Geophysical Modeling in Geodetic VLBI Analysis

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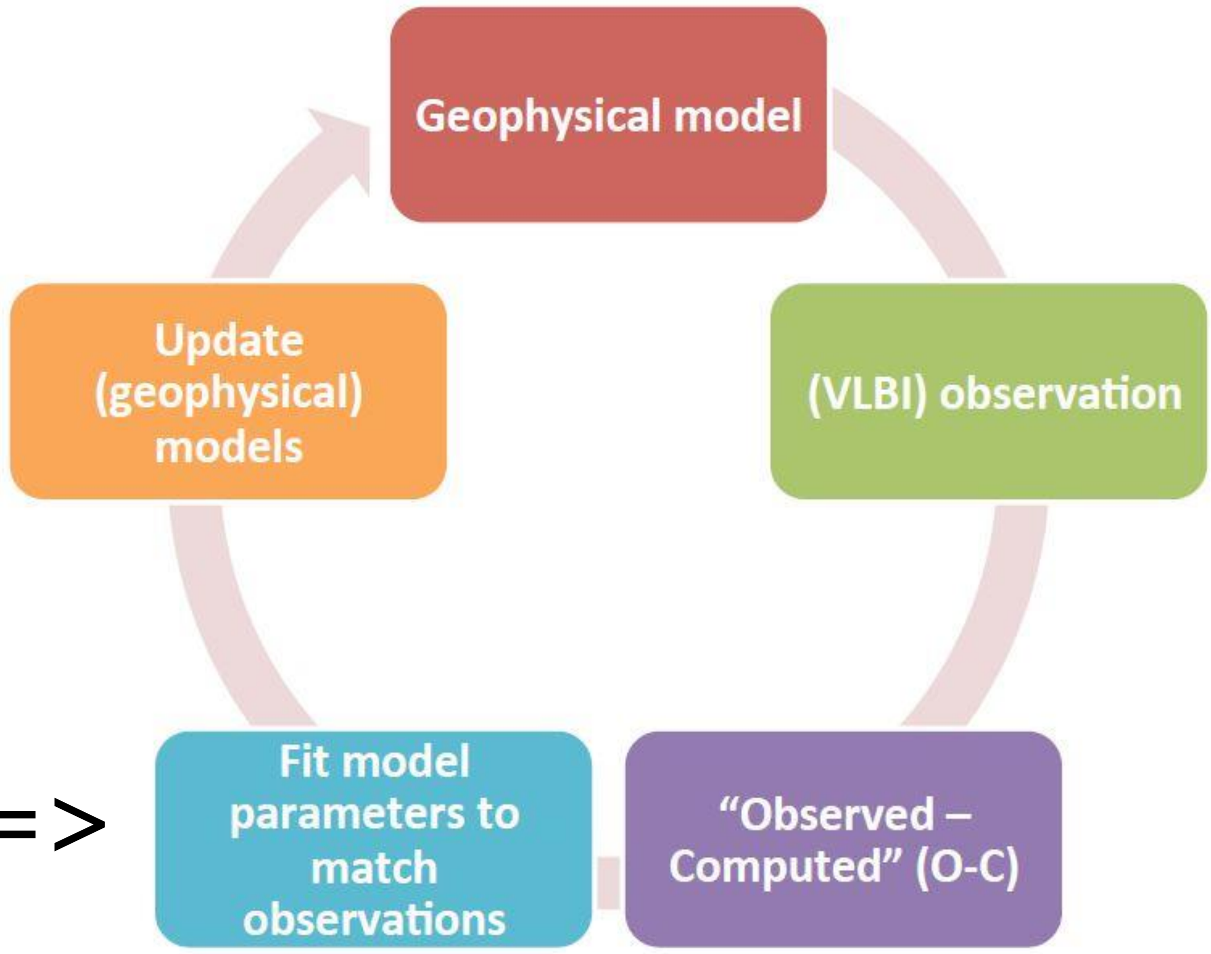
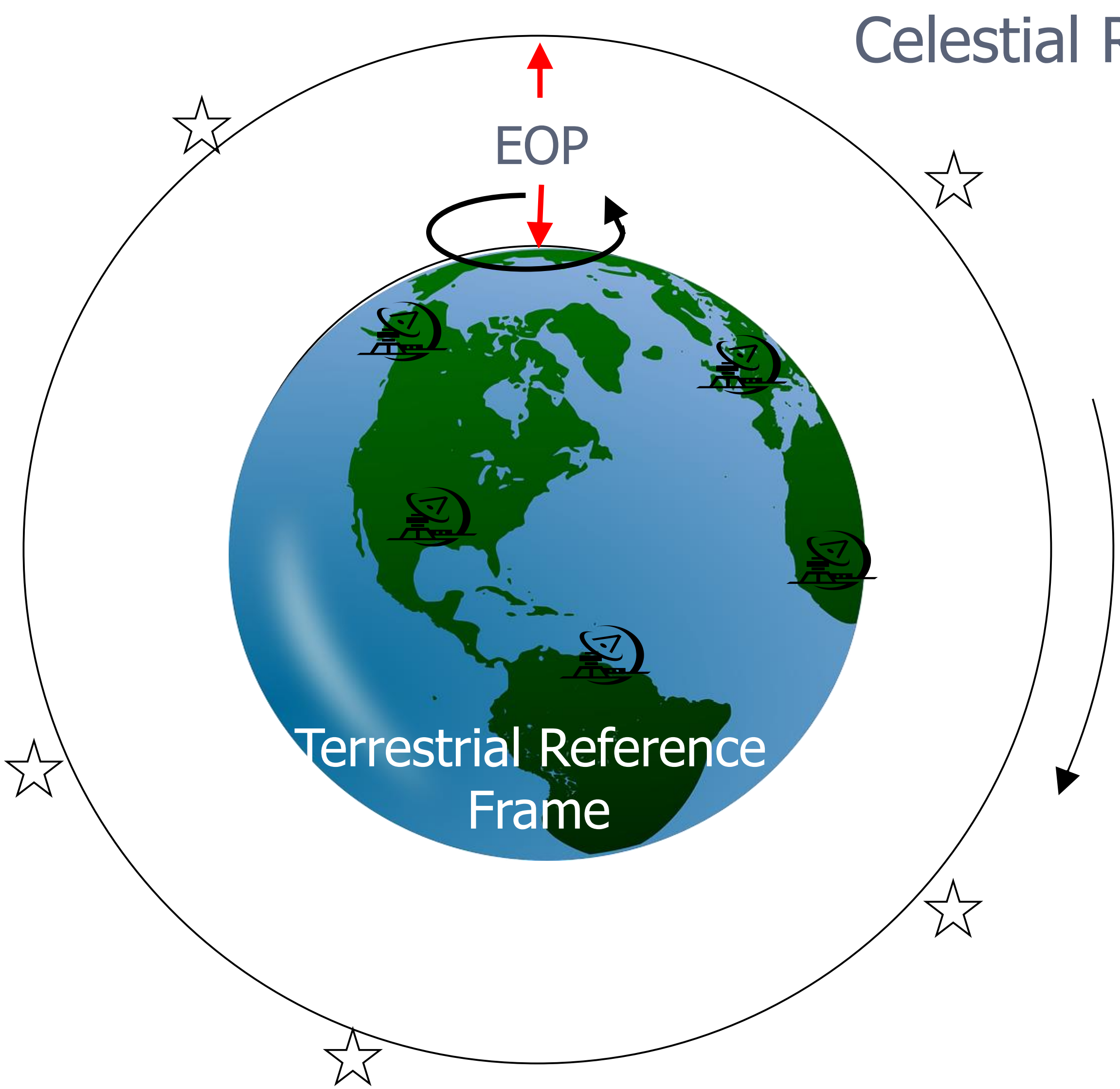
Geophysical Model Cycle



Use Analysis Software:
e.g., Calc/Solve, VieVS =>

Reference Frames



CRF realized by positions of a set of observed quasars

Earth Orientation Parameters (EOPs):
- Connect the two frames
- Describe irregularities of Earth rotation

TRF realized by positions of a set of observed antenna positions

Geophysical Modeling->Theoretical Delay

1. Get site coordinates in the TRF at observation epoch

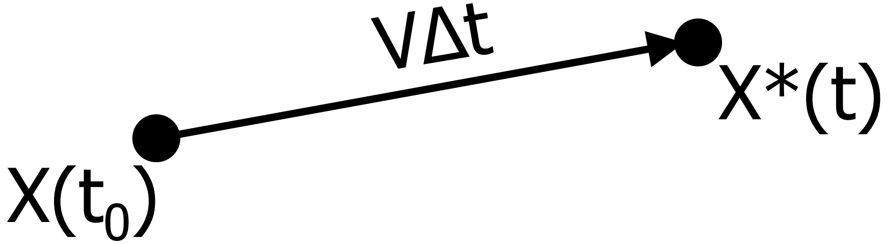
2. Add displacement models

3. Transform positions in crust-fixed terrestrial reference frame (TRF) to geocentric celestial reference system CRF (J2000) : Precession/Nutation, Polar Motion, UT1

4. Account for gravitational delay contribution due to the Sun and planets

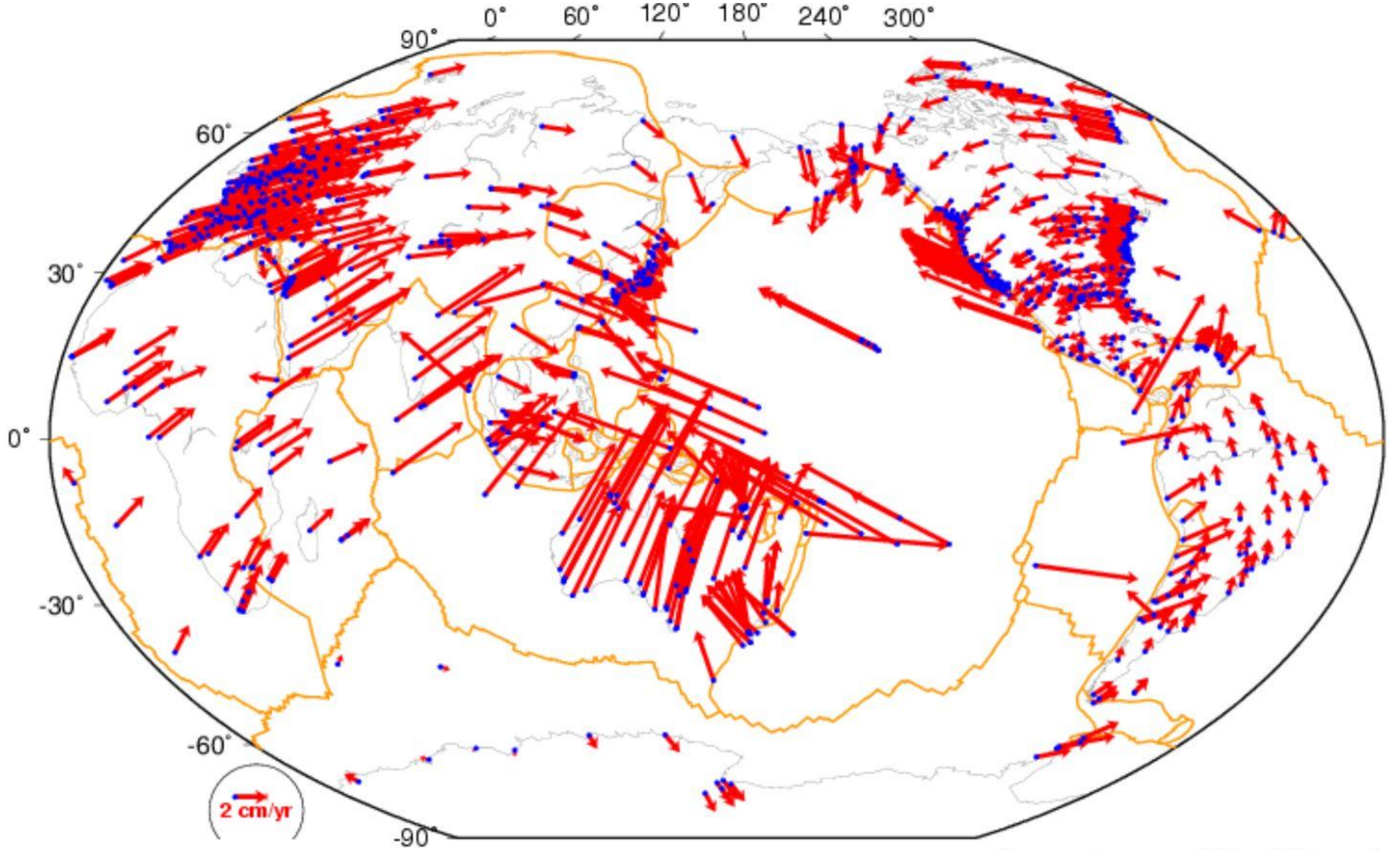
1. ITRF position at epoch

ITRF -> coordinates **X(t_0)** at reference epoch t_0
and velocities **V** for each site



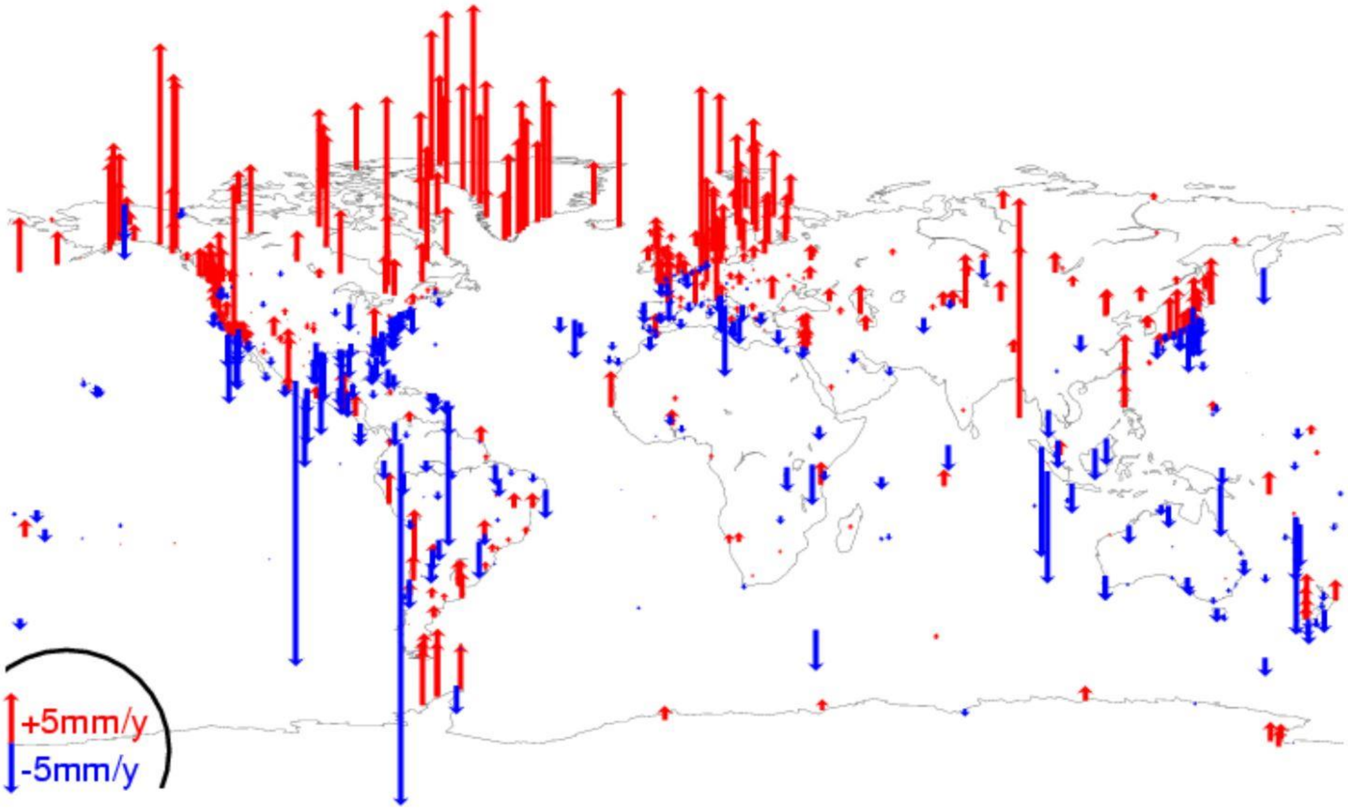
First find the ITRF station position at the epoch of observation.

ITRF2014 Horizontal Velocities



(courtesy of Z. Altamimi, IGN)

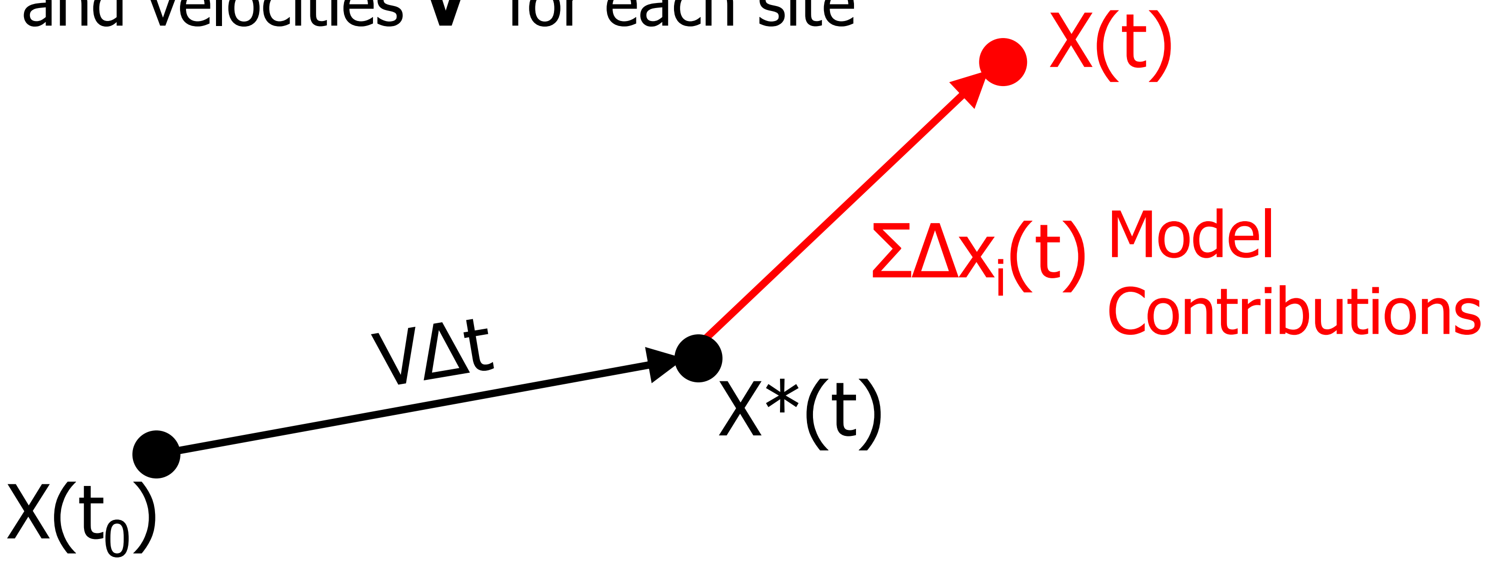
ITRF2014 Vertical Velocities



(courtesy of Z. Altamimi, IGN)

2. Model Displacements

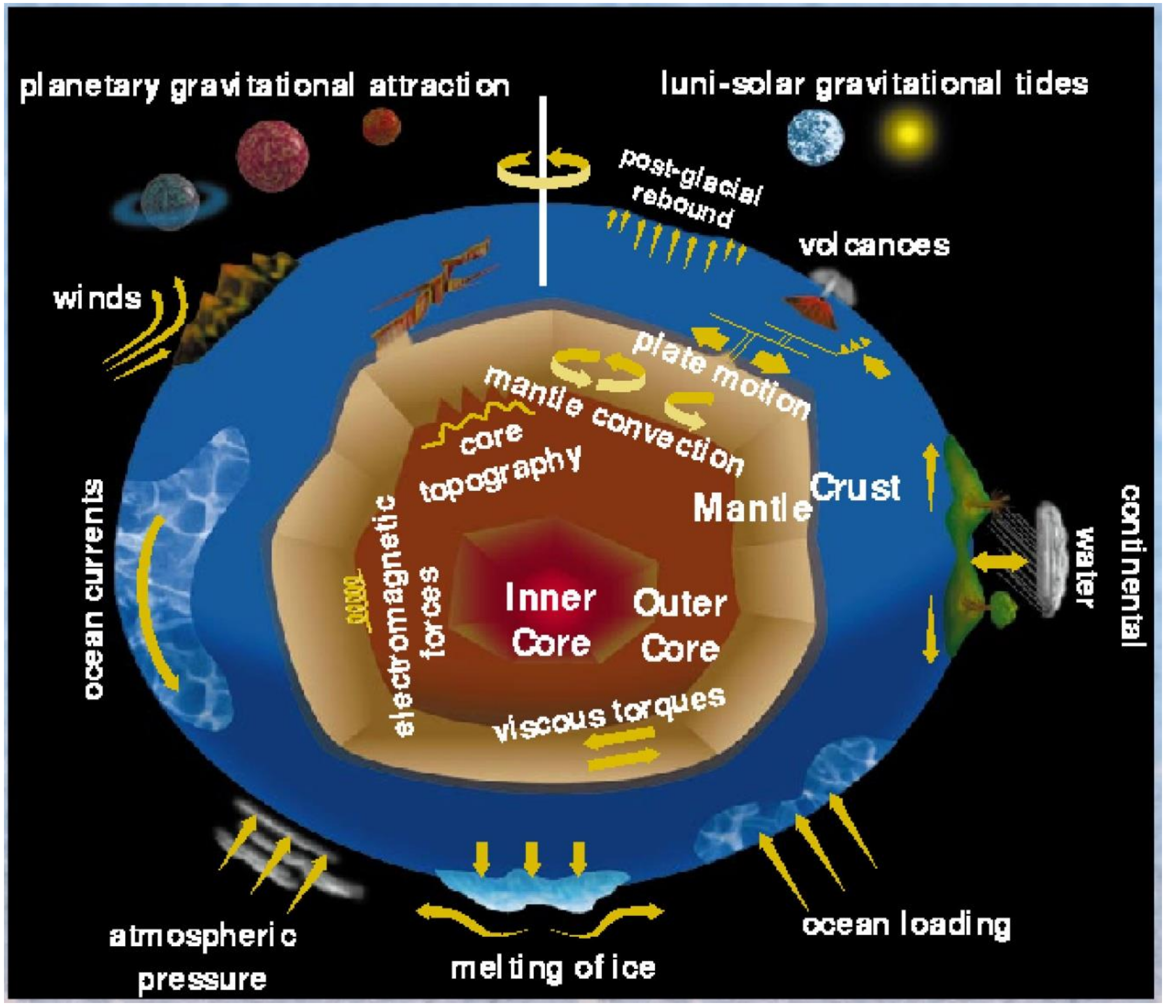
ITRF -> coordinates $\mathbf{X(t_0)}$ at reference epoch t_0
and velocities **V** for each site



First find the ITRF station position at the epoch of observation.

Then add the model displacements

Causes of Site Motion and Variations in Earth Orientation

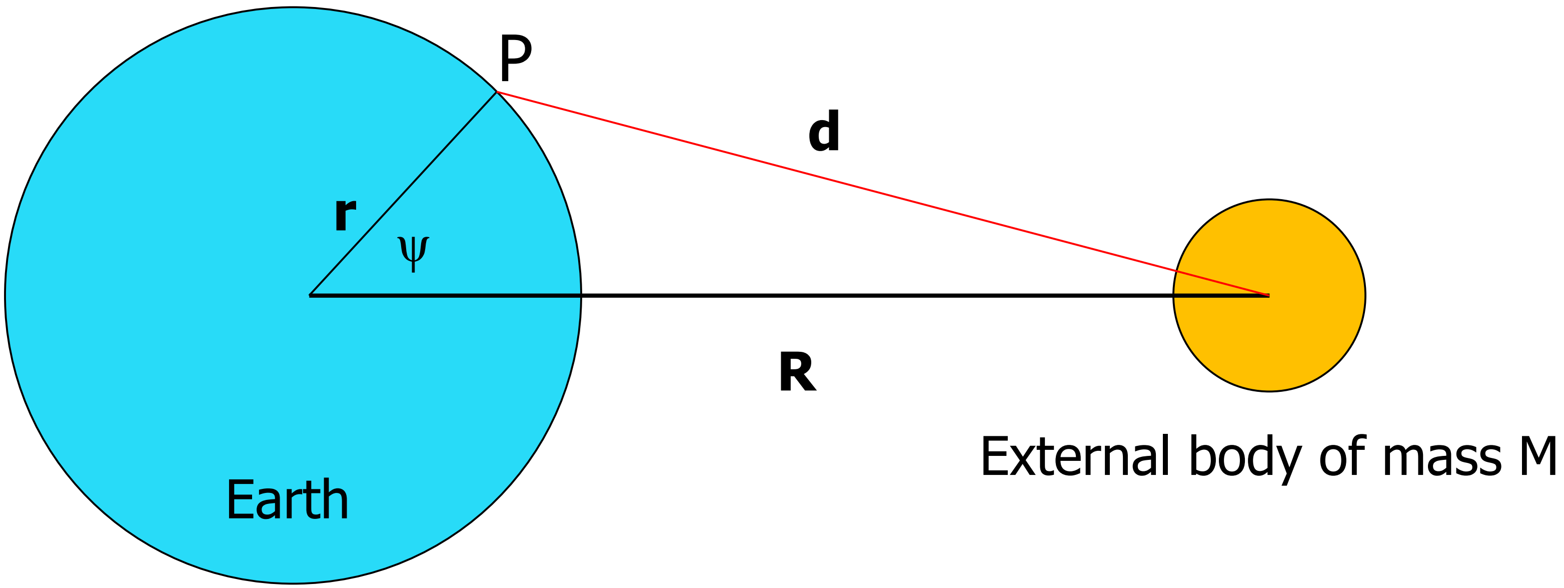


Displacement Models

- Solid Earth tide (largest for 12 h band, up to 40 cm in vertical)
- Ocean Loading (largest for 12 h band, mm-cm in vertical)
- Pole Tide (12,14 months period, mm-cm)
- Ocean Pole Tide Loading (mm)
- Tidal S1-S2 atmosphere loading (few mm)
- Non-tidal loading
  - Atmosphere Pressure Loading
  - Hydrology Loading

Tidal Deformations

Solid Earth tides are a deformation of the Earth caused mainly by the attraction of the Sun and Moon.



Potential at P: W = GM/d

Tide Generating Potential

Use the reciprocal distance expansion for the potential W = GM/d

$$W = \frac{GM}{R}\sum_{n=0}^{\infty} \frac{r^n}{R^n} P_n(\cos\psi) = W_0 + W_1 + W_2 + \cdots$$

Remember that force is the gradient of a potential

$$a_0 = \nabla W_0 = 0$$

$$W_1 = \frac{GM}{R^2}\, r cos\psi = \frac{GM}{R^2}\, x$$ In external body direction

$$a_1 = \nabla W_1 = \left(\frac{GM}{R^2}, 0, 0\right)$$ $\Rightarrow$ Independent of position on the Earth
No contribution to tidal force

Tide Generating Potential

Main Tidal Potential

$$W_2 = \frac{GM}{R}\left(\frac{r}{R}\right)^2 P_2(\cos\psi)$$

Third-degree Potential is much weaker!

$$W_3 = \frac{GM}{R}\left(\frac{r}{R}\right)^3 P_3(\cos\psi)$$

$$\frac{W_2}{W_3} = \left(\frac{R}{r}\right)\left(\frac{P_2}{P_3}\right) \geq 80$$

Relative strength of the lunar and solar tide-raising forces

$$\frac{a_L}{a_S} = \frac{M_{moon}}{M_{sun}}\left(\frac{R_{sun}}{R_{moon}}\right)^3 \approx 2.2$$

Tide Generating Potential

Expand the potential as function of
(θ, λ, r) : geocentric station coordinates
(θ', λ', R) : geocentric coordinates of the Sun, Moon or planet

using $\quad cos\psi = \cos\theta\cos\theta'(t) + sin\,\theta\,\sin\theta'(t)\,\cos(\lambda'(t) - \lambda)$

$$W_{Tidal}(t) = GM\sum_{n=2}^{\infty}\sum_{m=0}^{n}\frac{r^n}{R^{n+1}}P_{nm}(\cos\theta)P_{nm}(\cos\theta'(t))\,cos\left[m(\lambda-\lambda'(t))\right]$$

Second harmonic is just:

$$\frac{GMr^2}{R^3}\left\{\left(\left[\frac{3}{2}cos^2\theta-\frac{1}{2}\right]\left[\frac{3}{2}cos^2\theta-\frac{1}{2}\right]\right.\right.$$
$$+\,cos\theta sin\theta cos\theta' sin\theta'\cos(\lambda-\lambda')$$
$$\left.\left.+\,\frac{1}{4}sin^2\theta sin^2\theta' cos2(\lambda-\lambda'\right)\right\}$$

Long period m=0

Diurnal m=1

Semi-diurnal m=2

Full tidal potential expansion (Hartmann & Wenzel) -> See IERS Standards

Diurnal EOP

The effects of the tidal potential show up in many different places.

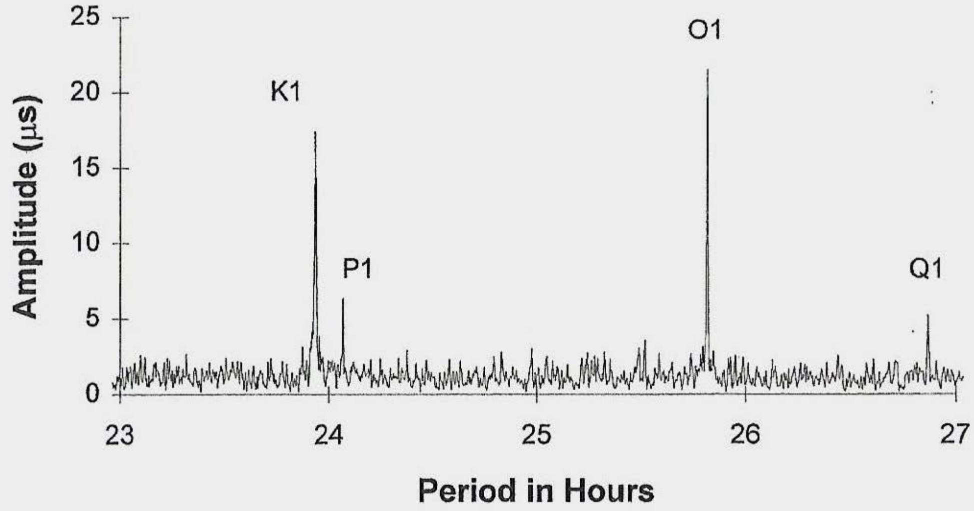


Figure 2. Frequency spectrum of UT1 derived from 15 years of hourly estimates (VLBI data). The "fast periodogram" algorithm of *Press and Rybicki* [1989] as implemented in *Press et al.* [1992] was used to extract the spectrum.

J.M. Gipson, JGR (1995)

Spectrum of UT1 (variations in Earth axial rotation rate) in the diurnal band. Similar spectra for semi-diurnal or for rotation about the X and Y axes.

Tide Generating Potential

Table 3.3: Selected tidal waves – long-period components (The potential amplitudes are taken from the Hartmann & Wenzel (1995) tidal potential catalogue and transformed to heights in mm).

Symbol	Doodson Nr.	Argument	Period [days]	Amplitude [mm]	Origin M/S
M_0	055.555	–	permanent	215	M
S_0	055.555	–	permanent	100	S
	055.565	N'	6798	28	M
Sa	056.554	$(h - p_S)$	365.26	5	S
Ssa	057.555	$2h$	182.62	31	S
Sta	058.554	$2h + (h - p_S)$	121.75	2	S
MSm	063.655	$(s - 2h + p)$	31.81	7	M
Mm	065.455	$(s - p)$	27.56	35	M
MSf	073.555	$2(s - h)$	14.77	6	M
Mf	075.555	$2s$	13.66	67	M
Mf'	075.565	$2s + N'$	13.63	28	M
$MStm$	083.655	$2s + (s - 2h + p)$	9.56	2	M
Mtm	085.455	$2s + (s - p)$	9.13	13	M
Mtm'	085.465	$2s + (s - p) + N'$	9.12	5	M
$Msqm$	093.555	$2s + 2(s - h)$	7.10	2	M

Table 3.4: Selected tidal waves – short-period components (The potential amplitudes are taken from the Hartmann & Wenzel (1995) tidal potential catalogue and transformed to heights in mm).

Symbol	Doodson Nr.	Argument	Period [hours]	Amplitude [mm]	Origin M/S
$2Q_1$	125.755	$(\tau - s) - 2(s - p)$	28.01	6	M
σ_1	127.555	$(\tau - s) - 2(s - h)$	27.85	8	M
Q_1'	135.645	$(\tau - s) - (s - p) - N'$	26.87	9	M
Q_1	135.655	$(\tau - s) - (s - p)$	26.87	50	M
ρ_1	137.455	$(\tau - s) - (s - 2h + p)$	26.72	10	M
O_1'	145.545	$(\tau - s) - N'$	25.82	49	M
O_1	145.555	$(\tau - s)$	25.82	262	M
	155.455	$(\tau - s) + (s - p)$	24.85	7	M
M_1	155.655	$(\tau + s) - (s - p)$	24.83	21	M
π_1	162.556	$(t - h) - (h - p_S)$	24.13	7	S
P_1	163.555	$(t - h)$	24.07	122	S
S_1	164.556	$(t + h) - (h - p_S)$	24.00	3	S
K_1^m	165.555	$(\tau + s)$	23.93	252	M
K_1^s	165.555	$(t + h)$	23.93	117	S
K_1'	165.565	$(\tau + s) + N'$	23.93	50	M
J_1	175.455	$(\tau + s) + (s - p)$	23.10	21	M
OO_1	185.555	$\tau + 3s$	22.31	11	M
OO_1'	185.565	$\tau + 3s + N'$	22.30	7	M
$2N_2$	225.855	$2\tau - 3(s - p)$	12.91	16	M
μ_2	237.555	$2\tau - 2(s - h)$	12.87	19	M
N_2	245.655	$2\tau - (s - p)$	12.66	121	M
ν_2	247.455	$2\tau - (s - 2h + p)$	12.63	23	M
M_2'	255.545	$2\tau - N'$	12.42	24	M
M_2	255.555	2τ	12.42	632	M
L_2	265.455	$2\tau - (s - p)$	12.19	18	M
T_2	272.556	$2t - (h - p_S)$	12.02	17	S
S_2	273.555	$2t$	12.00	294	S
K_2^m	275.555	$2(\tau + s)$	11.97	55	M
K_2^s	275.555	$2(t + h)$	11.97	25	S
K_2'	275.565	$2(\tau + s) + N'$	11.97	24	M
M_3	355.555	3τ	8.28	8	M

See S. Böhm

Solid Earth Tides

A.E.H. Love (1911) showed that the displacement response (deformation of the the Earth) is related to the tide generating potential V_T

$$\Delta uen = [\frac{h_n}{g} V_T \hat{r}, \frac{l_n}{g sin\theta} \frac{\partial V_T}{\partial \lambda} \hat{e}, \frac{l_n}{g} \frac{\partial V_T}{\partial \theta} \hat{n}]$$

h_n and l_n factors are called Love and Shida numbers
For degree n=2, l_2=0.0847 and h_2=0.6078

What are the most important terms?

deg	Moon	Sun
2	425 mm	173 mm
3	7.5 mm	0.008 mm
4	0.13 mm	0.000 mm

Remember from earlier:
Relative strength of forces

$W_2/W_3 > 80$

Solid Earth Tides

Other Contributions to Solid Earth Tides
[See IERS Conventions 2010]

- Necessary to reach the targeted accuracy better than 1 mm

- Requires additional correction terms to the in-phase terms described above:

  1) Out-of-phase correction arising from imaginary parts of Love numbers
  => models the anelastic component of deformation

  Anelastic deformation => earth response lags the time
  variation of the potential

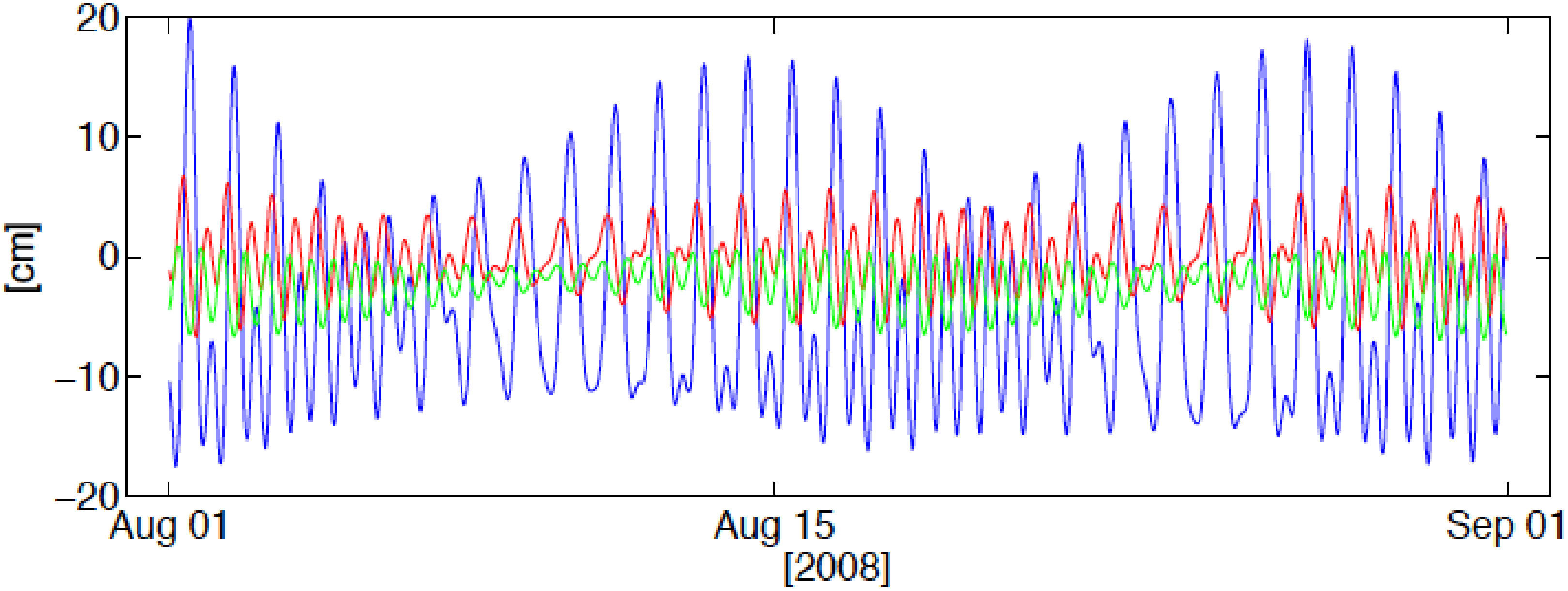
  2) Frequency domain corrections

  - in-phase correction for degree-2 in the diurnal and long-period bands
  - out-of-phase correction for degree-2 long-period band

Solid Earth Tides

Wettzell, Germany

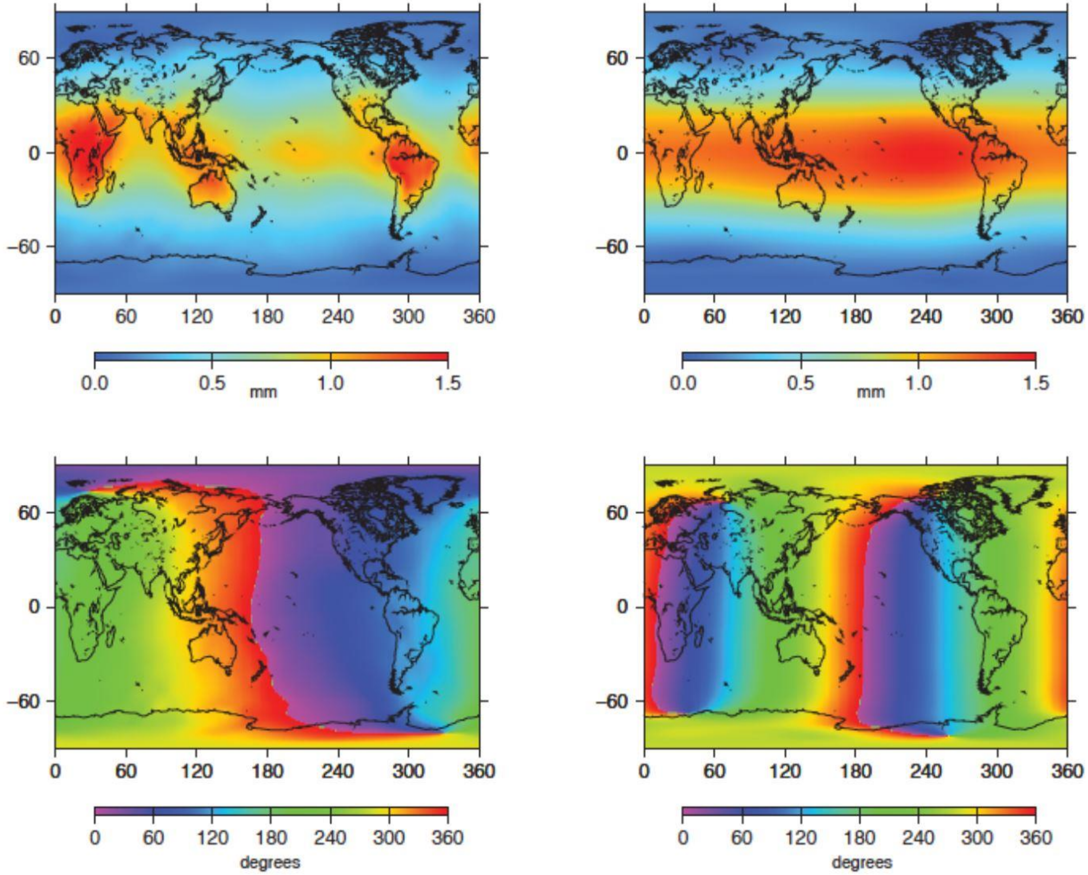
Radial, East, North



From Krasna, 2013,https://geo.tuwien.ac.at/fileadmin/editors/GM/GM91_krasna.pdf)

Atmosphere loading (S1/S2)

S1



S2

(from IERS Conv. 2010)

Tidal loading available as gridded correction

Deformation due to Polar Motion - Poletide

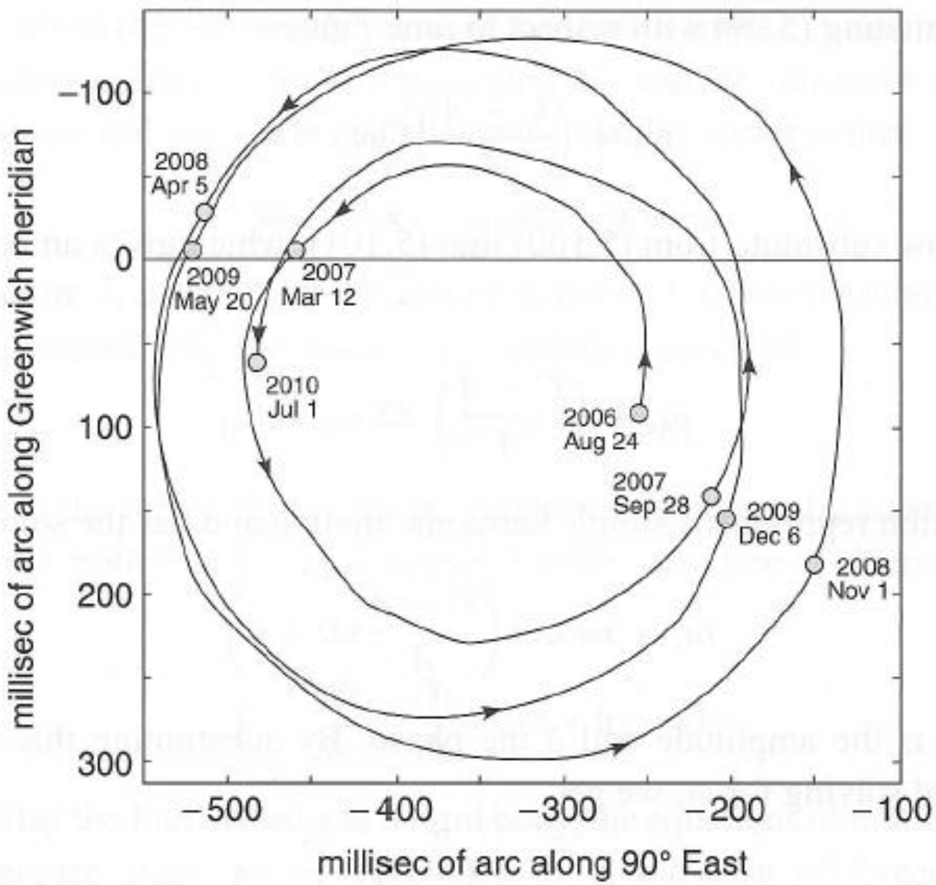


Fig. 5.8. The instantaneous rotation axis of the Earth exhibits a nearly circular motion with period 435 days – the Chandler wobble – and an annual circular motion. These motions are superposed on a slow drift of about 20 m per century along longitude 80 °W. Data source: International Earth Rotation and Reference Systems Service.

Motion of the rotation axis about the geographic pole:
Radius ~ 10-12m

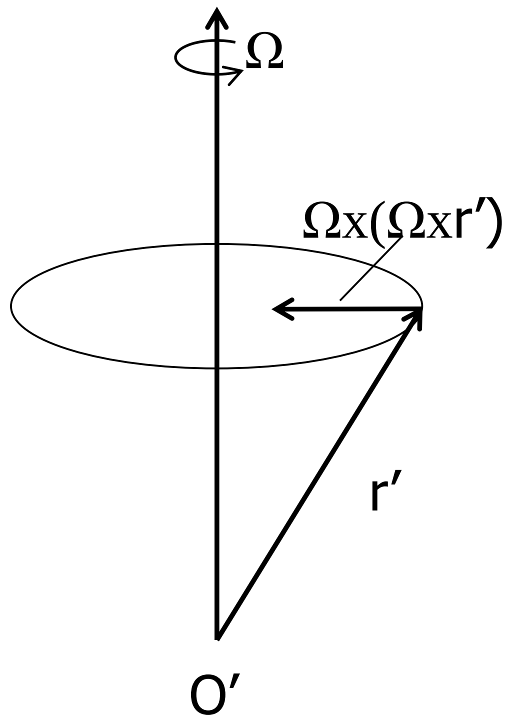
Deformation due to Polar Motion - Poletide

$$\vec{a} = \vec{a}' + 2\vec{\Omega}\times\vec{v}' + \vec{\Omega}\times(\vec{\Omega}\times\vec{r}')$$

Acceleration in the fixed frame, where the primed system is rotating

Coriolis acceleration

Centrifugal acceleration a_c



The centrifugal potential corresponding to this acceleration is

$$V_c = \frac{1}{2}\left[\Omega^2 r'2 \;-\left(\vec{\Omega}\cdot\vec{r'}\right)^2\right] = \frac{1}{2}\left|\vec{\Omega}\times\vec{r}'\right|^2$$

$$\vec{a}_c = -\nabla V_c$$

Deformation due to Polar Motion - Poletide

$$V_c = \frac{1}{2}\left[\Omega^2 r'2 \ - \left(\vec{\Omega}\cdot\overrightarrow{r'}\right)^2\right] = \frac{1}{2}\left|\vec{\Omega}\times\vec{r}'\right|^2$$

Angular rotation of the Earth

$$\vec{\Omega} = \Omega_0[m_x\hat{x} + m_y\hat{y} + (1+m_z)\hat{z}]$$

Average rotation rate

Time-dependent offsets of pole

Fractional variation in rotation rate

m_z variation ~ 1/100 m_x or m_y

$$V(\theta,\lambda) = -\frac{1}{2}\Omega_0^2 r^2[\sin 2\theta(m_x\cos\lambda + m_y\sin\lambda)]$$

See J. Wahr (1985)

Site displacement response to the potential via Love and Shida numbers:

$$\Delta U = \frac{h_2}{g}V \qquad \Delta E = \frac{l_2}{g}\frac{1}{\sin\theta}\frac{\partial}{\partial\lambda}V \qquad \Delta N = \frac{l_2}{g}\frac{\partial}{\partial\theta}V$$

$$h_2 = 0.6207, l_2 = 0.0836$$

Deformation due to Polar Motion - Poletide

$$\Delta U = -33 \sin 2\theta (m_x \cos\lambda + m_y \sin\lambda)$$

$$\Delta E = 9 \cos\theta (m_x \sin\lambda - m_y \cos\lambda)$$

in mm

$$\Delta N = -9 \cos 2\theta (m_x \cos\lambda + m_y \sin\lambda)$$

$$m_x = x_p - \bar{x}_p \qquad m_y = y_p - \bar{y}_p$$

polar motion

Mean pole from RS Conventions (2010): quadratic before 2010, linear after 2010

NEW linear model approved recently

Ocean Pole Tide Loading

Similar to pole tide loading

- Centrifugal potential causes ocean height to change
- Changes the distribution of the load on the crust
- Results in site displacement

Compute the site loading effect by convolving with loading Green's function

$$\Delta u(\theta,\lambda,t) = \int_{ocean} G(\theta,\lambda;\theta',\lambda')\rho_{seawater}\underbrace{[\eta(\theta',\lambda',t)-\overline{\eta}(t)]}_{\text{Ocean height difference}}d\Omega'$$

Subtract the average ocean height at each epoch
=> ocean mass conservation

See Wahr (1985) for first calculation

Ocean Pole Tide Loading

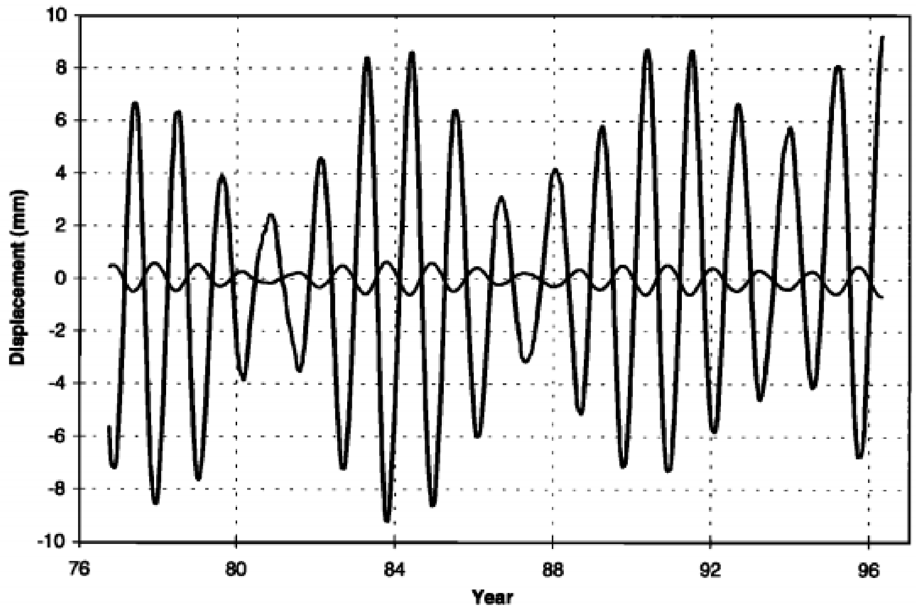


Figure 4. Up displacement at Haystack due to EOP variations. The larger-amplitude curve shows the direct effect of the pole tide. The smaller-amplitude curve shows the effect of polar motion induced ocean loading.

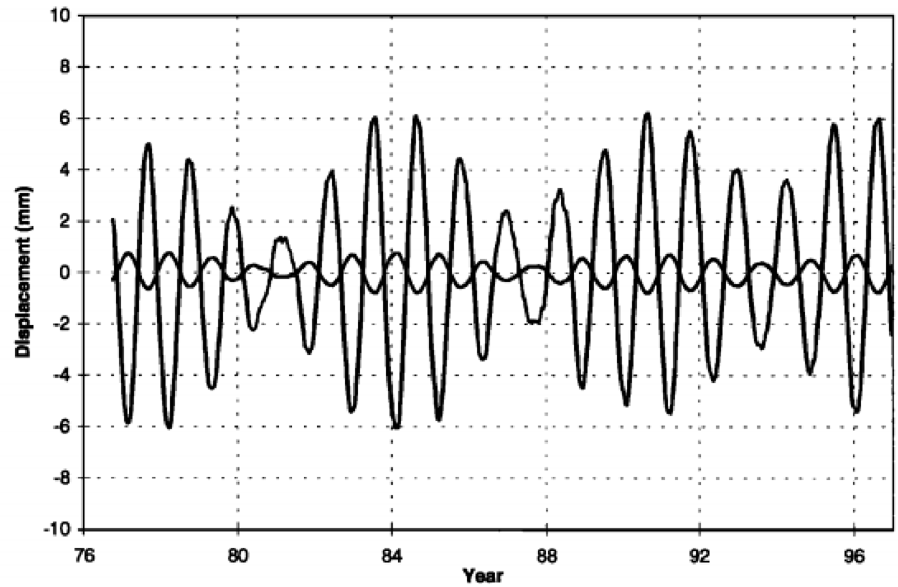


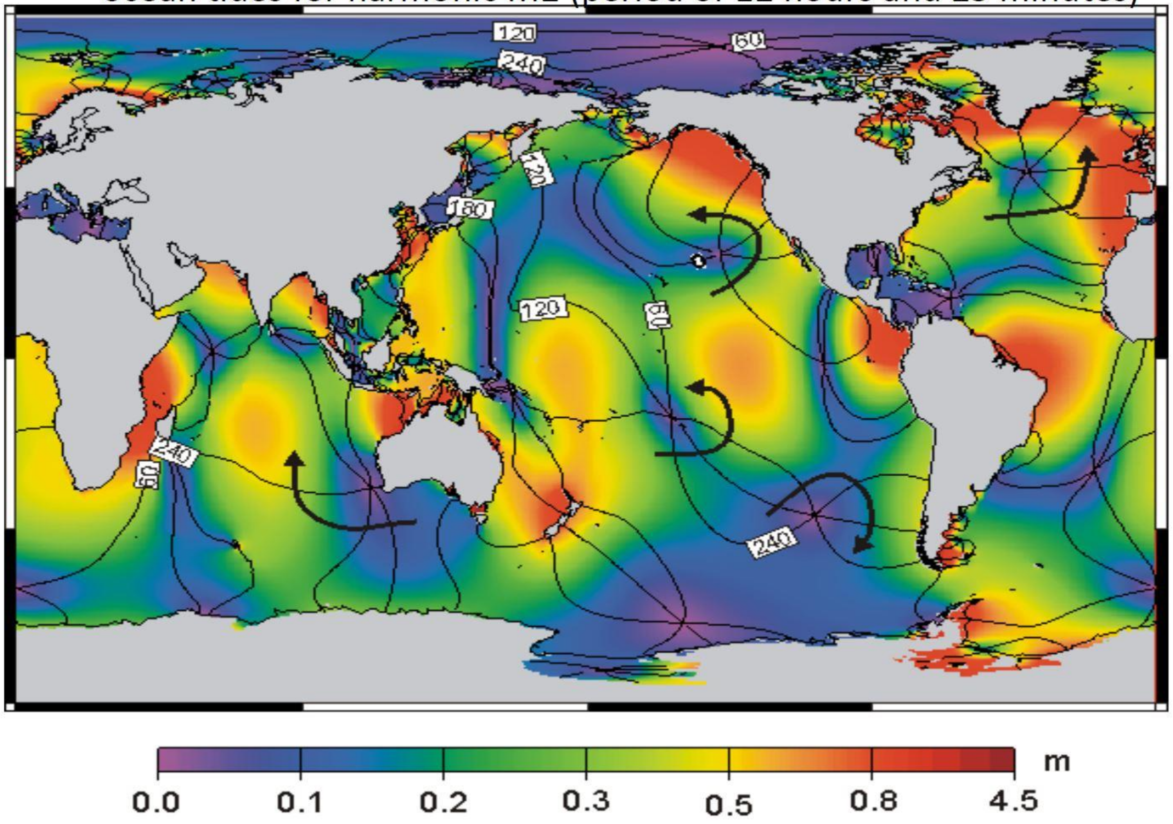
Figure 5. Up displacement at Kauai due to EOP variations. The larger-amplitude curve the direct effect of the pole tide. The smaller-amplitude curve shows the effect of polar motion induced ocean loading.

(Plots from Gipson and Ma, 1998)

- Pole tide effect is much larger than the polar motion induced loading
- The effects are 180 deg out of phase
- See S. D. Desai (2002) for version of model in IERS Conventions

Ocean Loading Tides

ocean tides for harmonic M2 (period of 12 hours and 25 minutes)



(from http://holt.oso.chalmers.se/loading/)

Ocean Tidal Loading

Computation of ocean tidal loading

- Tide elevation from global tide maps, where Z and δ are the amplitudes and phases of each specific partial tides k at (ϕ, λ)

$$\xi(k,\phi,\lambda,t) = Z_k(\phi,\lambda)\cos[\omega_k t + \chi_k - \delta_k(\phi,\lambda)]$$

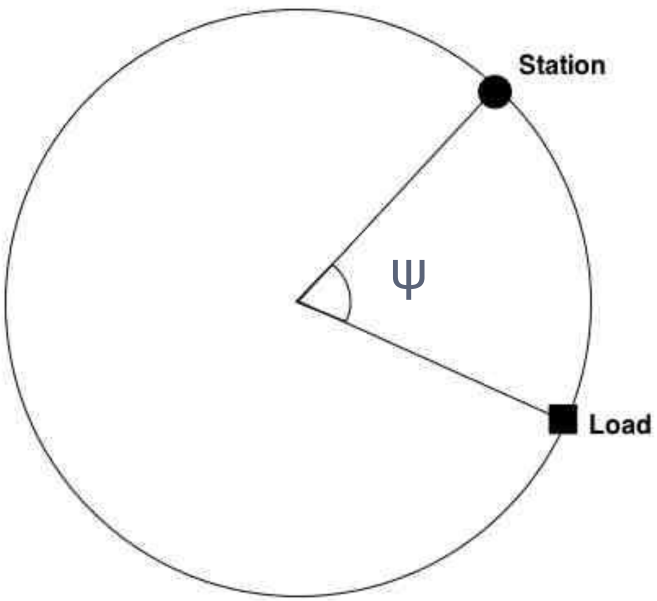
- Response of oceans to the Tide Generating Potential is much different than for solid earth

- Response depends strongly on local/regional conditions

- Loading at a site has to be computed by globally integrating the loading Green's function over the tide elevation mass for each tidal constituent k

$$\Delta x_k(\theta,\lambda,t) = \int_{ocean} G(\theta,\lambda;\ \theta',\lambda')\ \xi(k,\theta',\lambda',t)d\Omega'$$

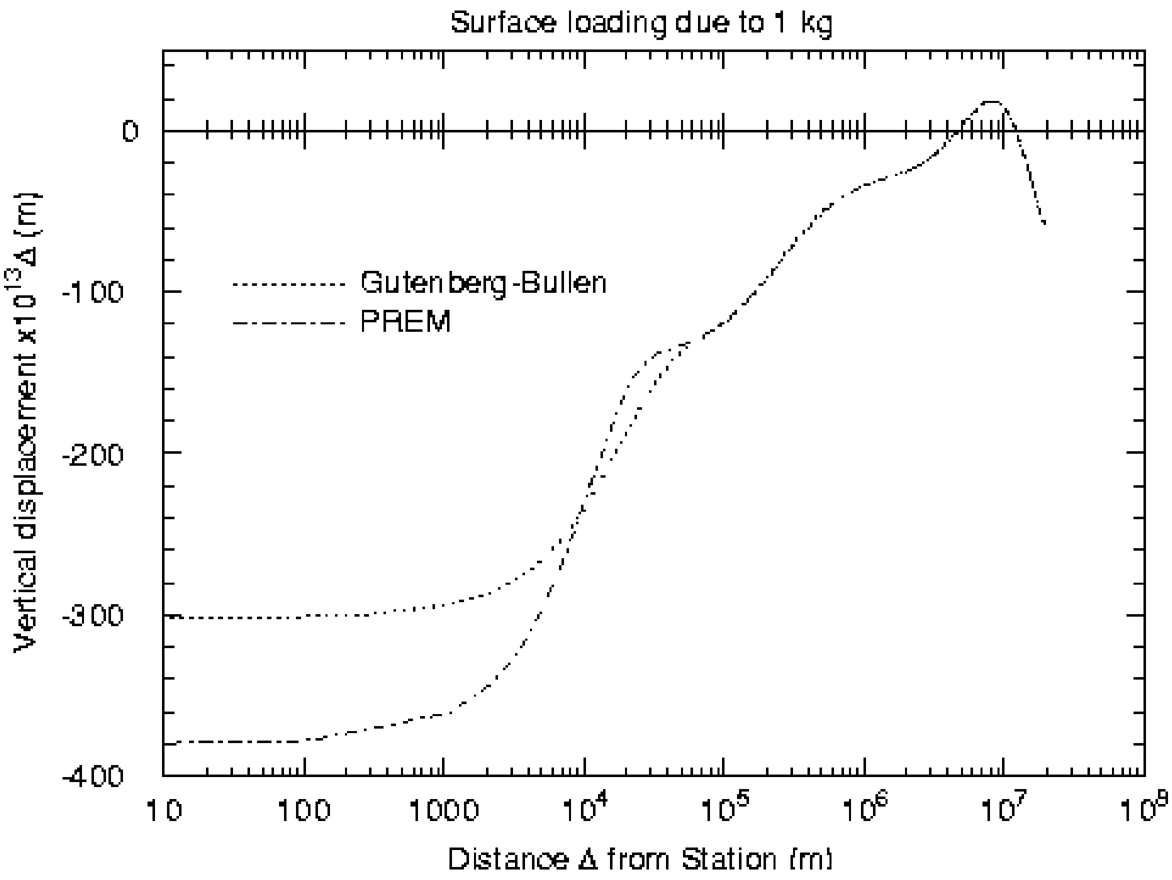
Loading Green's Functions



- Loading Green's function is the response at the station due to a mass load at an angular distance ψ from the station.
- Response is larger the closer the mass is to the station.
- Integration over the surface of the earth =>total adjustment of the station position caused by the surface mass distribution.
- Loading contribution is dominated by loading near the station as well as any large coherent regional loads far from the station.

Response to Ocean Tides

Globally integrate the loading Green's function over the tidal height elevation mass for each tidal constituent



(from http://holt.oso.chalmers.se/loading/)

Ocean Tidal Loading

Site displacement due to loading is given by a sum over 11 tides

$$x_k(t) = \sum_{j=1}^{11} A_{kj} \cos(\chi_j(t) - \Phi_{kj}), (k = U, E, N)$$

Astronomical argument of the tide

For each partial ocean tide, the tide crest occurs Φ_{kj} hours after the crest of the solid earth tide at the Greenwich meridian.

1) Loading UEN amp/phase are computed for 11 main tides (M2,S2,N2,K2,K1,O1,P1,Q1,Mf,Mm,Ssa) e.g., at Scherneck (Onsala) website

2) Better to also use HARDISP routine that computes loading based on 342 constituents found by interpolating tidal admittances based on the 11 main tides. (Error too large if keep only the 11 tide contribution) See IERS 2010 Conventions.

Ocean Loading Tides

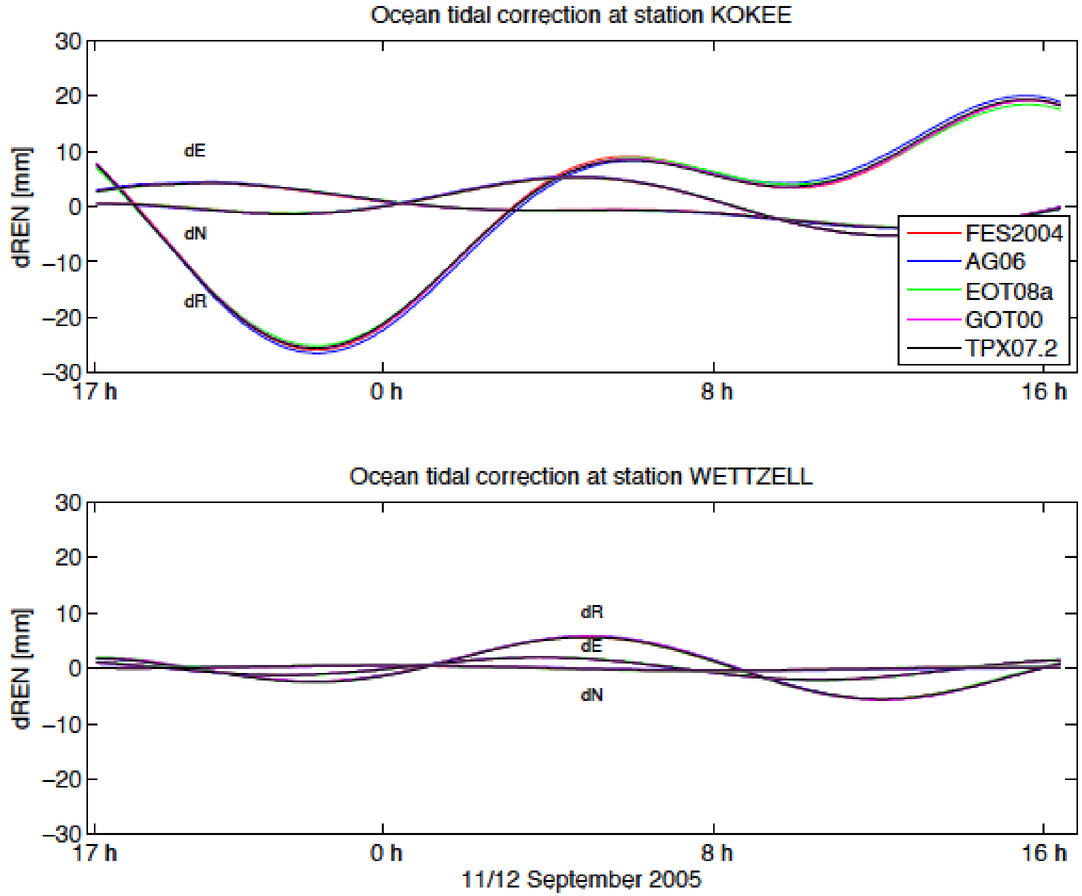
ONSALA											
$$ GOT4.10c_PP ID: 2019-03-10 20:16:27											
$$ Computed by OLMPP by H G Scherneck, Onsala Space Observatory, 2019											
$$ Onsala, RADI TANG lon/lat: 11.9264 57.3958 0.000											
Amplitudes (meters)											
.00368	.00131	.00083	.00219	.00233	.00121	.00071	.00124	.00093	.00049	.00043	U
.00153	.00036	.00037	.00105	.00043	.00041	.00013	.00110	.00014	.00006	.00007	EW
.00065	.00025	.00019	.00031	.00039	.00010	.00011	.00028	.00003	.00001	.00000	NS
Phases relative to Greenwich (degrees)											
-61.6	-34.9	-90.8	-88.5	-55.7	-109.7	-58.4	-41.3	10.1	6.0	2.1	U
86.4	122.2	56.0	-55.0	107.0	31.7	100.9	-57.5	-166.2	-169.8	-177.7	EW
116.6	157.3	102.0	101.0	54.4	-32.7	52.7	160.0	-37.7	-4.8	11.6	NS
$$ END TABLE											

Order: M2 S2 N2 K2 K1 O1 P1 Q1 MF MM SSA

Loading tables for other sites can be obtained at:

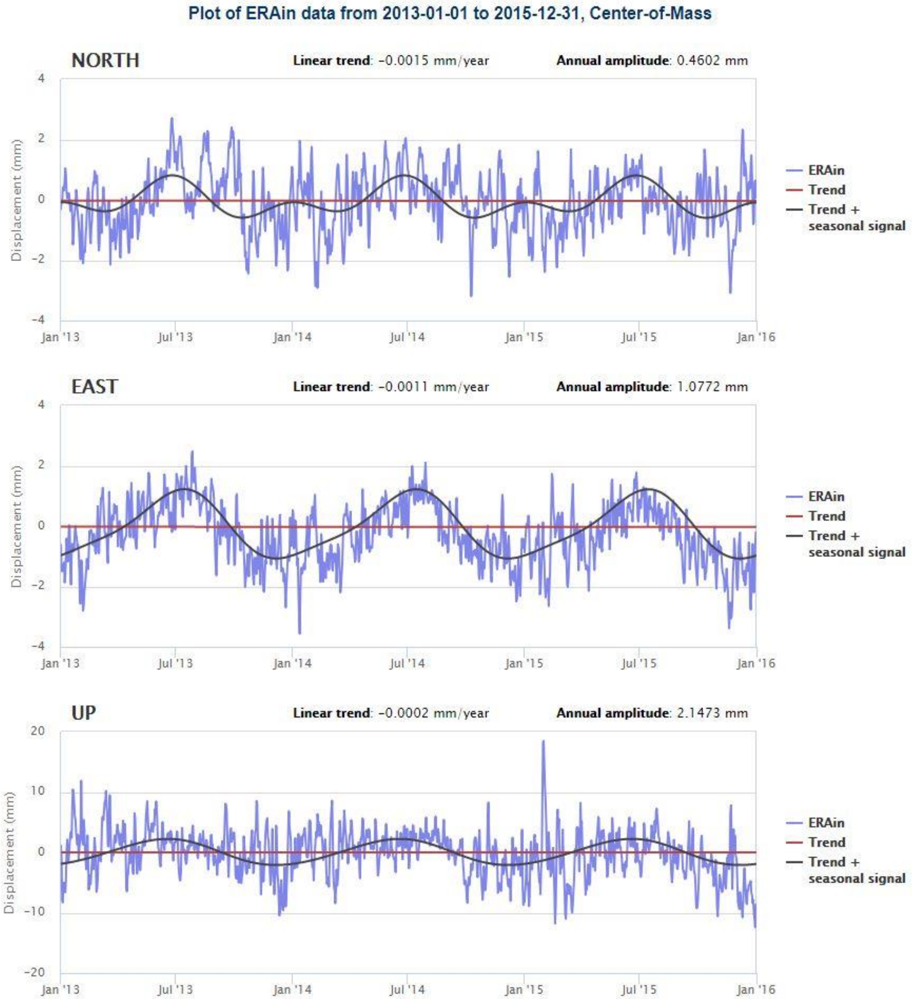
http://holt.oso.chalmers.se/loading
OR http://geodac.fc.up.pt/loading/index.html

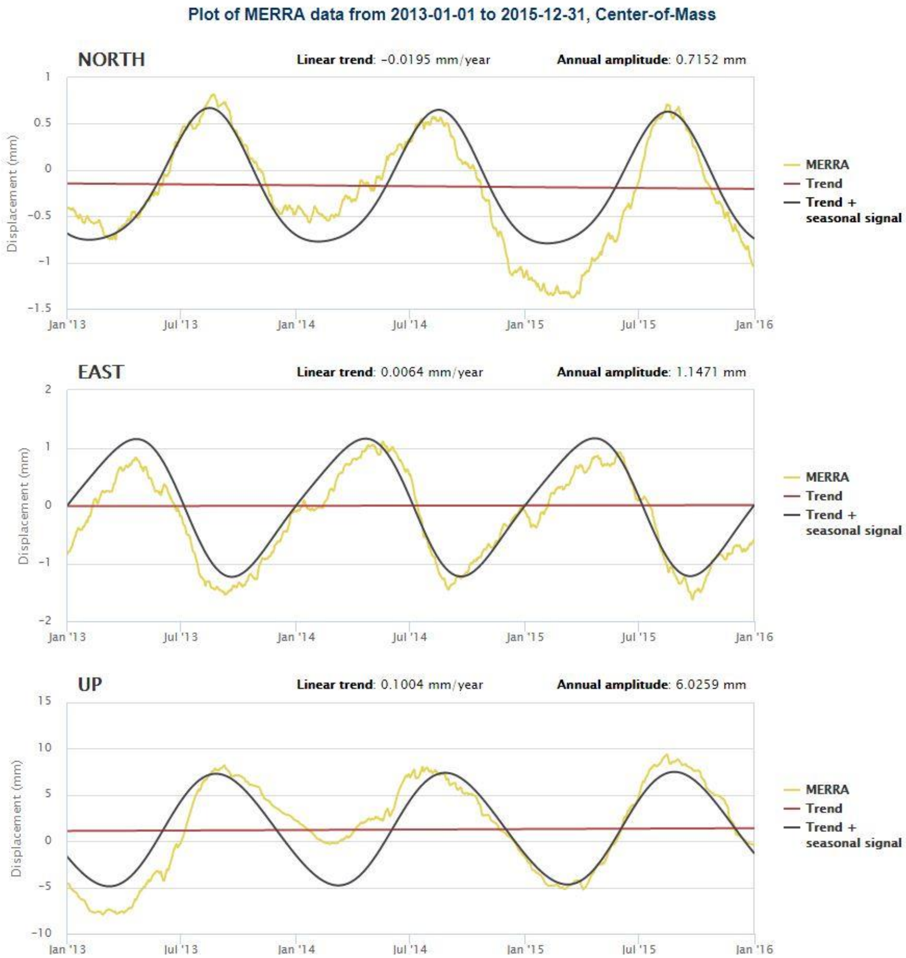
Ocean Tidal Loading



from Krasna, 2013:
https://geo.tuwien.ac.at/fileadmin/editors/GM/GM91_krasna.pdf)

Nontidal Loading at Wettzell Germany

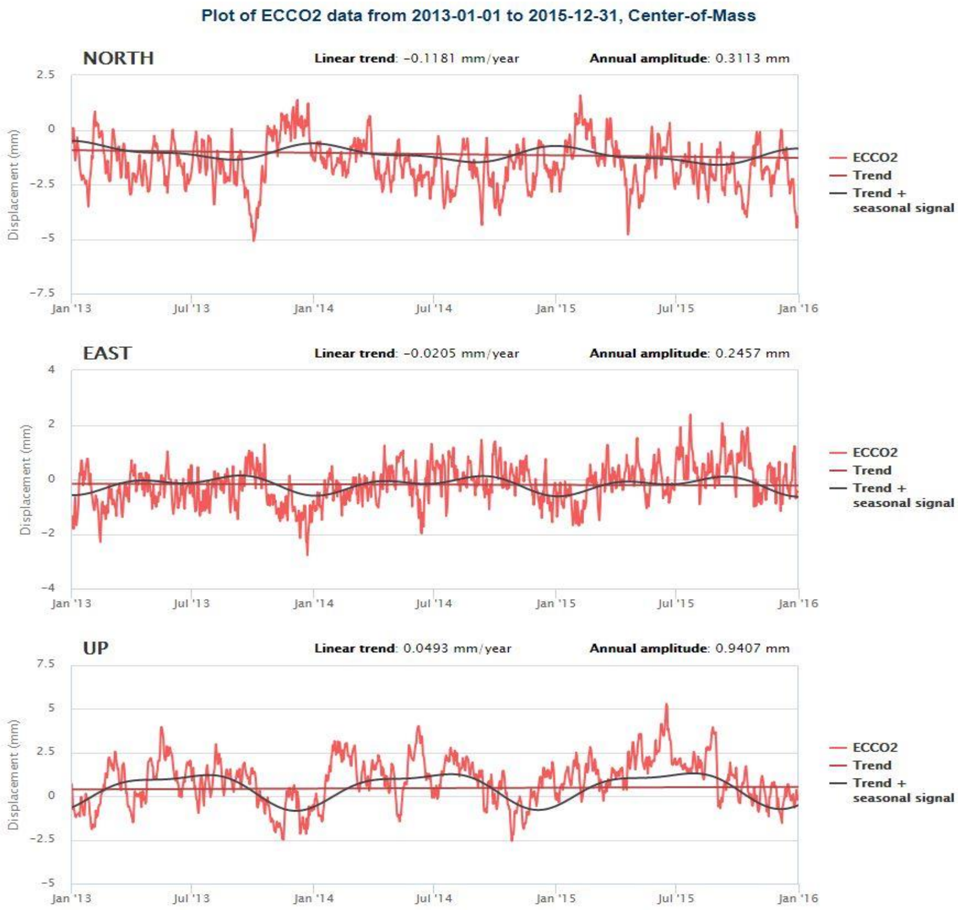




Atmospheric loading estimated from ECMWF Reanalysis surface pressure

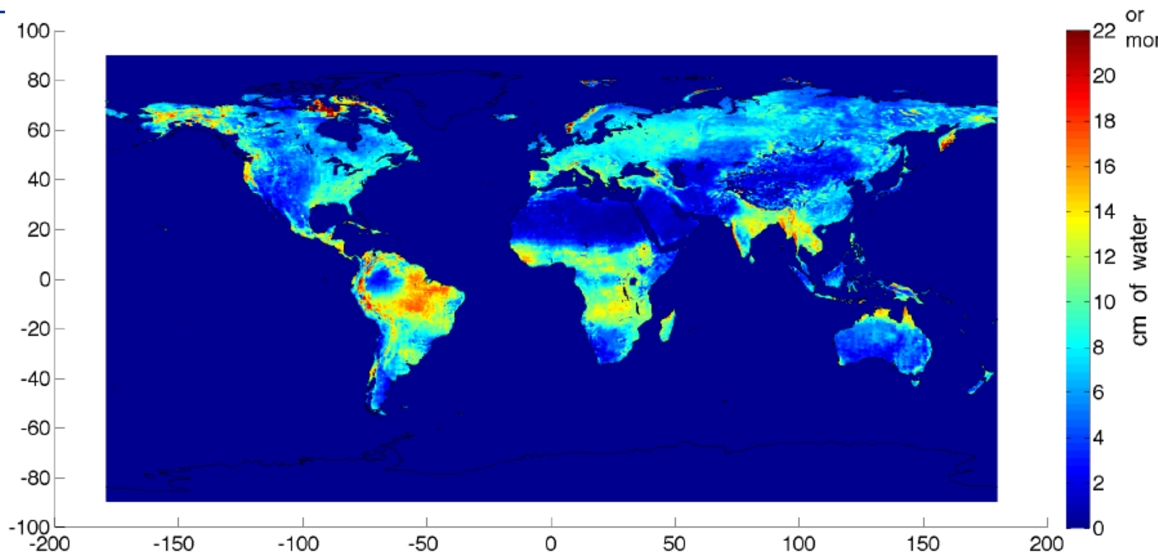
Hydrology loading (soil-moisture + snow) estimated from GSFC MERRA model

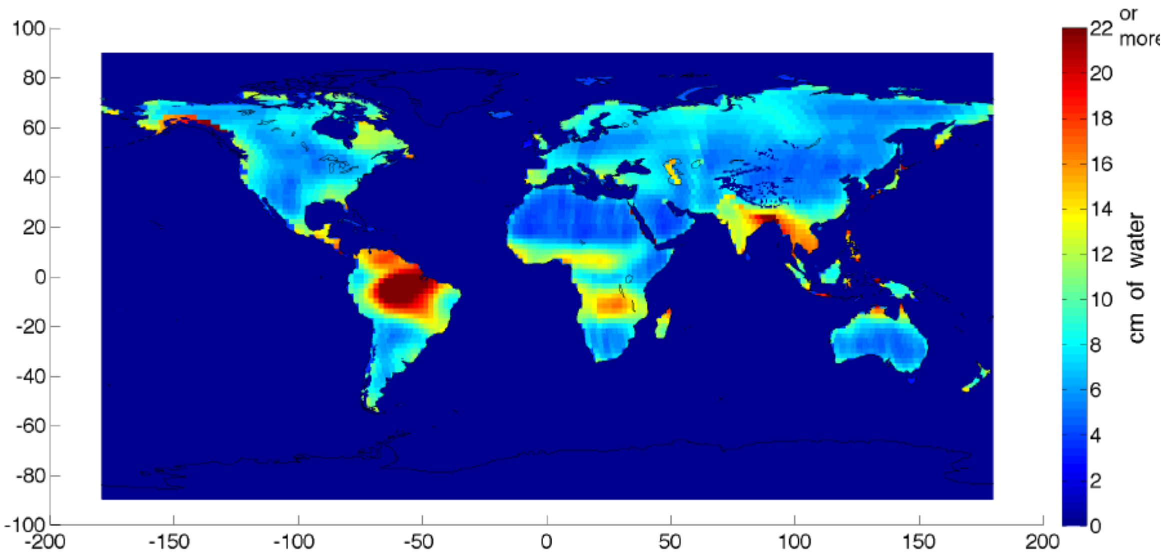
Nontidal Loading at Wettzell Germany



Non-tidal ocean loading estimated from ECCO2
(Menemenlis *et al.*, 2008) ocean bottom pressure (daily, 0.25°)
http://loading.u-strasbg.fr/displacements.php (J.P. Boy)

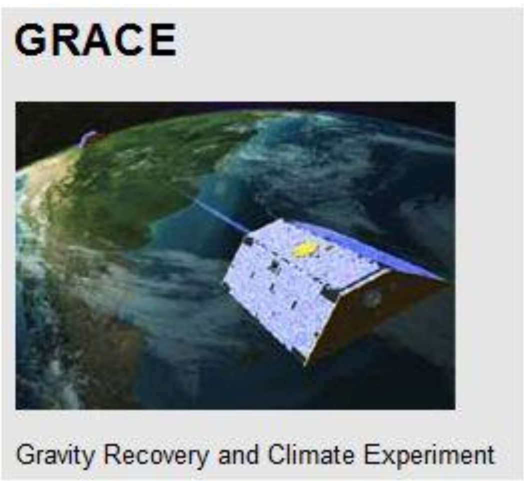
Hydrology Loading



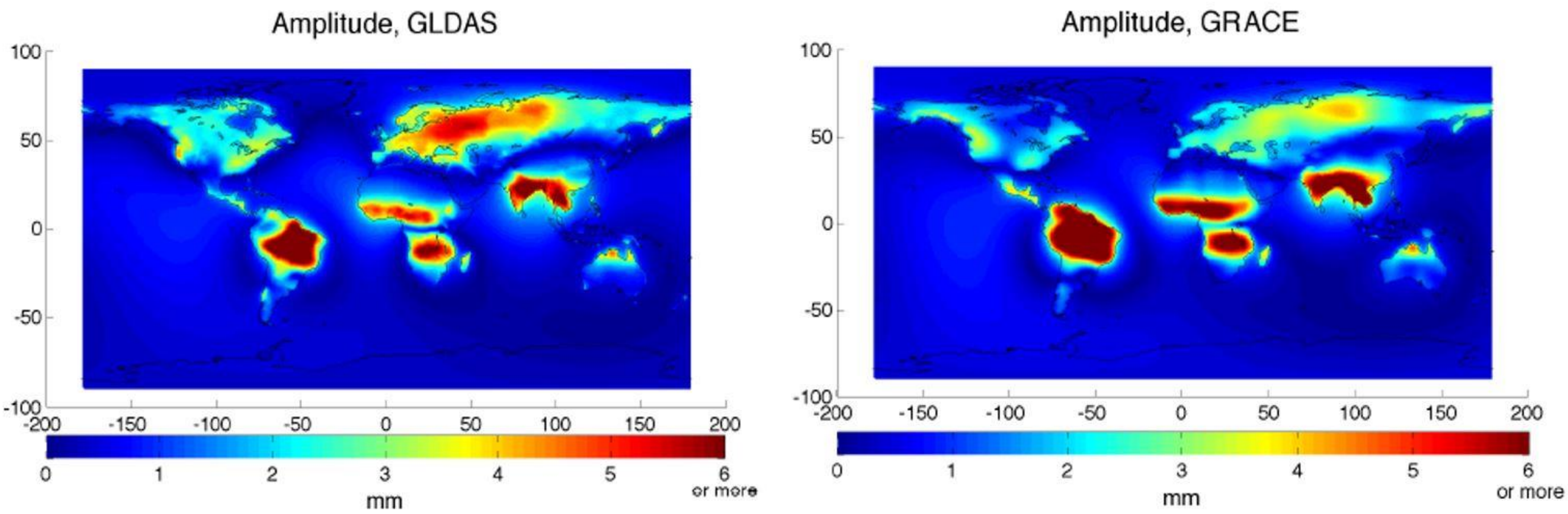


- NASA GLDAS hydrology model (Rodell et al. 2004)
- Contributions from soil moisture, snow water, plant canopy surface water storage

GRACE mass is in equivalent cm of water



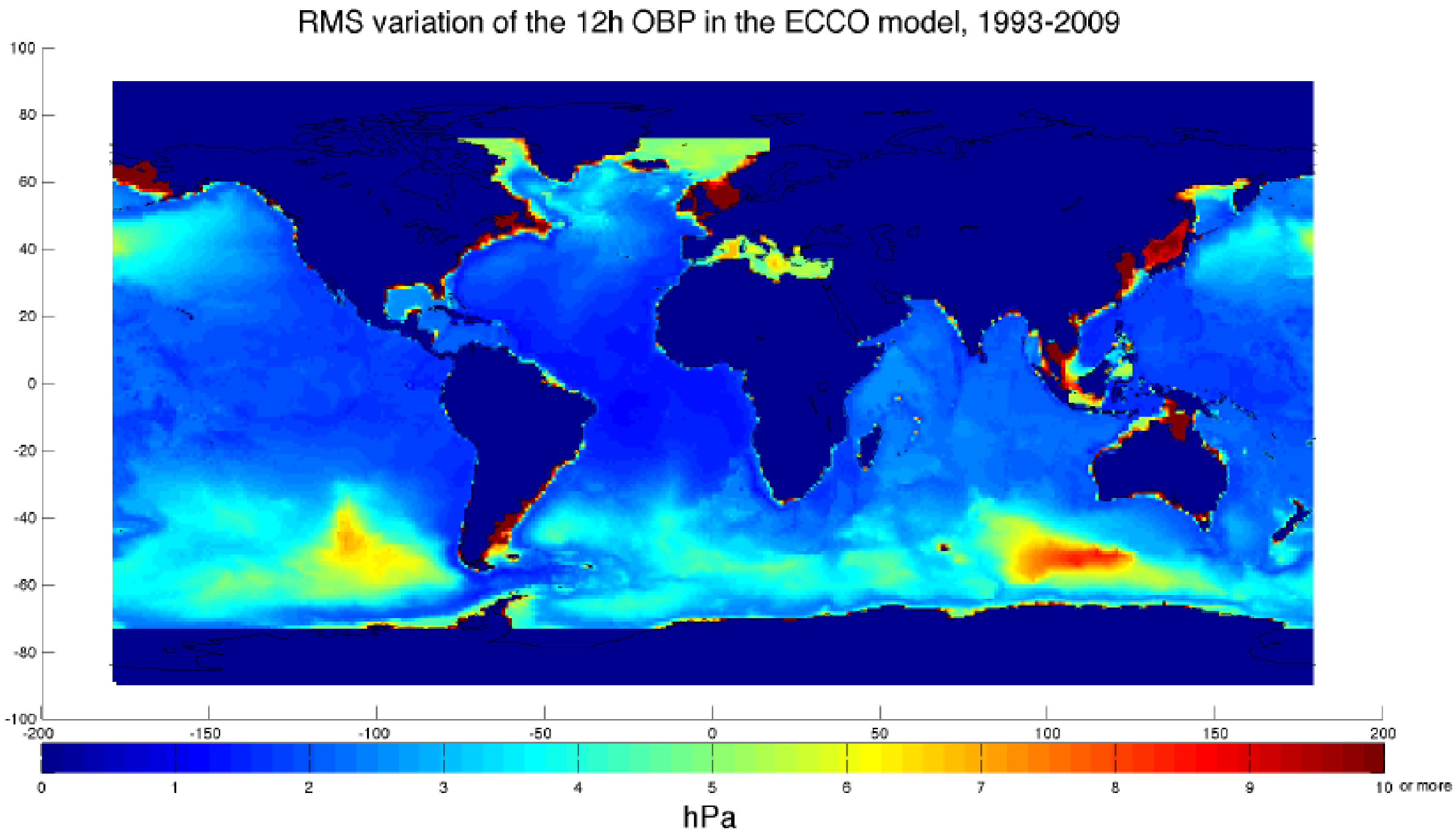
Hydrology Loading



- Compute site loading convolving over hydrology mass distributions
- Annual amplitude shown above

See Eriksson and MacMillan, J. Geodesy (2014)

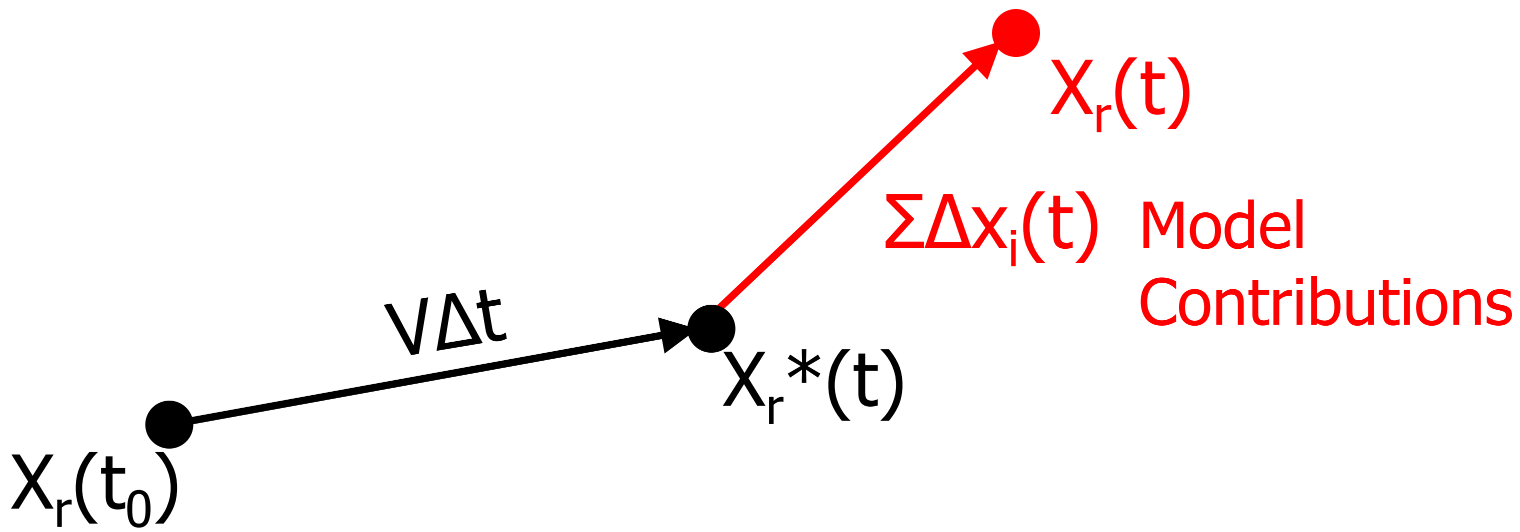
Nontidal Ocean Loading



- JPL ECCO ocean model
- Used 12-hour ocean bottom pressure since 1993
- Oceanic volume (not mass) conserving
- Site displacement loading computed by usual Green's function approach

3. Transform to Geocentric CRF

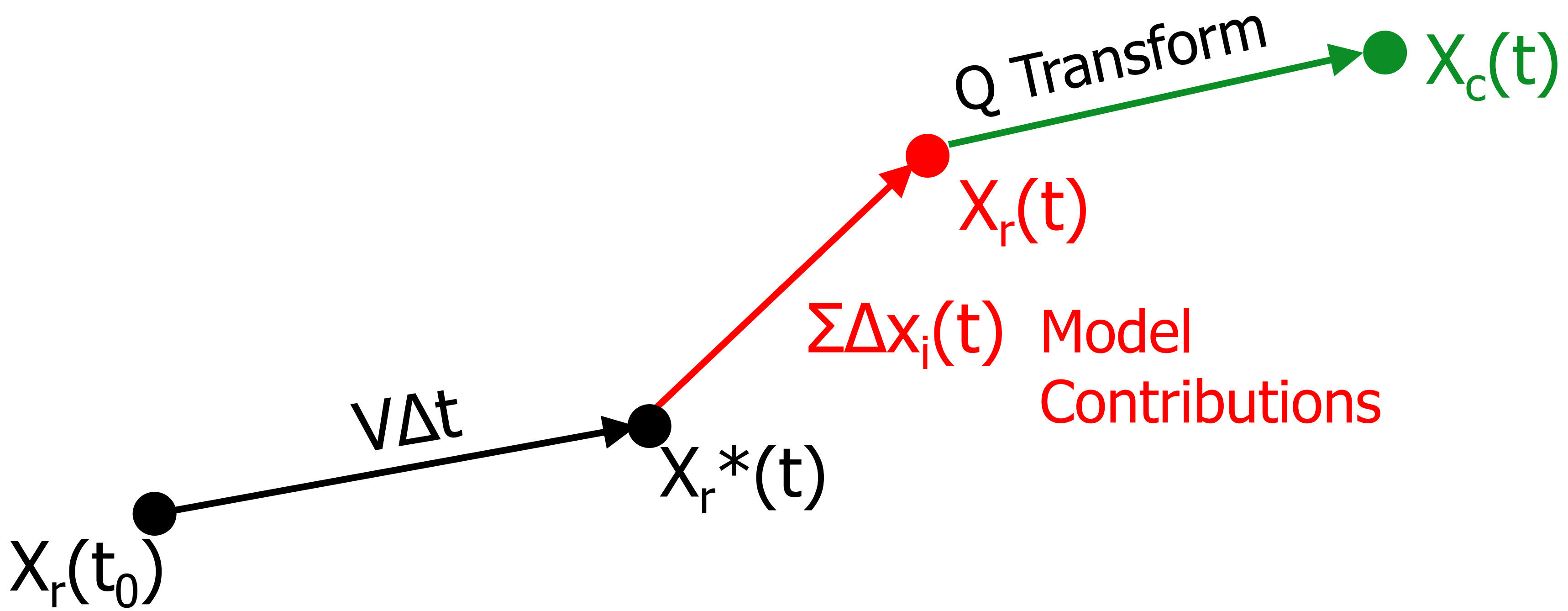
ITRF -> coordinates $\mathbf{X(t_0)}$ at reference epoch t_0
and velocities $\mathbf{V}$ for each site



First find the ITRF station position at the epoch of observation.
Then add the model displacements

3. Transform to Geocentric CRF

ITRF -> coordinates **X(t_0)** at reference epoch t_0
and velocities **V** for each site



First find the ITRF station position at the epoch of observation.
Then add the model displacements
Transform X(t) to Geocentric J2000 Celestial Reference Frame

Transformation TRF -> CRF

Transform positions in crust-fixed terrestrial reference frame (TRF) to geocentric celestial reference system at J2000

$$Q = PNUXY$$

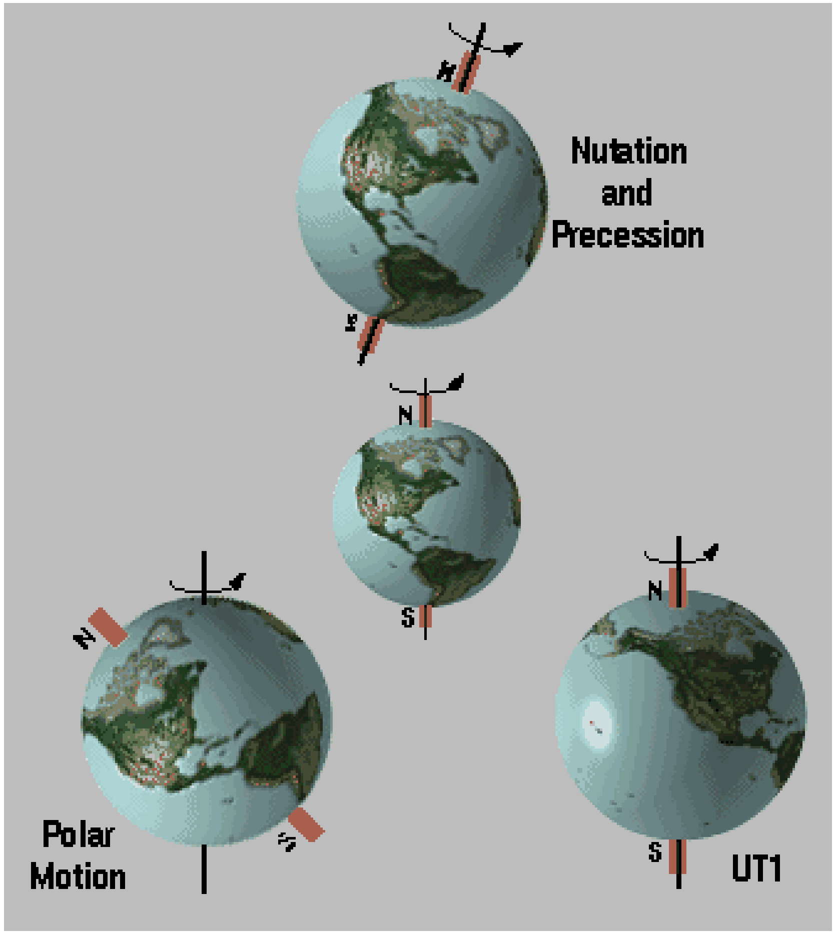
Precession Nutation Spin(UT1) Polar Motion

$$\vec{r}_c = Q\vec{r}_t$$

The theoretical delay is calculated by programs like CALC (input to SOL

There are several steps that are involved in this procedure:

Earth Orientation Parameters (EOP)

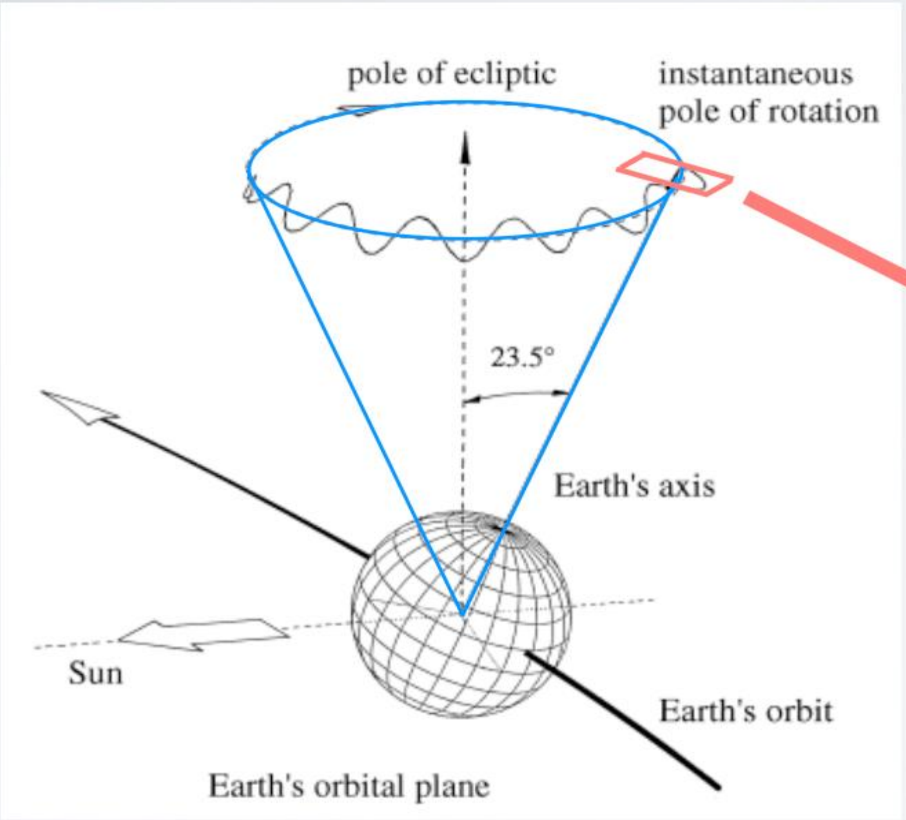


Nutation/precession: periodic and long-term motion of the spin axis in space relative to CRF

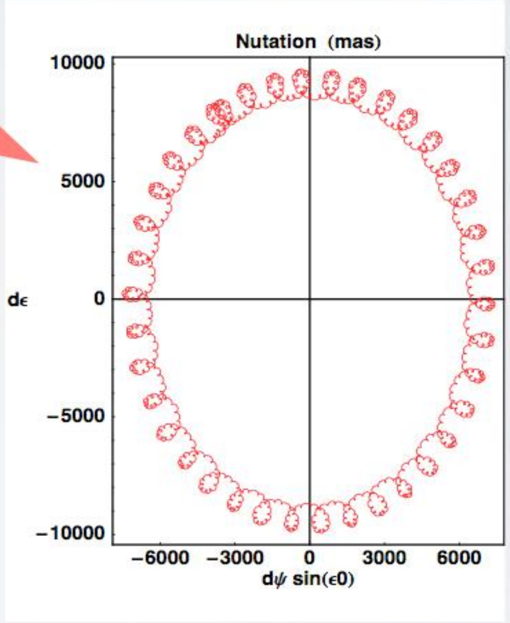
Polar motion: motion of the geographic pole relative to the spin axis

UT1: describes the non uniform daily rotation of the Earth

Precession/Nutation



Main cause: gravitational torque from the Moon, the Sun and the planets

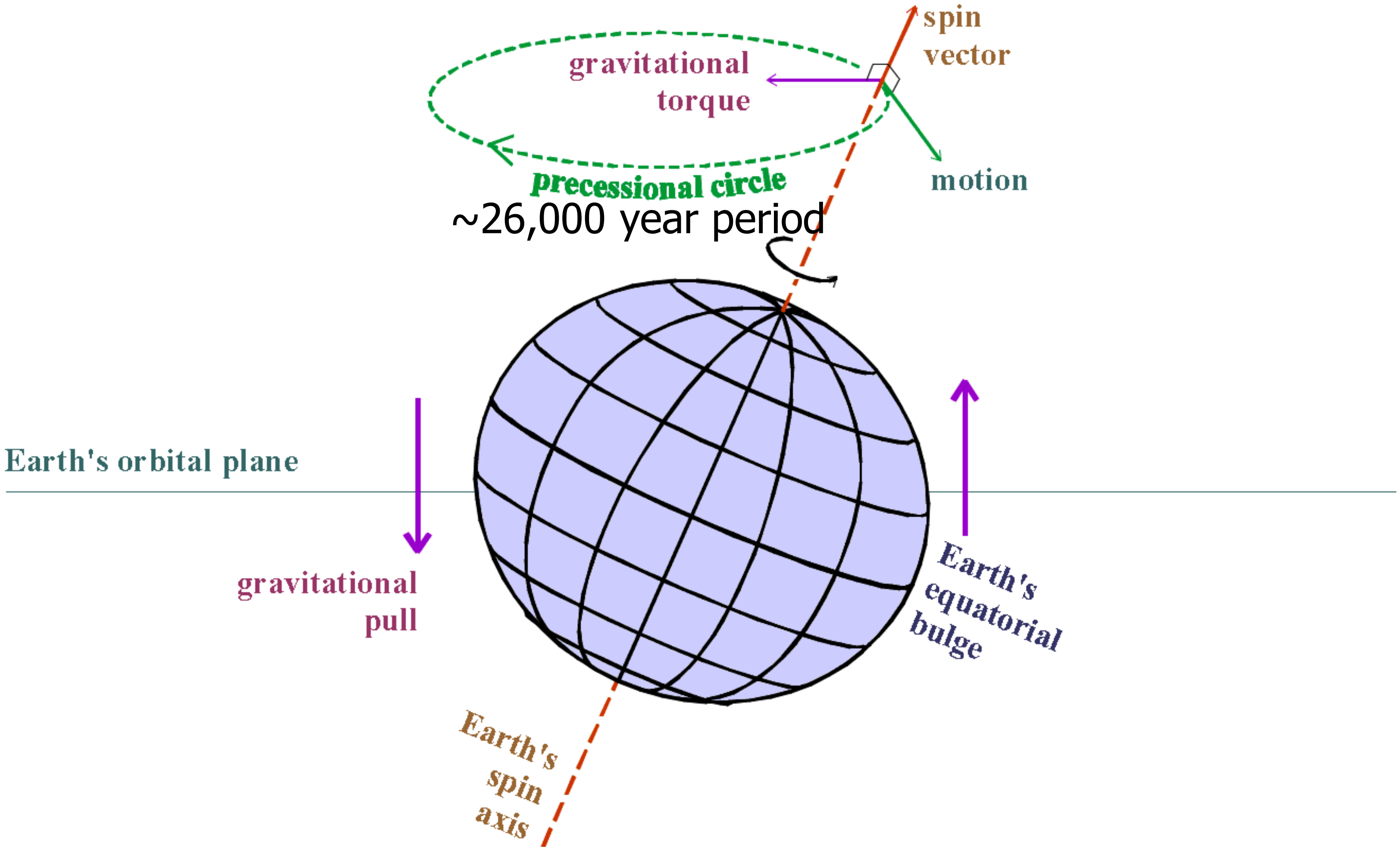


18.6 yr cycle

(source: hfps://www.ualberta.ca/~dumberry/nuta2on.jpg)

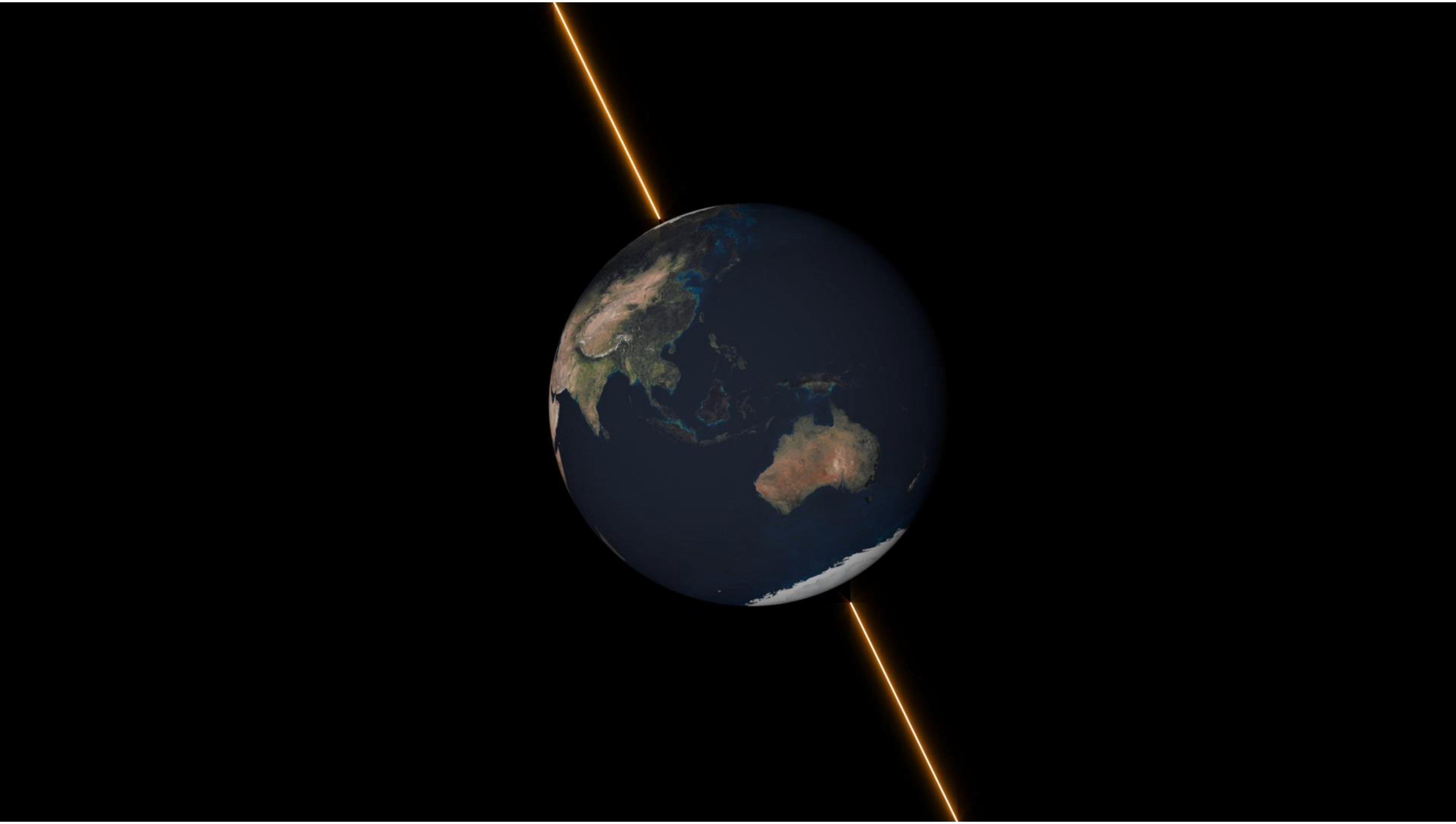
This motion is driven by the gravity of the Moon and the Sun acting on the Earth's equatorial bulge. However, because the Moon orbits the Earth once a month, in a tilted, elliptical orbit, the spin axis also undergoes a smaller set of motions on much shorter time scales (days to years).

Precession

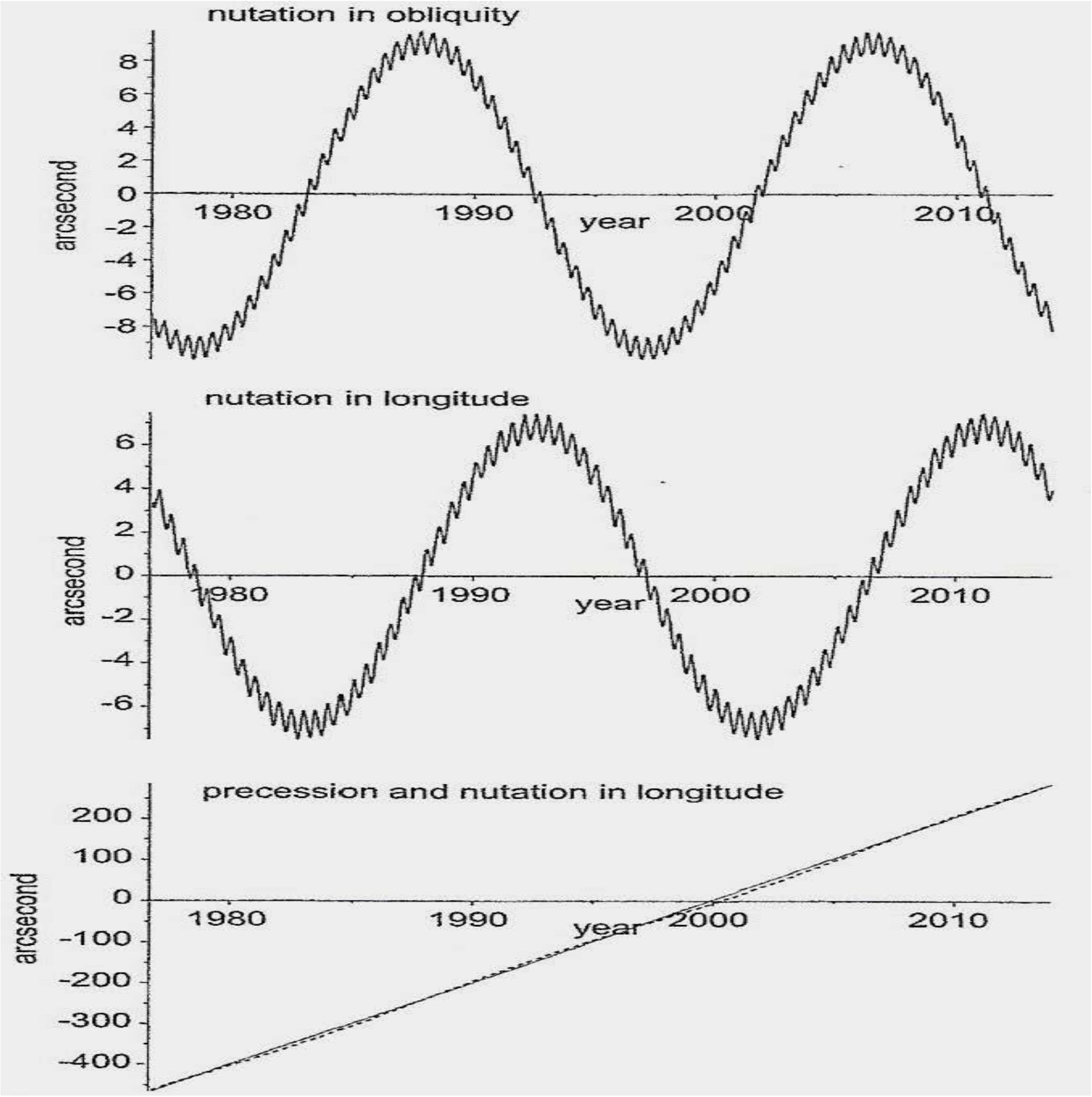


(Source hfp://star-www.stand.ac.uk/~fv/webnotes/PRECESS1.GIF)

Nutation and Precession



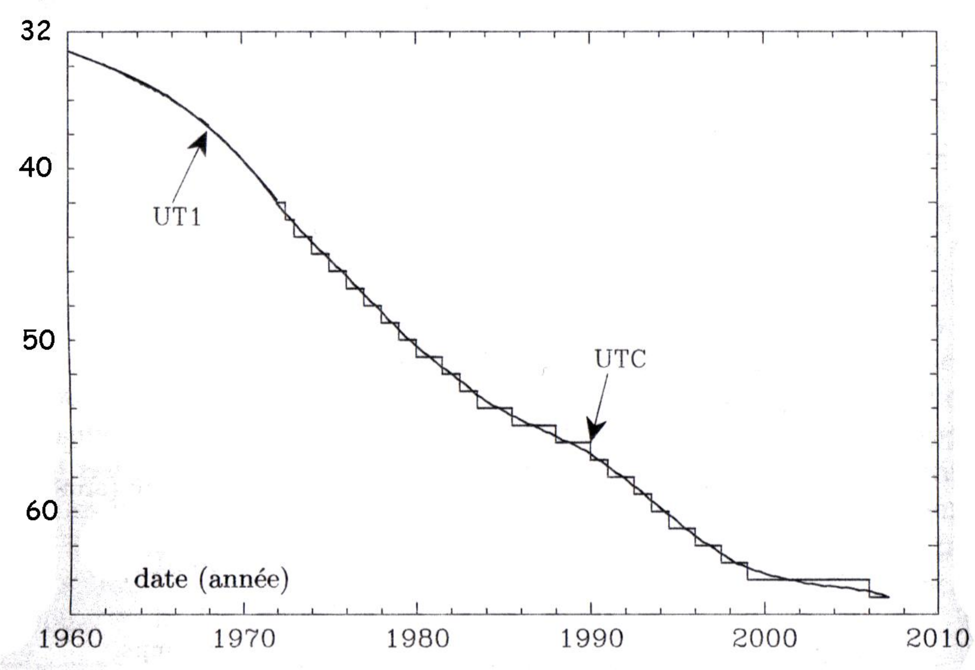
Nutation and Precession



18.6 yr nutation period

From Dehant and Mathews (2016)

UT1



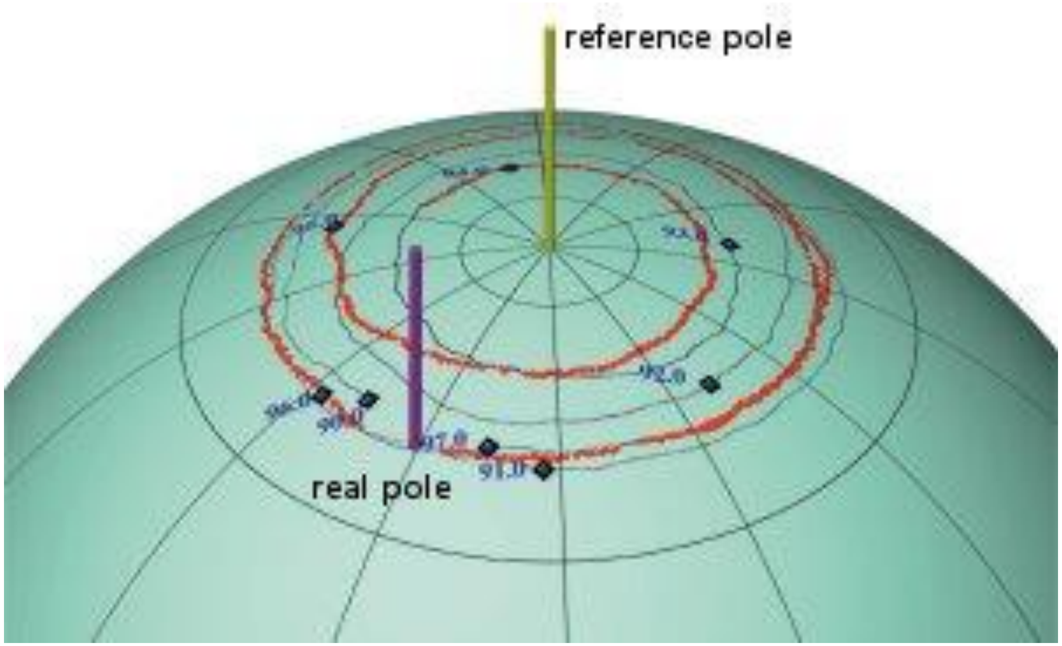
Universal Time (UT1)

- UT1 is related to the Greenwich mean sidereal time (GMST) by a conventional relationship
- gives access to the direction of the ITRF zero- meridian in the ICRS.
- Is expressed as the difference

– UT1-TAI or UT1-UTC

leap seconds !!

Polar Motion



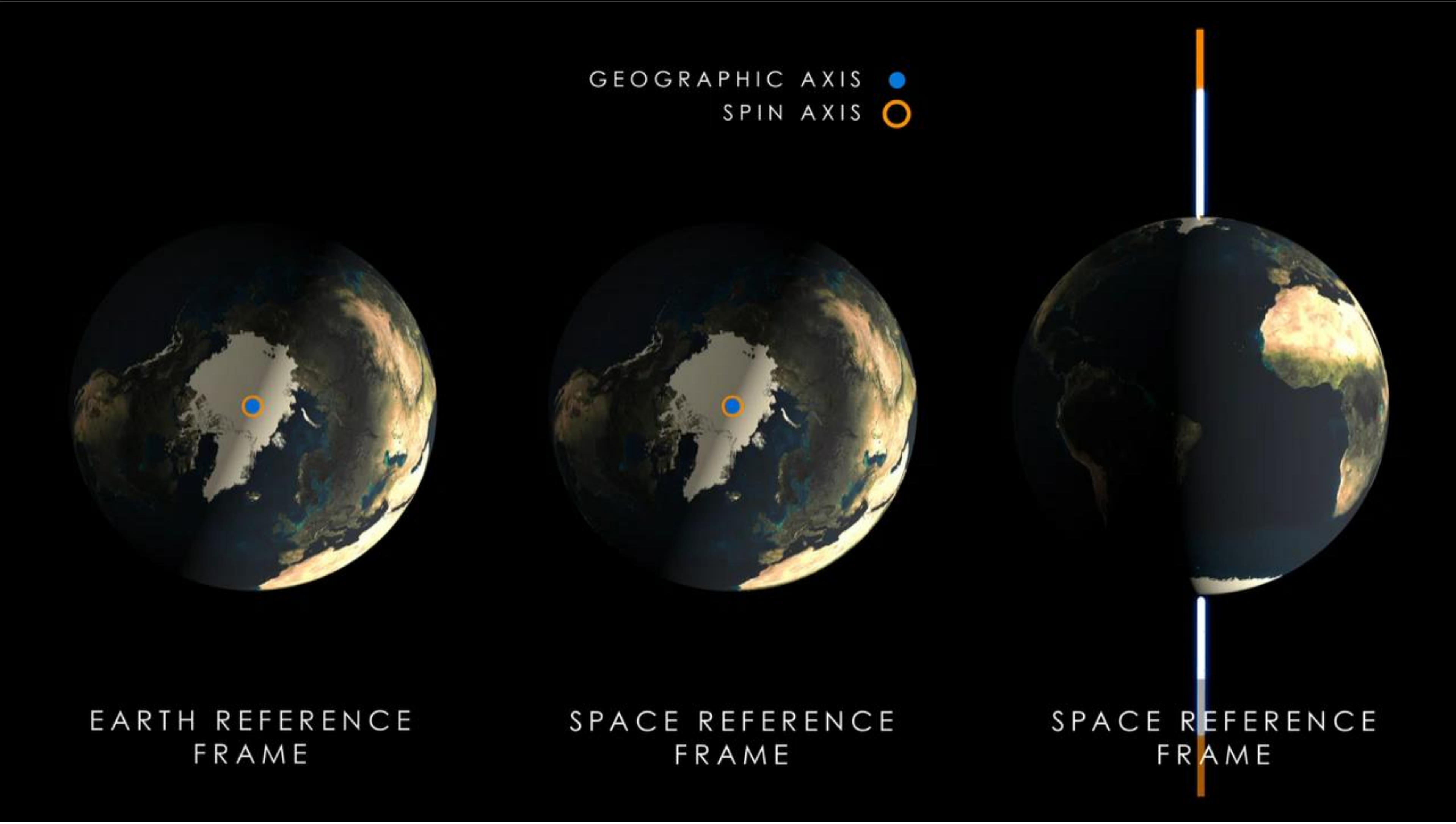
Motion of rotation (spin) axis relative to the geographic pole

http://www.aviso.altimetry.fr/uploads/pics/200112_pole_3d_uk.jpg)

3 major components

– Long-term drift ("mean pole")
– free oscillation with period about 435 days (Chandler wobble)
– an annual oscillation forced by the seasonal displacement of air and water masses

Polar Motion



Polar Motion

Y-pole

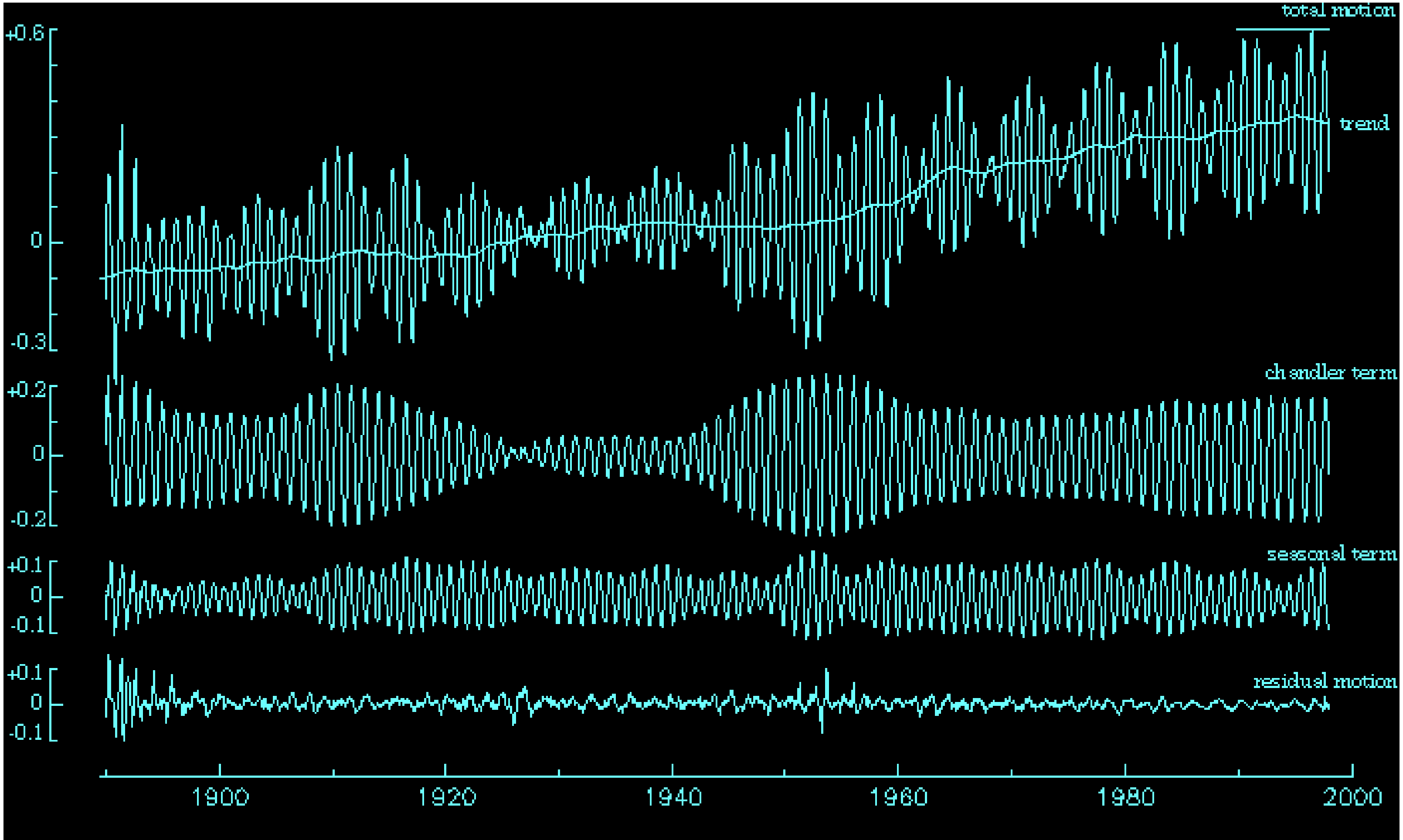


Figure II-3. Y - coordinate of the pole   Unit: 1".

EOP Variations

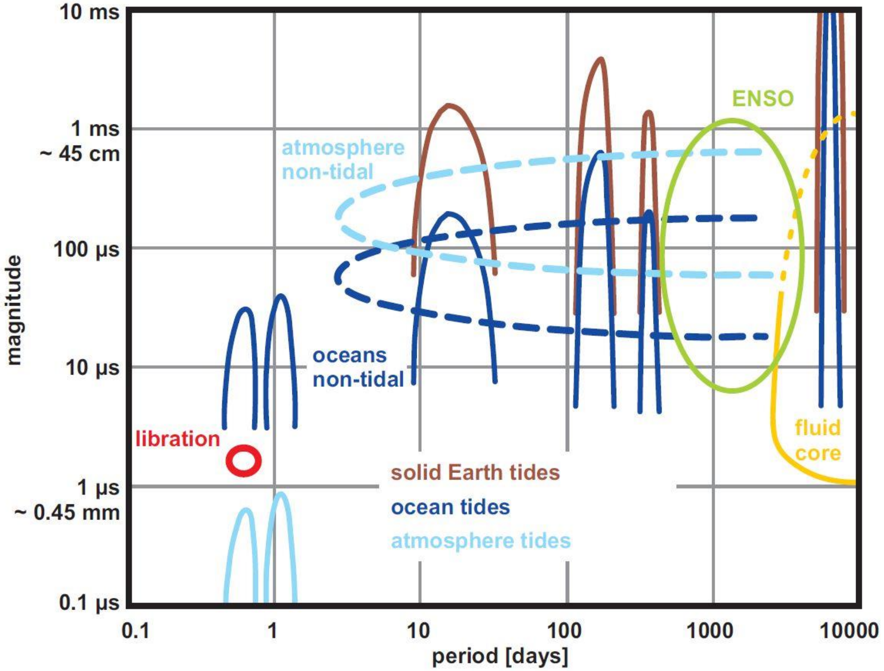


Figure 2.3: Signal components of universal time. (The figure is an adaption of ftp://gemini.gsfc.nasa.gov/pub/core/fig1.ps)

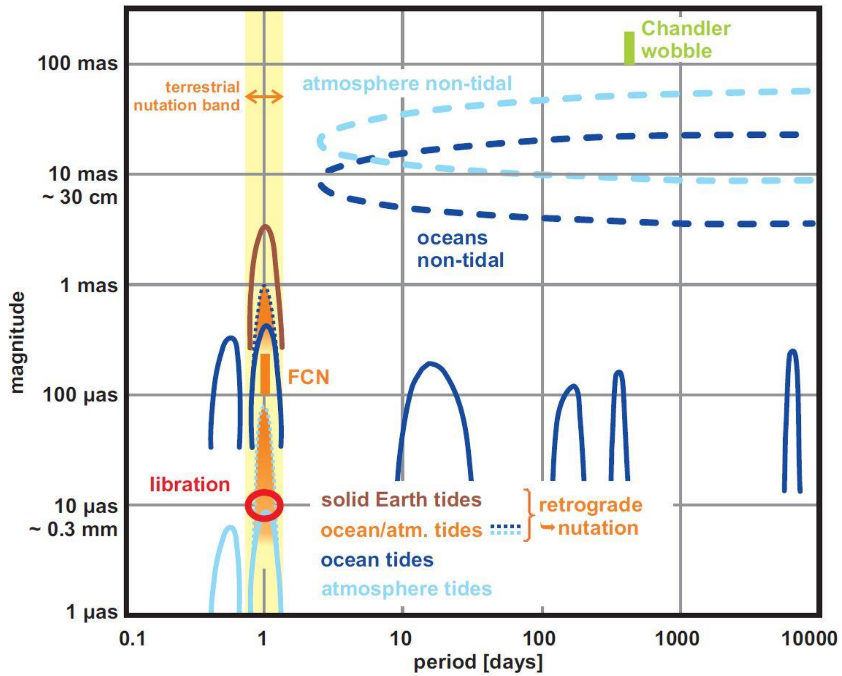


Figure 2.2: Signal components of polar motion. (The figure is an adaption of ftp://gemini.gsfc.nasa.gov/pub/core/fig2.ps)

Many geophysical effects contribute to EOP variations and are available as models => Too much to discuss here

4. Gravitational Delay

Compute Geometric Vacuum Delay

• Use the Eubanks ‘Consensus’ Model. [see IERS Conventions 2010]; Other models exist, e.g., M.Soffel, S. Kopeikin, and W. B. Han (2016)

• Lorentz transform station coordinates from Geocentric CRF to SSB (solar system barycentric) coordinates

• Acccount for general relativistic gravitational delay due to the sun, moon, Earth, other planets.

• Lorentz transform Solar System Barycentric (SSB) vacuum delay back from SSB to Geocentric CRF

Antenna Axis Offsets

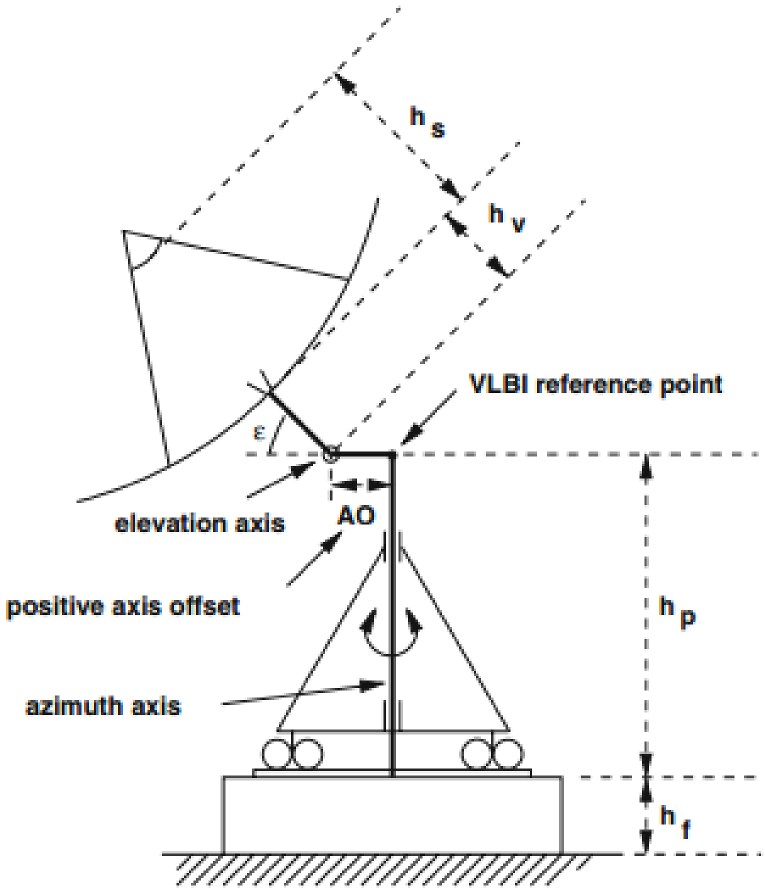


Fig. 1 Alt-azimuth telescope mount with positive axis offset

VLBI reference point is defined as the perpendicular projection of the moving axis onto the fixed axis

- Axis offsets is either physically measured at the site or estimated as a parameter in analysis solution

Antenna Thermal Deformation

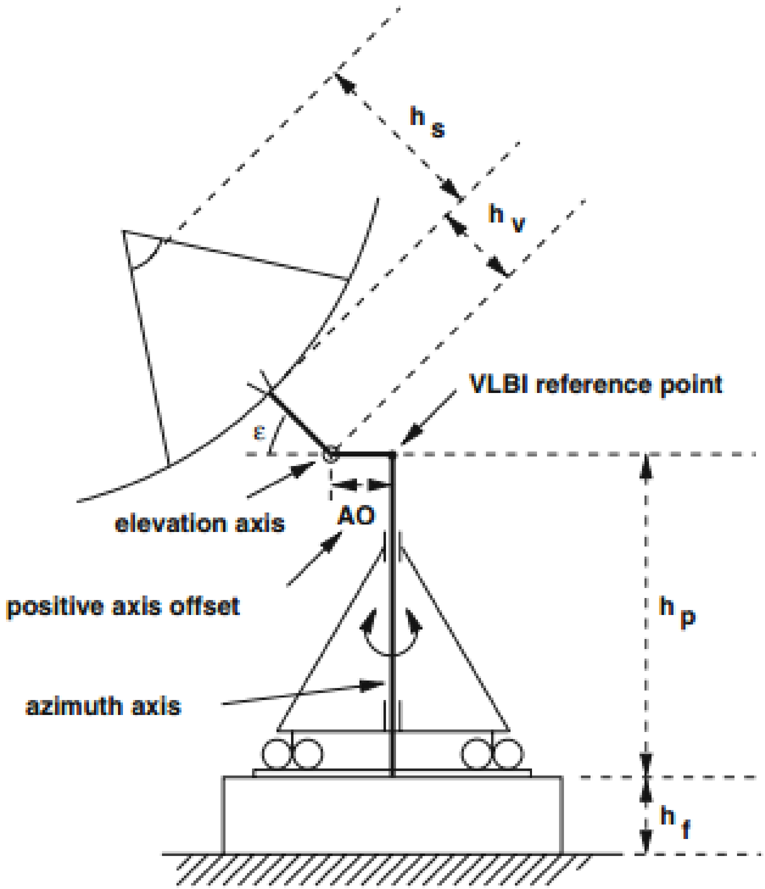


Fig. 1 Alt-azimuth telescope mount with positive axis offset

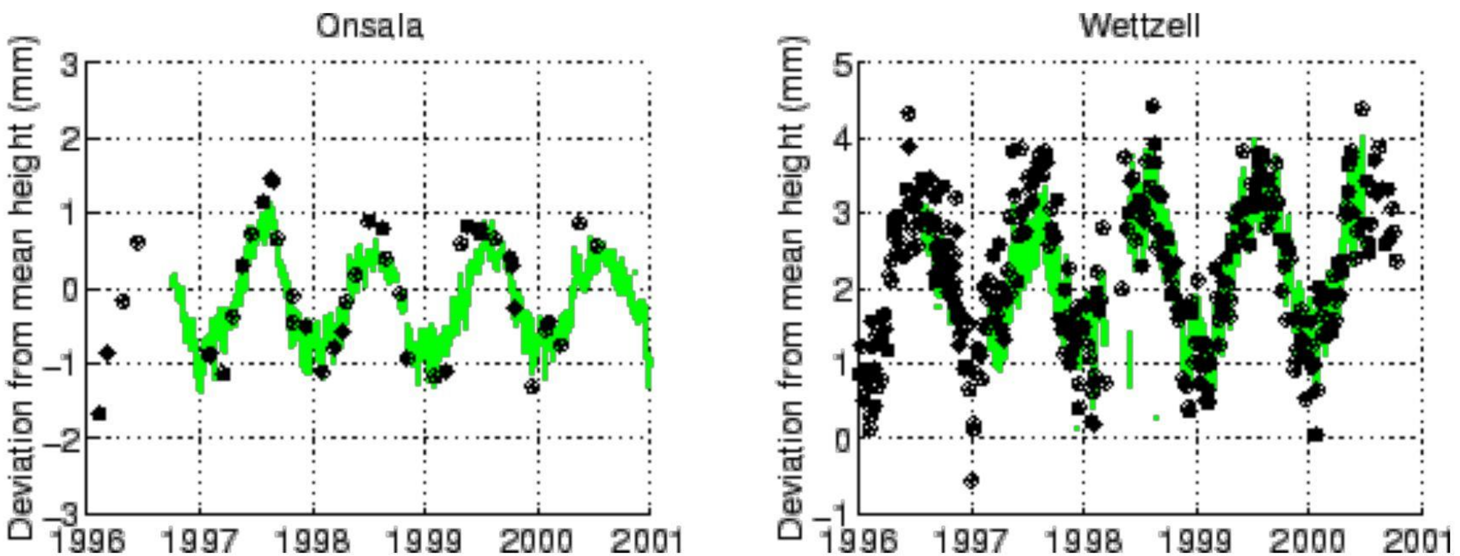
(a) For alt-azimuth mounts

$$\Delta\tau_{\text{therm},i} = \frac{1}{c} \cdot \left[\gamma_f \cdot (T(t - \Delta t_f) - T_0) \cdot (h_f \cdot \sin\varepsilon) \right.$$
$$+\gamma_a \cdot (T(t - \Delta t_a) - T_0) \cdot (h_p \cdot \sin\varepsilon$$
$$\left. +AO \cdot \cos\varepsilon + h_v - F_a \cdot h_s)\right] . \qquad (2)$$

Expansion coefficients $\gamma \sim 1.0\text{-}1.2 \times 10^{-5}/^{o}C$

See A. Nothnagel (2009), Conventions on thermal expansion modelling of Radio telescopes for geodetic and astrometric VLBI

Antenna Thermal Deformation



(source: http://ivscc.gsfc.nasa.gov/publications/ar2000/acoso/)

Vertical height at Onsala and Wettzell:
solid green line - measured by the invar rod measuring systems;

black points - a simple model based on daily mean temperature from the VLBI data base, thermal expansion coefficient, and the telescope dimensions.

Geophysical Modeling->Theoretical Delay

Review

1. Get site coordinates in the TRF at observation epoch

2. Add displacement models and delay corrections

3. Transform positions in crust-fixed terrestrial reference frame (TRF) to geocentric celestial reference system CRF (J2000) : Precession/Nutation, Polar Motion, UT1

4. Account for gravitational contribution to the delay due to the Sun and planets

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