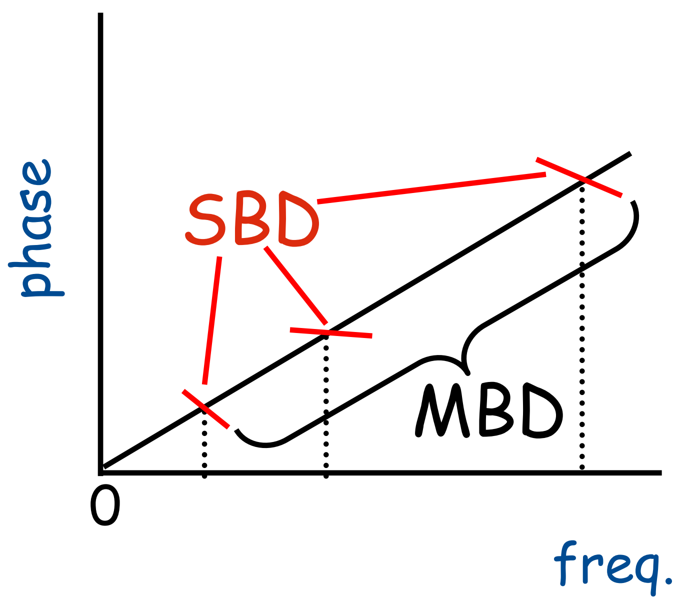






Fringe Fit: SBD vs MBD

- SBD = slope of phase across each frequency channels.
- MBD = slope of phase vs whole RF band (e.g. 720 MHz).
- SBD is not corrected by pcal (since fourfit uses only one tone).
- MBD is corrected by pcal.



MBD is more precise than the SBD

0215+015.twxiza, 336-1706, TV
TSUKUB32 - WETTZELL, fgroup X, pol RR

Ref-Rem, Band, Polarization

Fringe quality 9

SNR 109.1
PFD 0.0e+00
Intg.time 39.673
Amp 24.757
Phase -153.2
Sbdelay (us) -0.111190
Mbdelay (us) -0.005444
Fr. rate (Hz) -0.001816
Ref freq (MHz) 8212.9900
AP (sec) 2.000
Exp. R1355
Exper # 2251
Yr:day 2008:336
Start 170616.00
Stop 170656.00
FRT 170634.00
Corr. date: 2008:344:142836
Fourfit date: 2008:353:050322
Position (J2000) 02h17m48.9548s +1°44'49.699"

SNR = Peak amp / σ

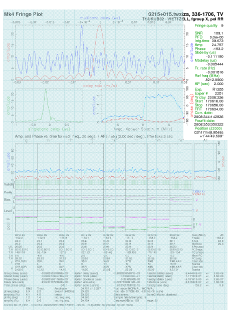
Depends on amp. & phase rms vs frequency and vs time.

Prob. of false detection. i.e. that a noise spike exceeds the signal amp.

Mean visibility amp. & phase

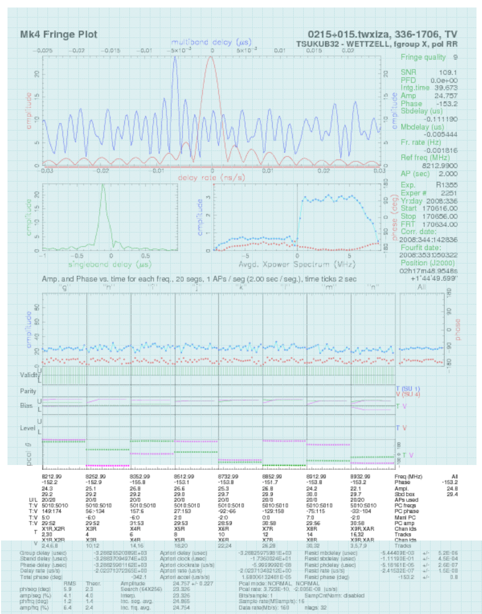
Residual SBD (µs)
Residual MBD (µs)
Residual FR (Hz)

Accumulation Period length, Fourfit Reference Time, ...

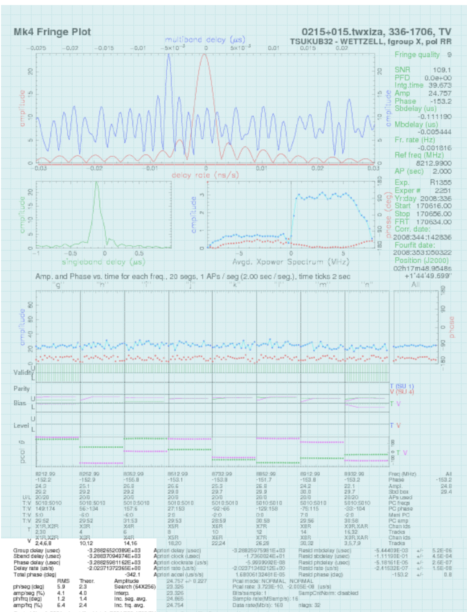


Fourfit: Wordy Part II

	8210.99	8220.99	8250.99	8570.99	Freq (MHz)	All
	-93.0	-95.7	-99.0	-96.7	Phase	-96.5
	291.1	304.7	308.8	301.2	Ampl.	301.9
	35.6	35.6	35.8	35.7	Sbd box	35.7
U/L	13/13	13/0	13/0	13/13	APs used	
B:N	2010:2010	2010:2010	2010:2010	2010:2010	PC freqs	
B:N	-145:143	-147:30	-14:69	-33:-172	PC phase	
B:N	0:0	0:0	0:0	0:0	Manl PC	
B:N	33:96	33:94	33:93	35:72	PC amp	
B	X1R,X2R	X3R	X4R	X9R,XAR	Chan ids	
	2,4,6,8	10,12	14,16	3,5,7,9	Tracks	
N	X1R,X2R	X3R	X4R	X9R,XAR	Chan ids	
	2,4,6,8	10,12	14,16	3,5,7,9	Tracks	



Correlator model + residual = total



Group delay (usec)	5.61237419104E+03
Sband delay (usec)	5.61251768088E+03
Phase delay (usec)	5.61234964601E+03
Delay rate (us/s)	-4.99305351947E-01
Total phase (deg)	186.9

	RMS	Theor.
ph/seg (deg)	1.4	0.3
amp/seg (%)	0.9	0.5
ph/frq (deg)	3.5	0.2
amp/frq (%)	1.9	0.3

Amplitude	301.439 +/- 0.394
Search (32X256)	290.158
Interp.	290.158
Inc. seg. avg.	301.531
Inc. frq. avg.	301.938

rms values of phases & amps. vs frequency: measure of how stable the visibilities are within the total band spanned.

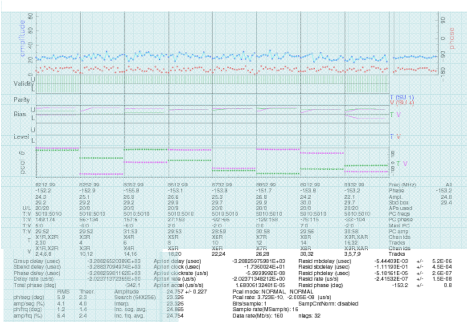
Fourfit: Wordy Part IV

Correlator model applied to the scan

Apriori delay (usec)	5.61234967866E+03
Apriori clock (usec)	3.1904583E+00
Apriori clockrate (us/s)	3.0000003E-08
Apriori rate (us/s)	-4.99305122619E-01
Apriori accel (us/s/s)	-3.38021266504E-05

Resid mbdelay (usec)	2.45124E-02	+/-	1.5E-06
Resid sbdelay (usec)	1.68002E-01	+/-	1.3E-04
Resid phdelay (usec)	-3.26489E-05	+/-	3.6E-08
Resid rate (us/s)	-2.29328E-07	+/-	3.4E-09
Resid phase (deg)	-96.5	+/-	0.1

Pcal mode: NORMAL, NORMAL
Pcal rate: -3.693E-08, -1.556E-08 (us/s)
Bits/sample: 1 SampCntNorm: disabled
Sample rate(MSamp/s): 8
Data rate(Mb/s): 80 nlags: 32



Residual correlator model errors calculated by fringe fit.

Fringe Fit: Control File

Fourfit's parameters are controlled through a control file:

- Scan start and stop time offset for the data to be considered valid.
- DR, MBD and SBD search window.
- Lower sideband offset: additive phase between LSB and USB when correlating VLBA data against Mark4 data.
- Phase cal frequency tone to be extracted.
- Phase cal mode: manual or normal or AP by AP.
- Phase cal phases specify a list of phases to be added to the visibility phases in each BBC/VC channel (if phase cal mode is normal).

Fringe Fit: Control File

cf_1234 is fourfit control file.

It tells fourfit what to do.

Basic layout:

`pc_mode normal` (pcal applied)

`sb_win -256.0 256.0 mb_win -2.0 2.0 dr_win -30.e-4 30.e-5`

- sb_win: sbd search window bounds (μs)
- mb_win: mbd search window bounds (μs)
- dr_win: delay rate search window bounds

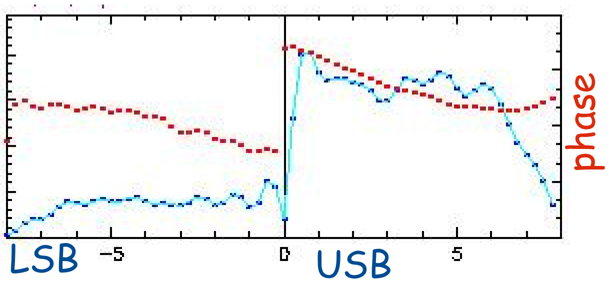
Keep the parameters as above to have a huge window.
If not specified fourfit defaults to a small window !

Fringe Fit: Control File

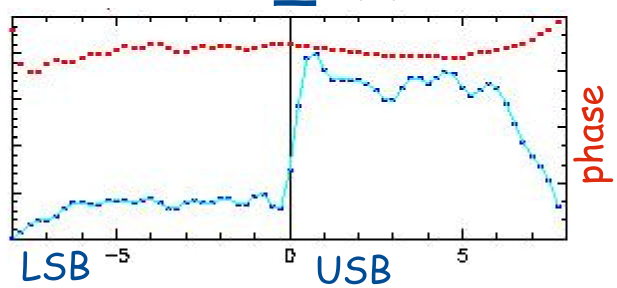
```
if station K
 lsb_offset 260.
```

LSB/USB offset for different backends

no lsb_offset



with lsb_offset



Phase cal tones extracted for X-band in kHz:

```
if f_group X
 ref_freq 8212.99
 pc_freqs ghijklmn 5010 5010 5010 5010 5010 5010 5010 5010
```

pivot frequency for fringe fit

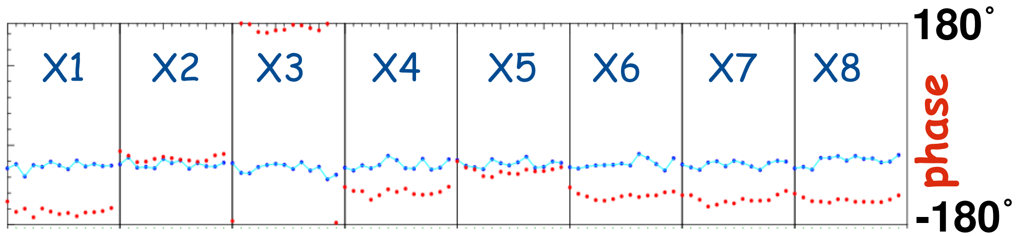
Phase cal tones extracted for S-band in kHz:

```
if f_group S
 ref_freq 2225.99
 pc_freqs abcdef 3010 3010 3010 3010 3010 3010
```

Fringe Fit: Control File

Manual phase cal:

```
if station J and f_group S
 pc_mode manual
 pc_phases abcdef -110 -127 -130 -69 -155 -100
if station J and f_group X
 pc_mode manual
 pc_phases ghijklmn 78 123 148 78 115 116 70 104
```

⟺ Manual pcal required!

Additive phase (self cal)

```
if station L and f_group S
  pc_phases abcdef -3.2 0.6 3.6 0.4 0.5 -1.5
if station L and f_group X
  pc_phases ghijklmn -4.0 4.3 4.4 1.1 -0.5 0.8 -6.2 2.0
```