Signal Propagation

Johannes Böhm

Third IVS VLBI School

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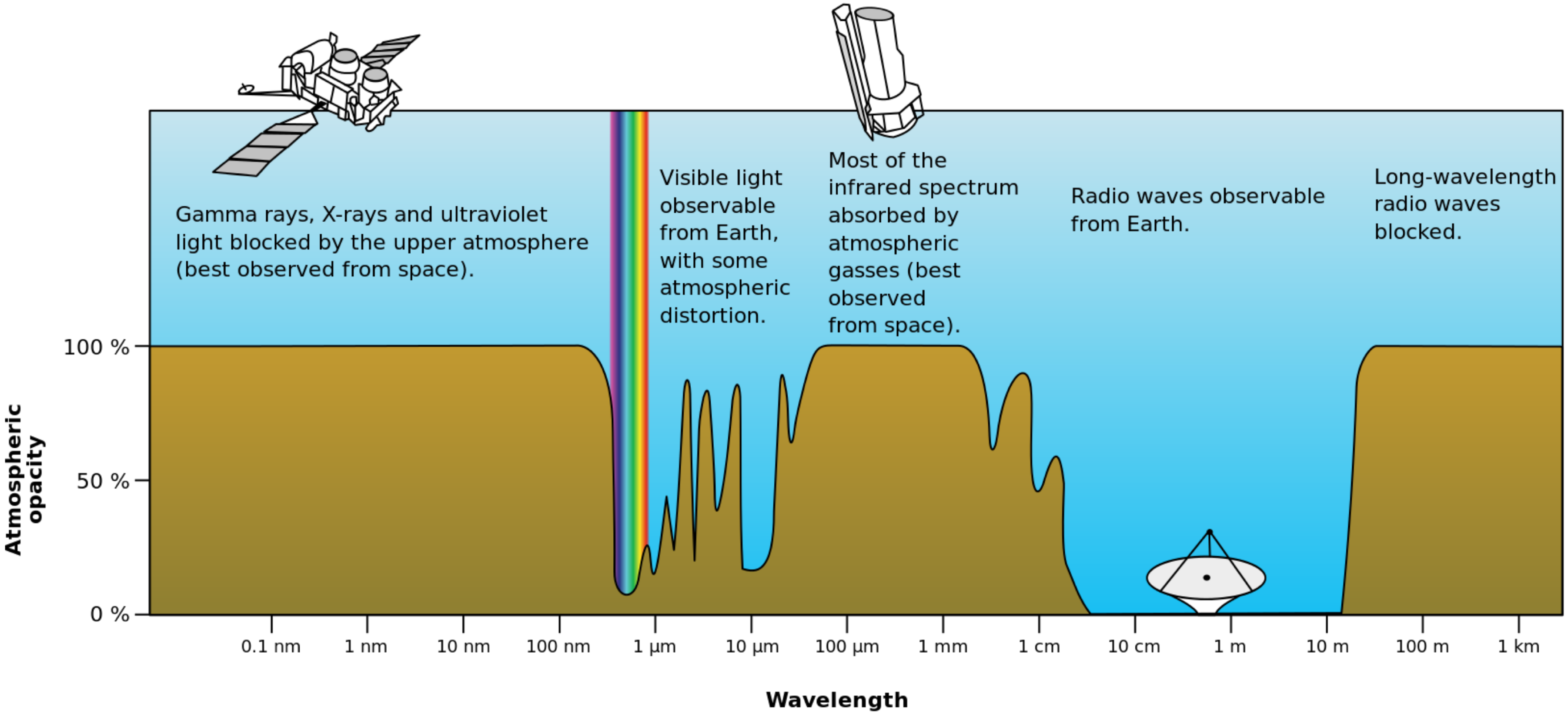
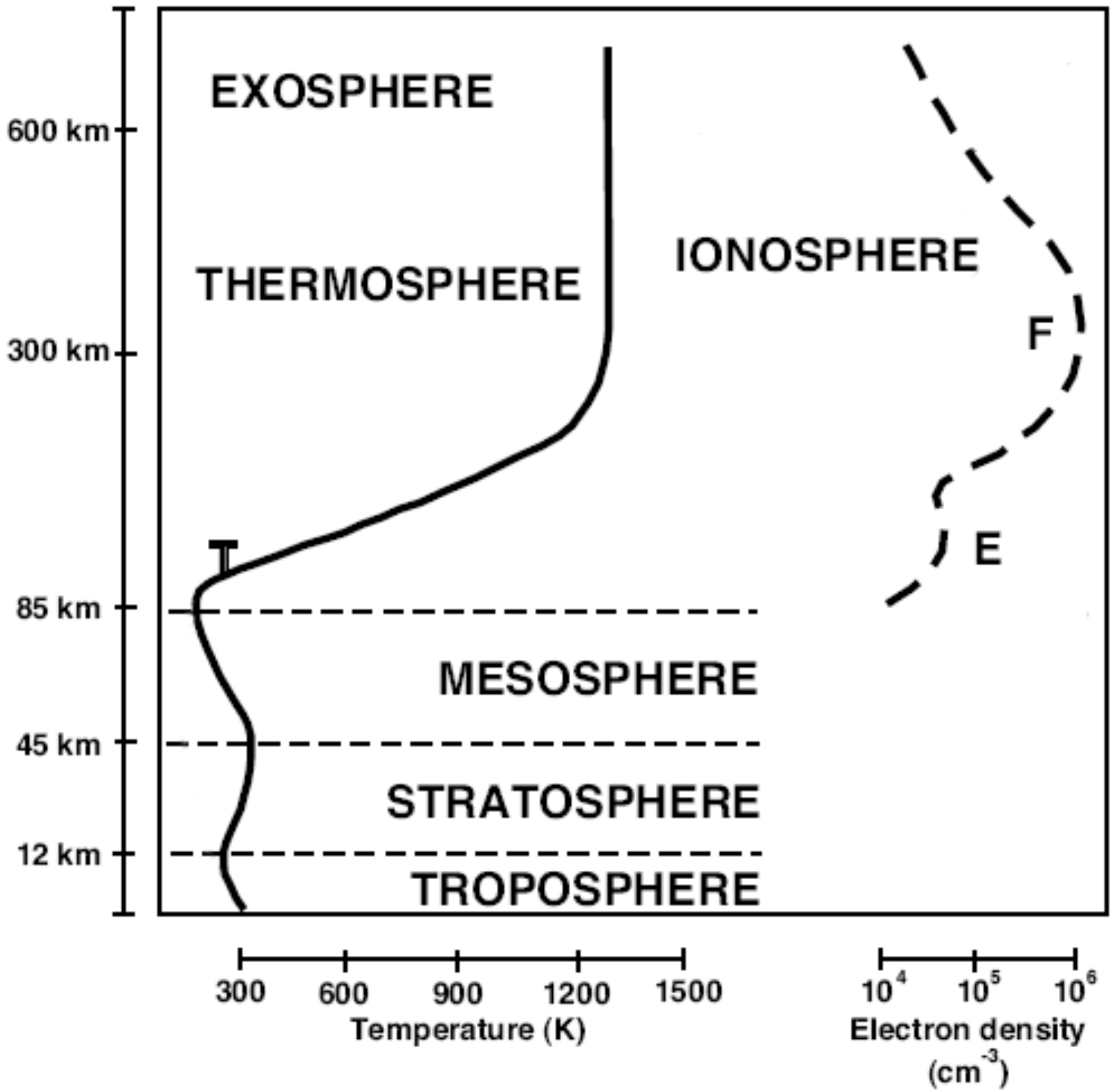
Atmosphere

- Atmospheric opacity



Wikipedia.de

Atmosphere



Wikipedia.de

Ionosphere

- Upper part of the atmosphere from about 60 km to 2000 km with the main concentration of particles between 300 and 400 km
- The electron production in the ionosphere is a direct consequence of the interaction of the solar radiation with atoms and molecules in the Earth's upper atmosphere
- Definition of the ionosphere:
  - Number of free electrons and ions is large enough to affect propagation of electromagnetic waves

Ionosphere

- Dispersive medium: propagation velocity of an electromagnetic wave is dependent on its frequency
- In such a medium the velocity of a sinusoidal wave and a wave group are different (phase vs. group velocity)

$$\nu_{ph} = \frac{c}{n_{ph}} \qquad \nu_{gr} = \frac{c}{n_{gr}} \qquad \nu_{gr} = \nu_{ph} - \lambda \frac{d\nu_{ph}}{d\lambda}$$

- Refractive index

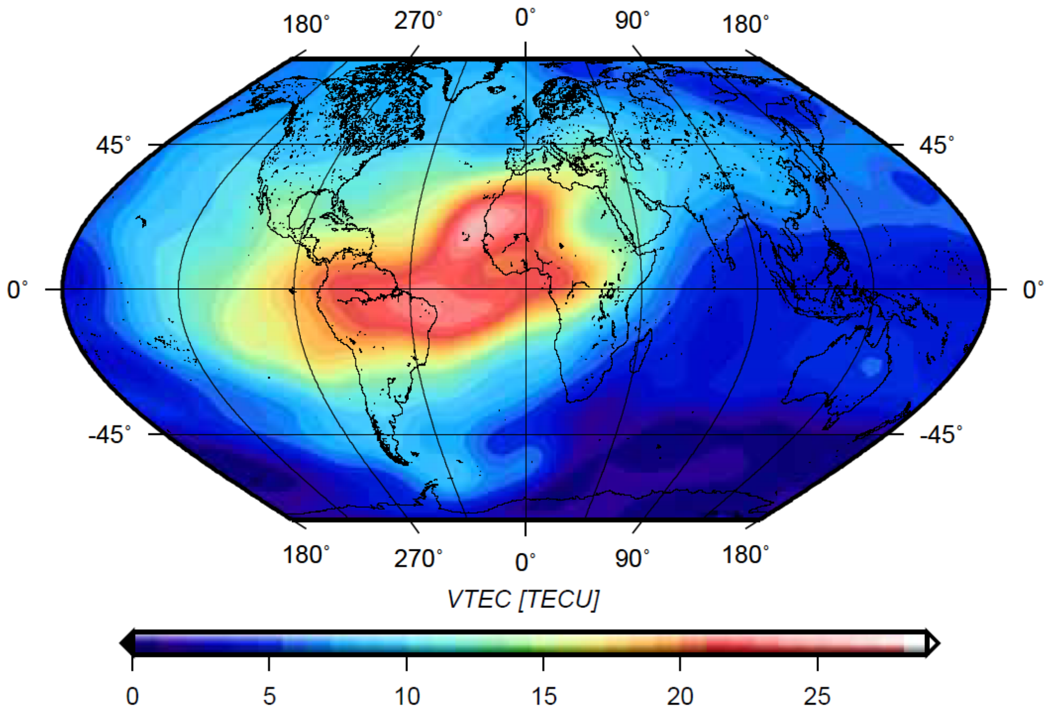
$$n_{gr}^{ion} = 1 + C_2 \frac{N_e}{f^2} = 1 + 40.31 \frac{N_e}{f^2}$$

TEC - Total Electron Content

- Represents the total amount of free electrons in a cylinder with a cross section on 1 m^2 and a height equal to the slant signal path

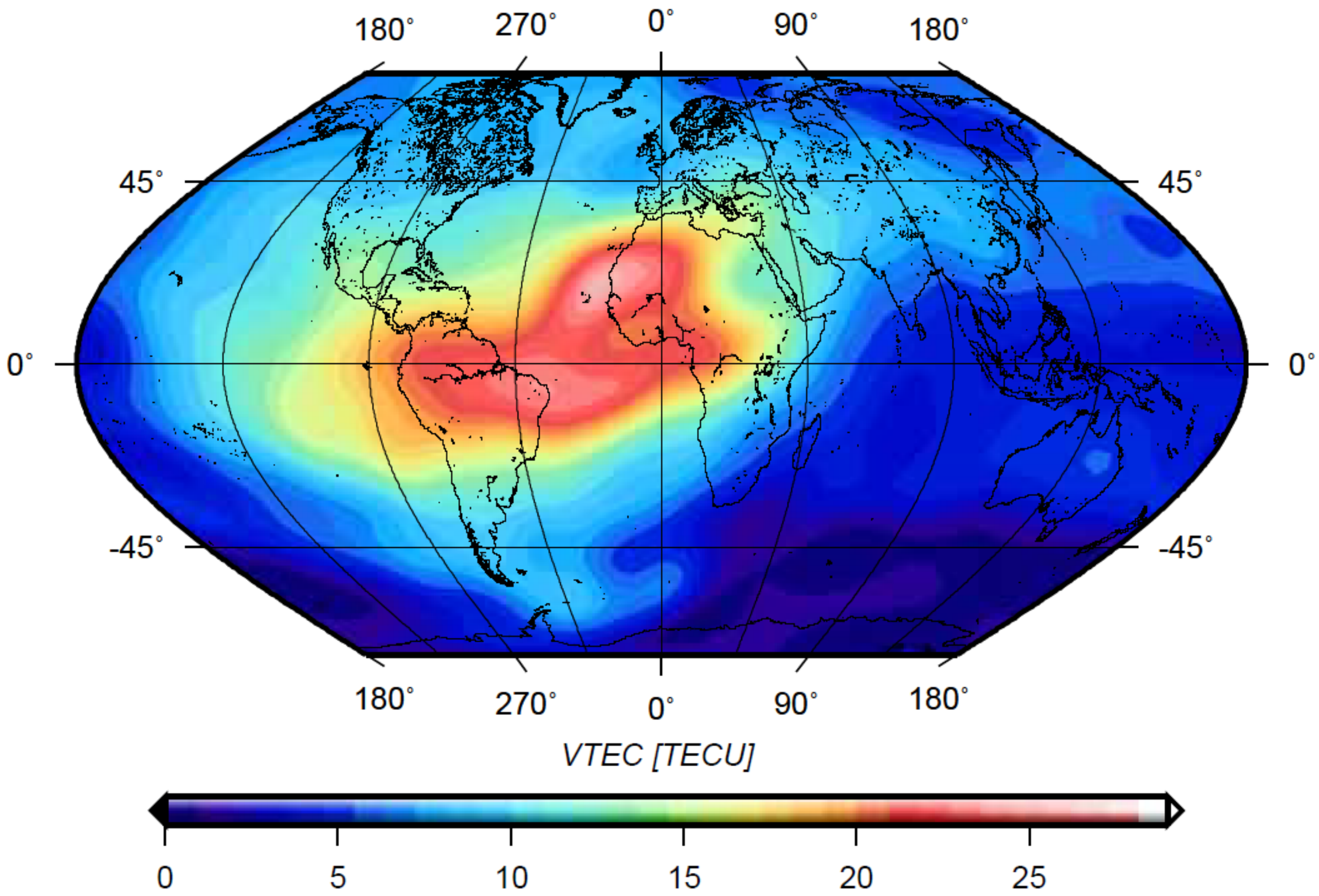
$$STEC = \int N_e(s)ds$$

- Measured in TEC Units (TECU)
    - 1 TECU is equivalent to 10^{16} electrons/m^2



TEC - Total Electron Content

- 1 TECU corresponds to
  - 7.6 cm at S-band (2.3 GHz)
  - 0.6 cm at X-band (8.4 GHz)

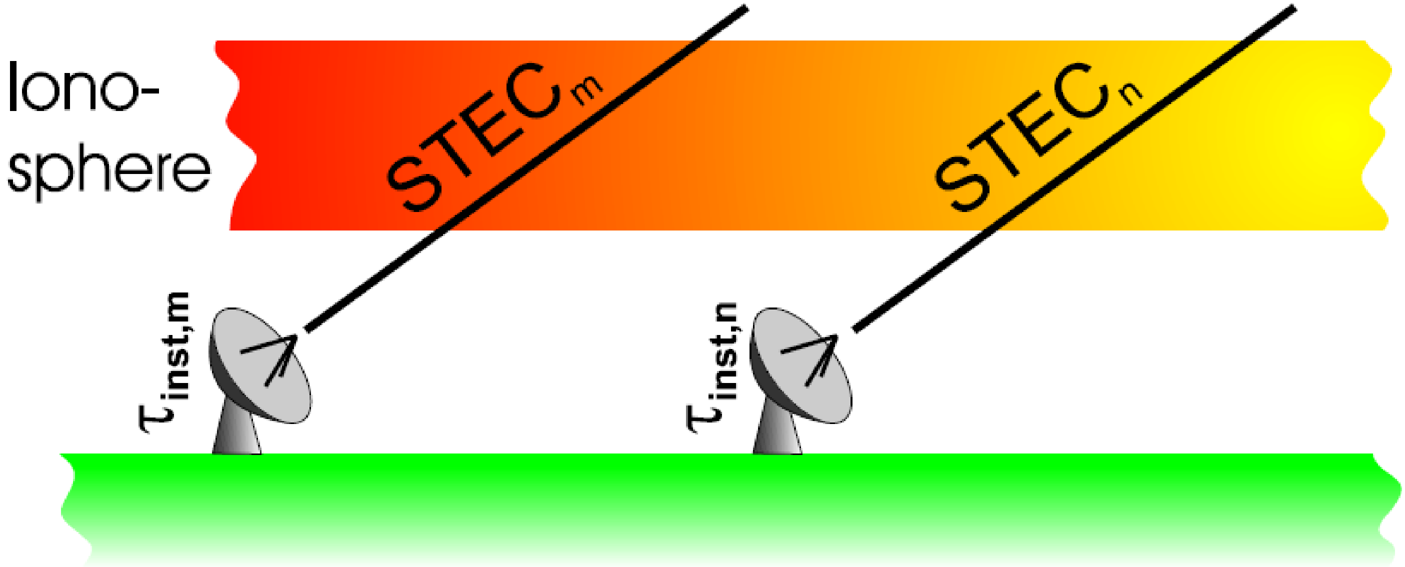


X/S VLBI and the ionosphere

- Only relative values of STEC can be determined

$$\tau_{gx} = \tau_{if} + \frac{\alpha}{f_{gx}^2} \qquad \tau_{gs} = \tau_{if} + \frac{\alpha}{f_{gs}^2}$$

$$\alpha = \frac{40.31}{c}\left(\int N_e ds_1 - \int N_e ds_2\right) = \frac{40.31}{c}(STEC_1 - STEC_2)$$



Hobiger, 2005

X/S VLBI and the ionosphere

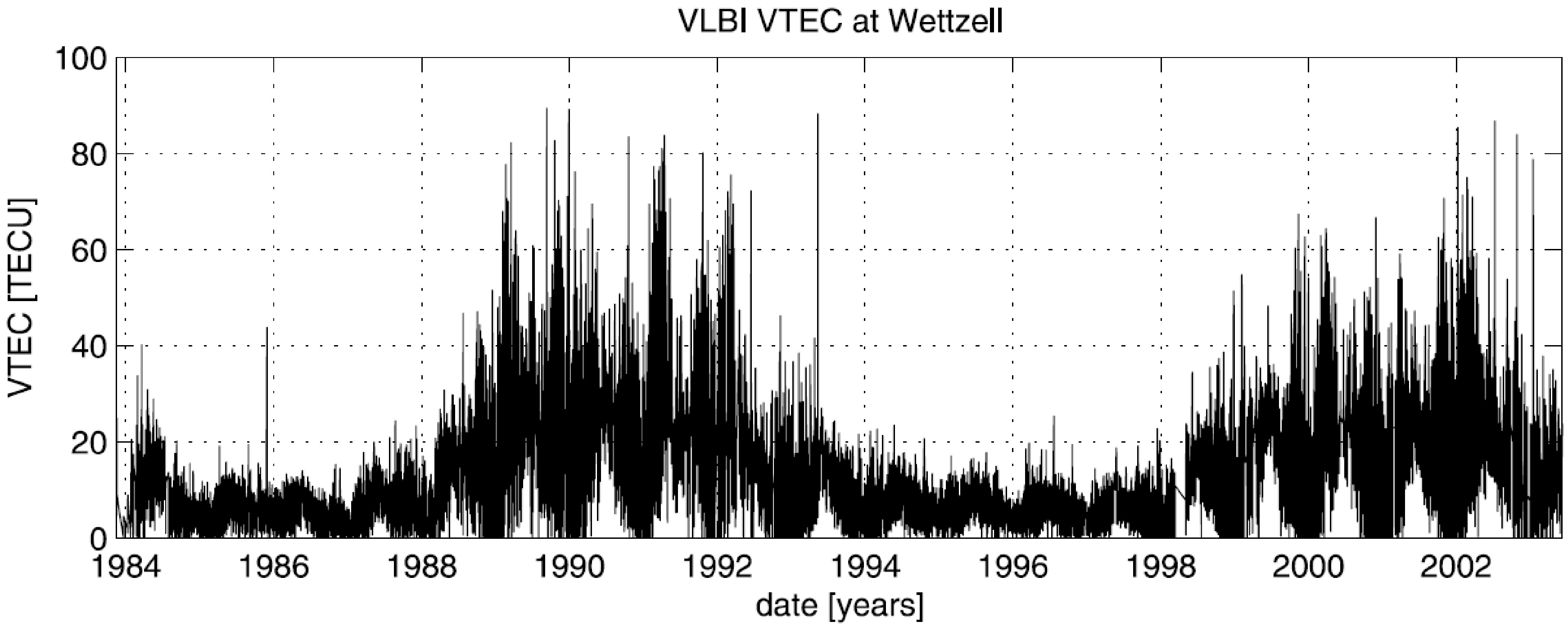
- Ionosphere-free group delay based on effective frequencies

$$\tau_{if} = \frac{f_{gx}^2}{f_{gx}^2 - f_{gs}^2}\tau_{gx} - \frac{f_{gs}^2}{f_{gx}^2 - f_{gs}^2}\tau_{gs}$$

- Instrumental biases are included (estimated w clocks)

X/S VLBI and the ionosphere

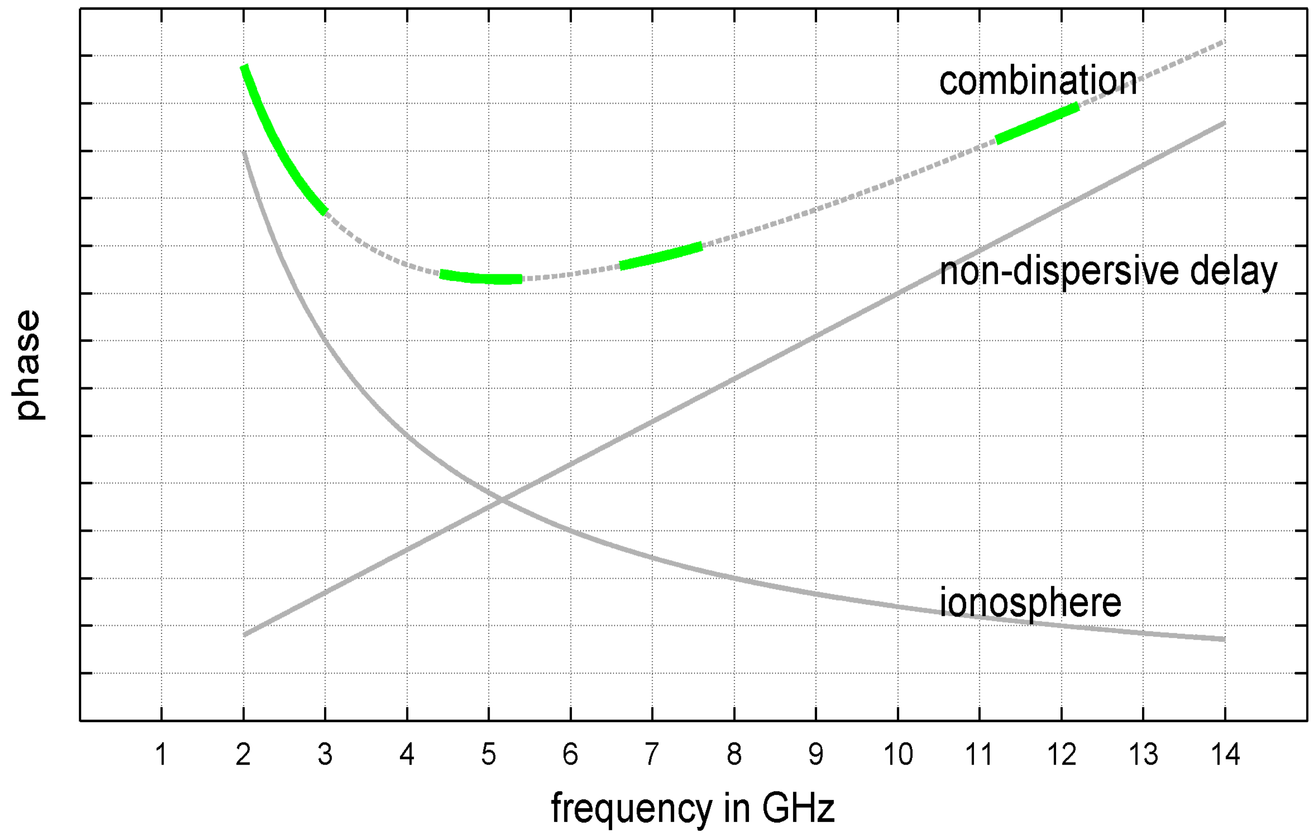
- Vertical TEC estimation from VLBI
  - only possible with appropriate use of constraints



Hobiger

VGOS and the ionosphere

- Phases are connected across the whole band
- Ionosphere delays are estimated together with the group delays in the fringe-fitting process



Troposphere

- Troposphere delays: strictly speaking delays in the neutral atmosphere (up to 100 km)
- Essentially no frequency dependency across microwave regime
- Refractivity N versus refractive index n

$$N = (n - 1) \cdot 10^6$$

  - $N \approx 300$, $n \approx 1.0003$
  - Units of N: ppm, mm/km, "Neper"

Refractivity

- Refractivity as a function of pressure, temperature and humidity, (and liquid water)

$$N = k_1 \frac{p_d}{T} + k_2 \frac{e}{T} + k_3 \frac{e}{T^2}$$

- Dry - wet $\rightarrow$ hydrostatic - "wet"
- Wet delay larger than "wet" delay by about 3 %

$$\underbrace{N = k_1 \frac{R}{M_d} \rho}_{\text{hydrostatic}} \; \Big| \; \underbrace{+ k_2' \frac{e}{T} + k_3 \frac{e}{T^2}}_{\text{“wet”}}$$

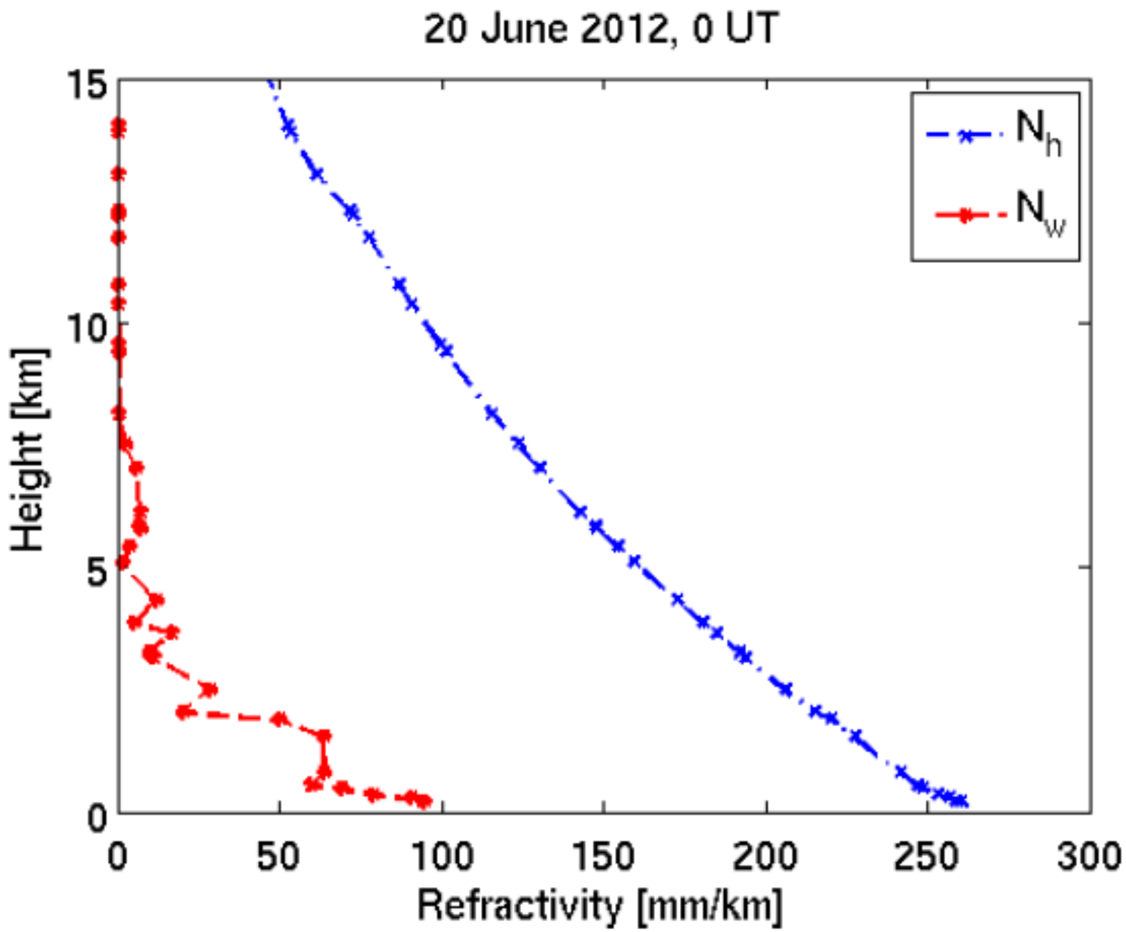
hydrostatic

“wet”

$$k_2' = k_2 - k_1 \frac{M_w}{M_d}$$

Refractivity

- Wet part: surface values not representative for the upper air conditions

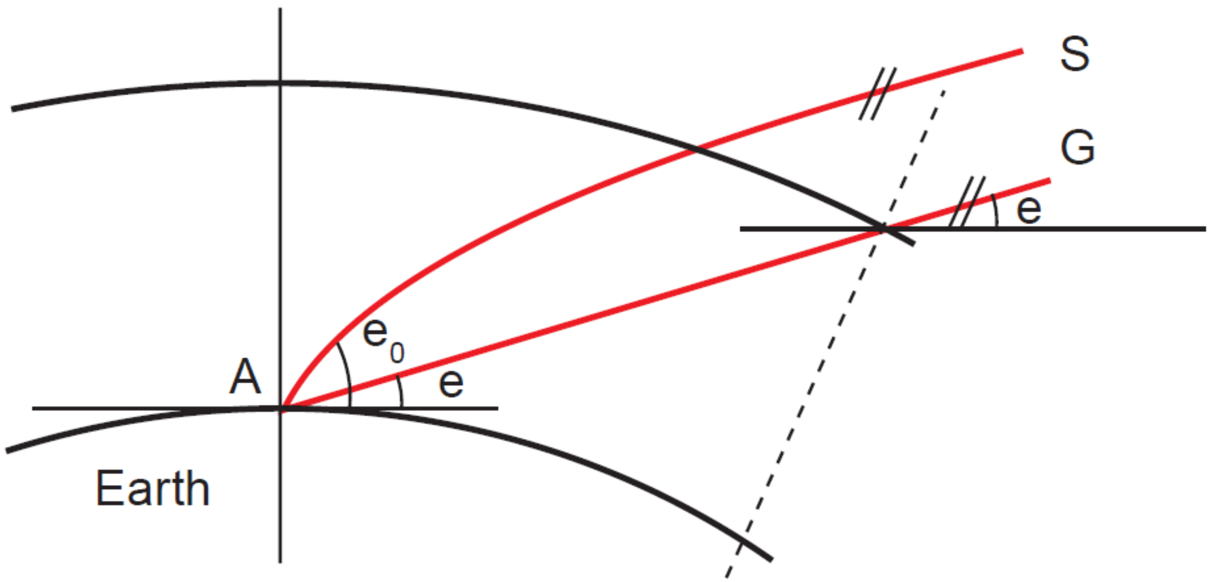


Radiosonde profile Vienna

Path delay

- Electric path length L is minimized

$$L = \int_S n(s)\,\mathrm{d}s$$



$$\Delta L = L - G = \int_S n(s)\mathrm{d}s - G = \Delta L_h + \Delta L_w + S - G$$

- Bending effect S – G about 2 dm at 5 degrees elevation

Hydrostatic zenith delay

- Equation by Saastamoinen (1972)

$$\Delta L_h^z = 0.0022768 \frac{p_0}{f(\theta, h_0)}$$

≈ 2.3 m at sea level

- Consequently we need the pressure at the site to determine the hydrostatic zenith delay very accurately
  - local recordings at the site (preferable if available)
  - gridded values from numerical weather models
  - empirical (blind) models like GPT

Wet zenith delay

- Estimated from VLBI observations
- Could be determined from
  - Ray-tracing through numerical weather models
  - Water vapour radiometry
  - GNSS analysis

Konrad (Elgered et al., 2012)

Mapping functions

- Elevation dependent mapping functions used for a priori hydrostatic delay and estimating zenith wet delays

$$\Delta L(e) = \Delta L^z \cdot mf(e)$$

- Zenith wet delays estimated every 20 to 60 minutes
- Correlation between height, clocks and zenith delays
- Partials are sin(e), 1, and mf(e)
- Separation into hydrostatic and wet part

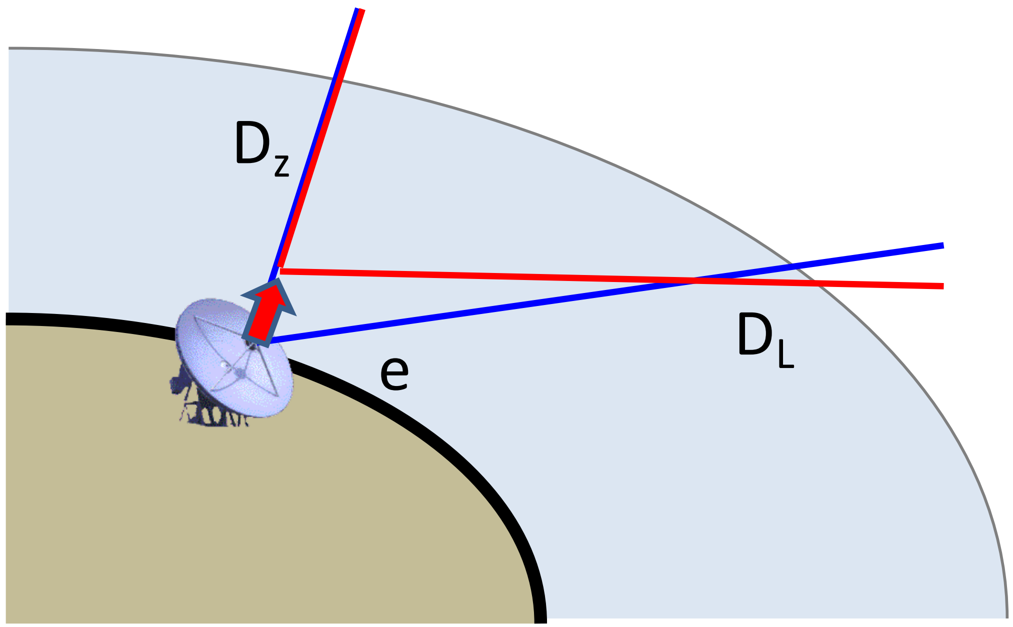
$$\Delta L(e) = \Delta L_h^z \cdot mf_h(e) + \Delta L_w^z \cdot mf_w(e)$$

Mapping functions

- Mapping function not perfectly known
- Errors via correlations also in station heights (and clocks)
- Low elevations necessary to de-correlate heights, clocks, and zenith delays
- Trade-off $\rightarrow$ about 5 degrees cut off elevation angle (sometimes with down-weighting)

Mapping functions

- The station height error is about 1/5 of the delay error at 5 degrees elevation (if cutoff is at 5 degrees)
- The corresponding decrease of the zenith delay is about half of the station height increase



$D_L(e) = D_z \cdot m(e)$

$D_L(e) = D_z' \cdot m(e)'$

Mapping functions

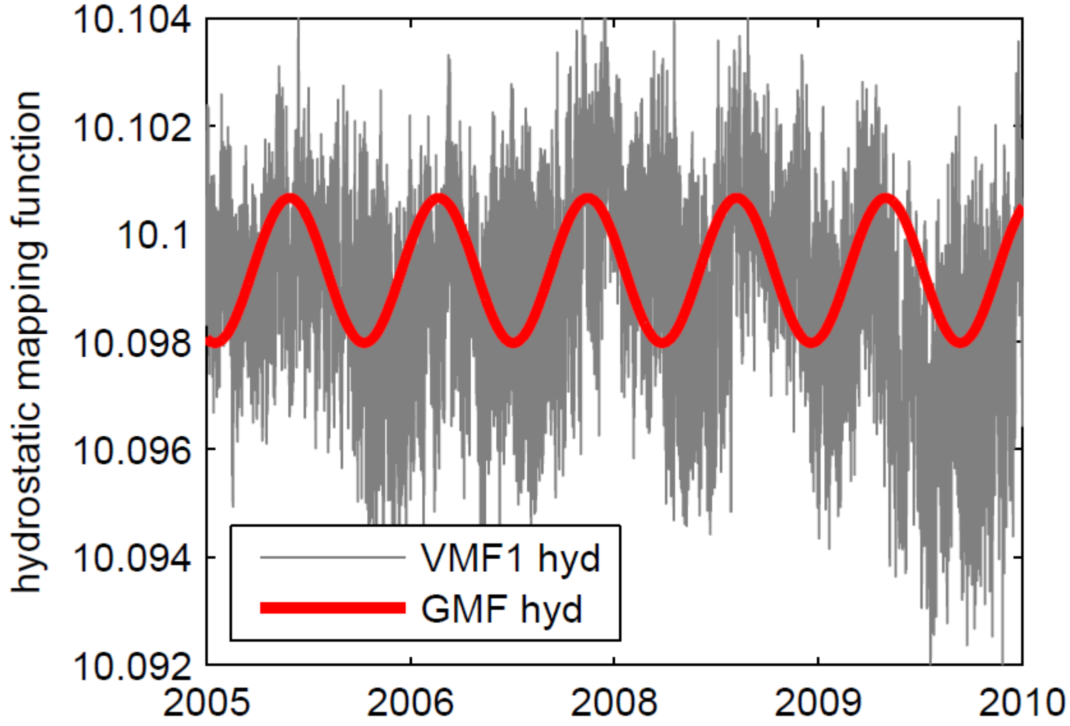
- Continued fraction form (Herring, 1992)

$$mf(e) = \frac{1 + \cfrac{a}{1 + \cfrac{b}{1 + c}}}{\sin(e) + \cfrac{a}{\sin(e) + \cfrac{b}{\sin(e) + c}}}$$

- Example Vienna Mapping Functions
  - Empirical functions for b and c coefficients
  - Coefficients a by ray-tracing and inversion using 6h data of the ECMWF
  - Available for all VLBI sites and on global grid

Mapping functions

- VMF1 versus GMF at Fortaleza (Brazil) at 5 deg. elevation



Tropospheric gradients

- Chen and Herring (1997)

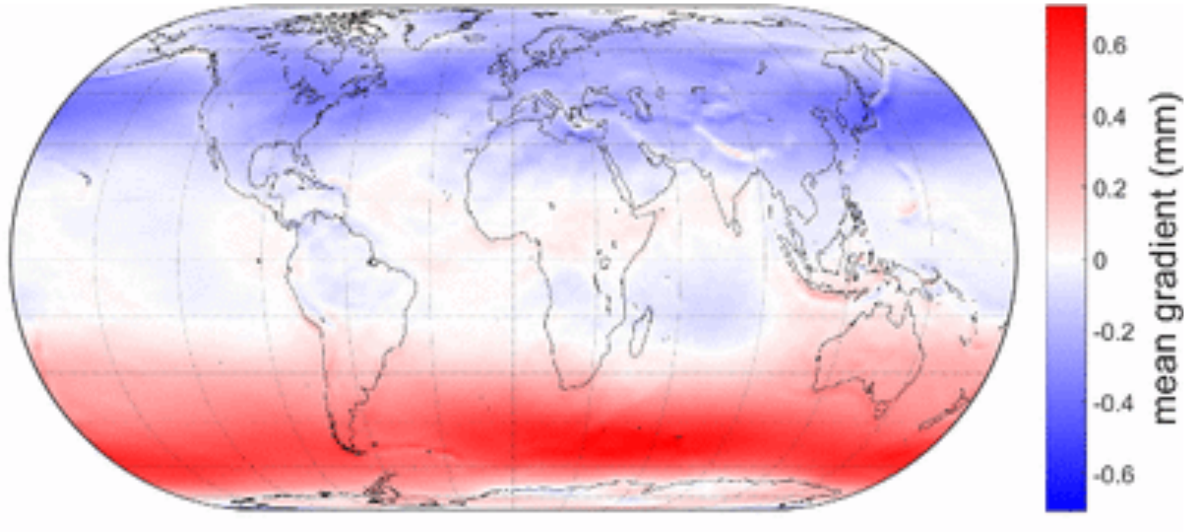
$$\Delta L(a,e) = \Delta L_0(e) + mf_g(e)(G_n \cos(a) + G_e \sin(a))$$

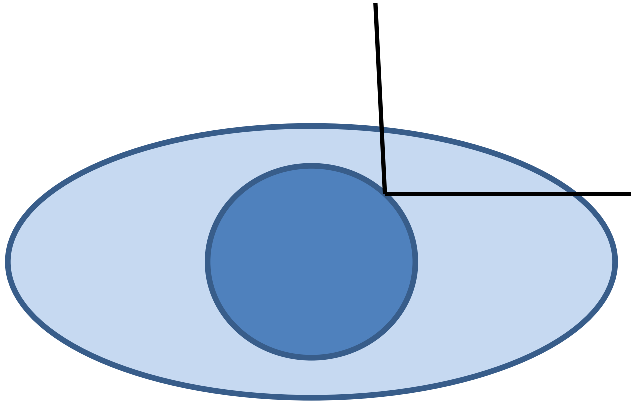
$$mf_g(e) = \frac{1}{\sin(e)\tan(e) + C},$$

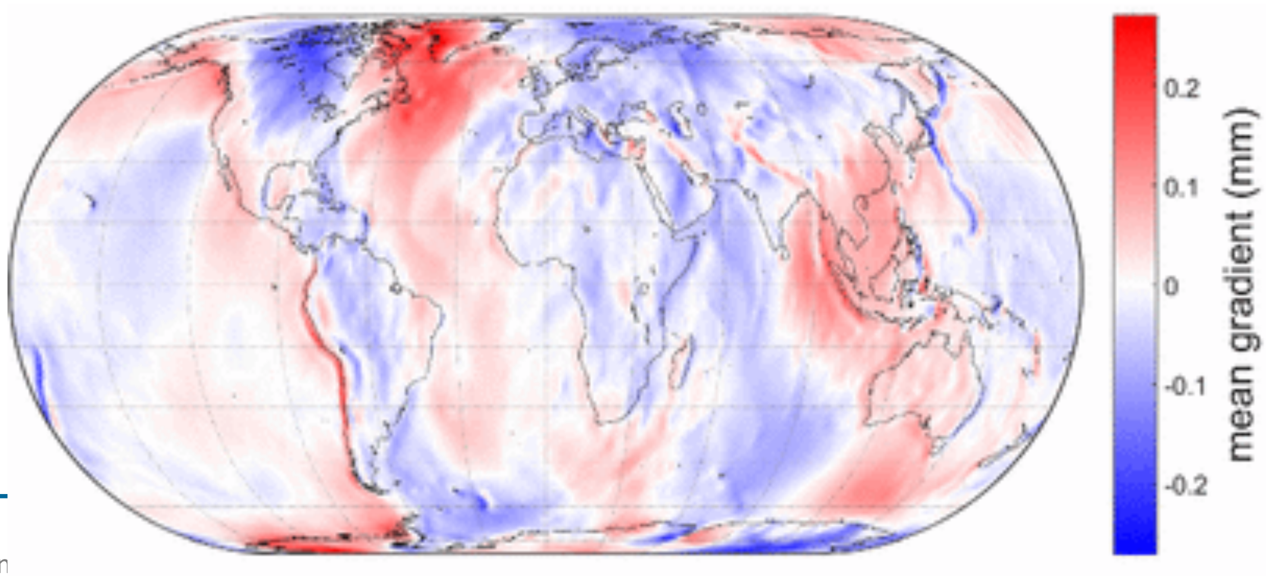
- Typical gradient: 1 mm (corresponds to 1 dm at 5 deg. elevation)
- Estimated e.g. every 3 hours
- Caused by weather fronts, coastal situations, atmospheric bulge, ..

Tropospheric gradients

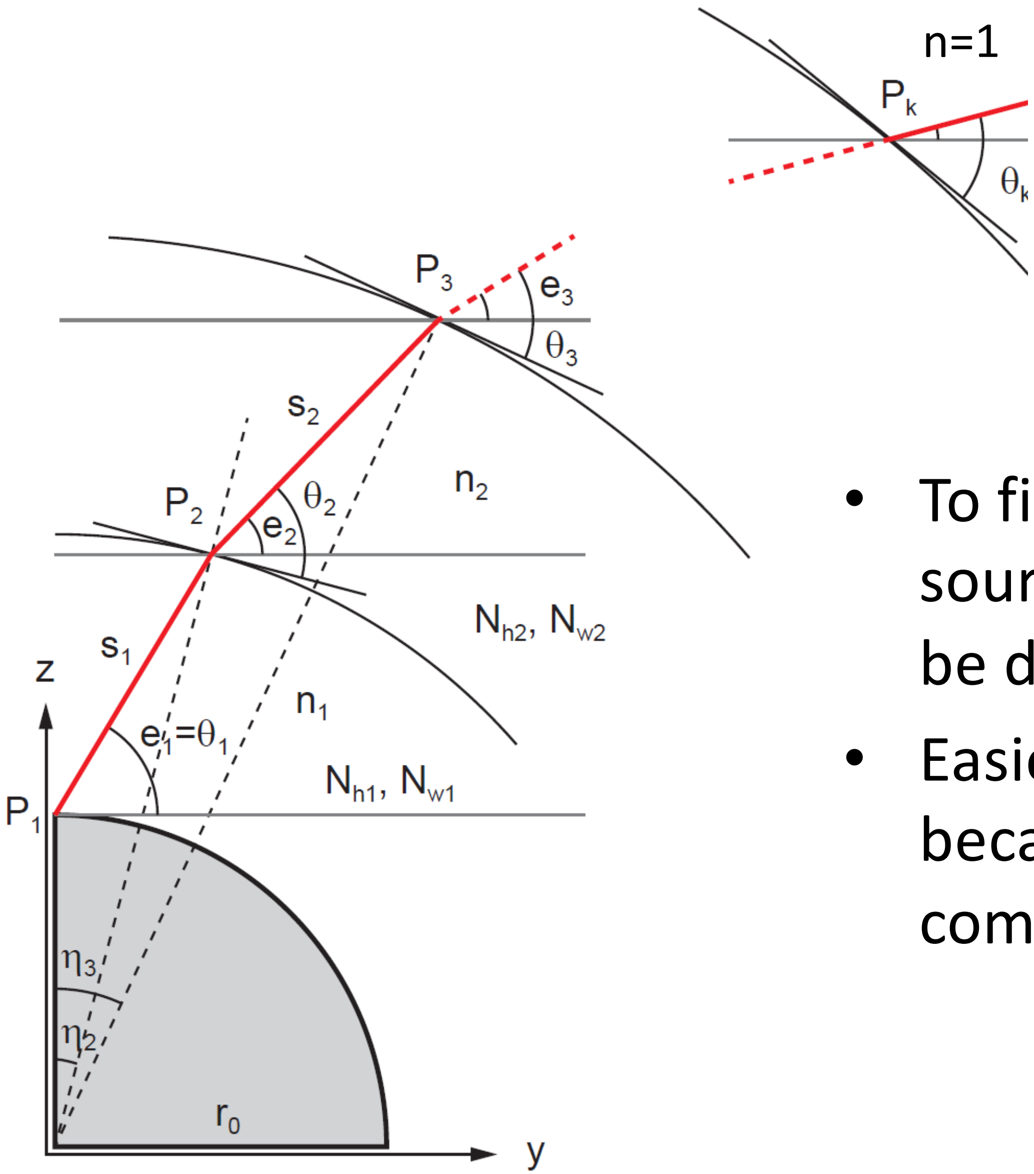
- Mean hydrostatic north and east gradients (Landskron, 2018)







Ray-tracing



- To find the ray-path from the source to the telescope (has to be done iteratively, "shooting")
- Easier in 2D case (6 equations), because no out-of-plane components

VMF Open Access Data

- http://vmf.geo.tuwien.ac.at/

- Ray-traced delays for the complete history of VLBI observations available there
- Online tool to do your own ray-tracing at VLBI sites
- Vienna Mapping Functions coefficients (from analysis and forecast data)
- 6h hydrostatic and wet gradients
- Empirical “backup” mapping functions, e.g. GPT3

Atmospheric turbulence

- Random fluctuations in refractivity distribution
- Structure function as modified by Treuhaft and Lanyi (1987)

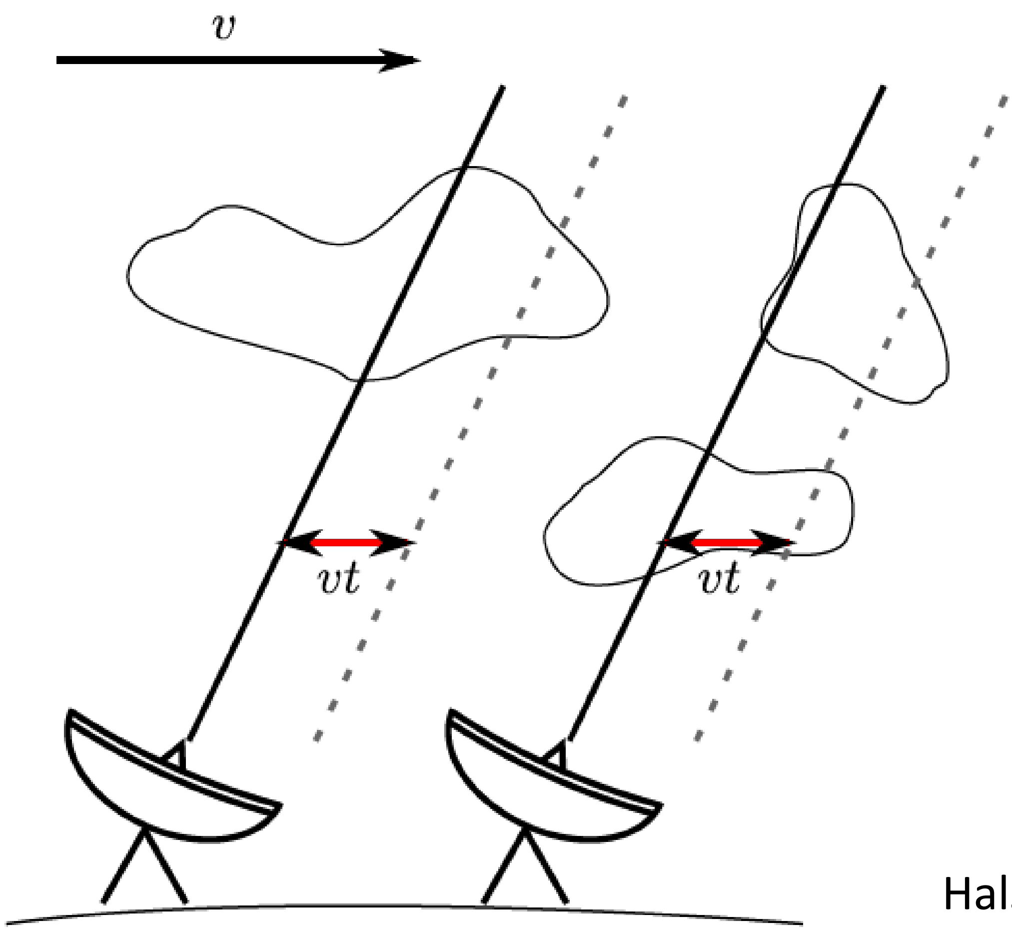
$$D_n(\mathbf{R}) = \left\langle [n(\mathbf{r}) - n(\mathbf{r}+\mathbf{R})]^2 \right\rangle = C_n^2 \frac{\|\mathbf{R}\|^{2/3}}{1 + \left[\frac{\|\mathbf{R}\|}{L}\right]^{2/3}}$$

  - C_n^2 is the refractive index structure constant
  - L is the saturation length scale

- Close observations in space and time are correlated

Atmospheric turbulence

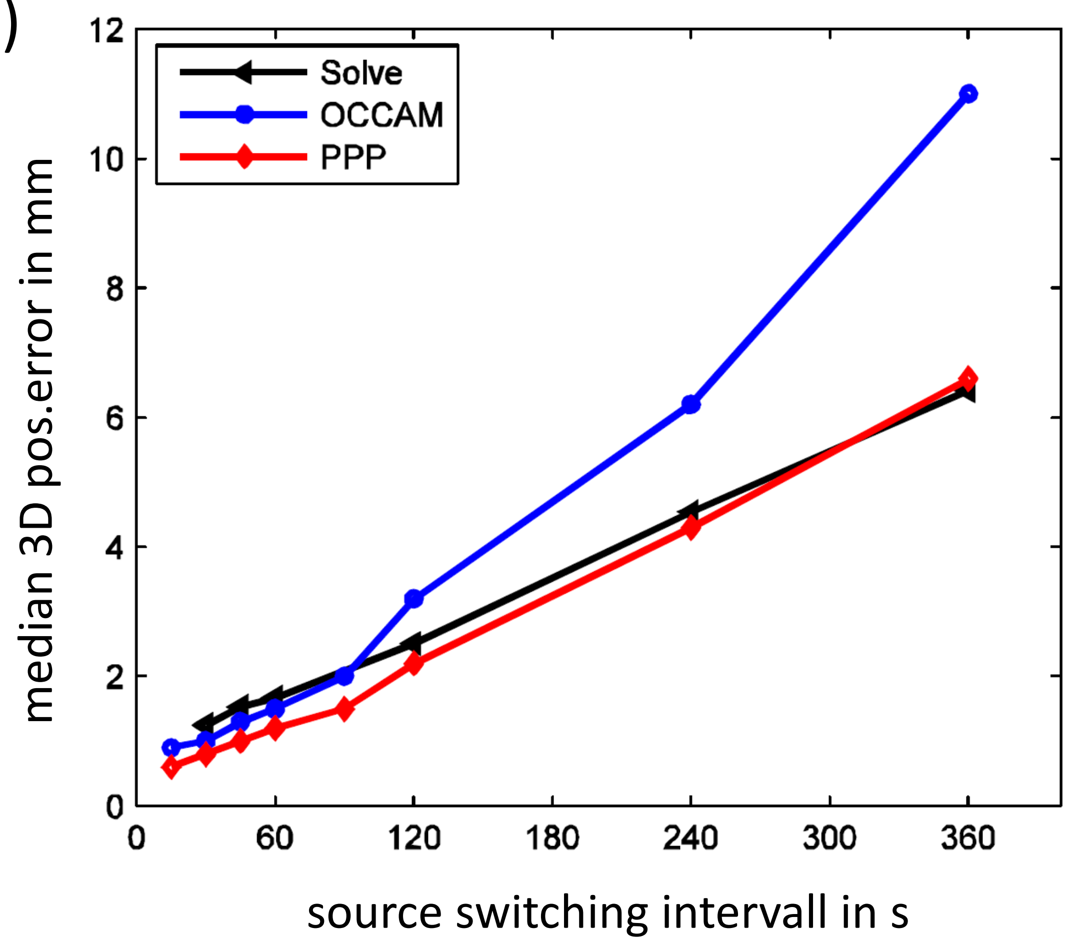
- Frozen flow theory for equivalence of correlation in space and time



Halsig, 2018

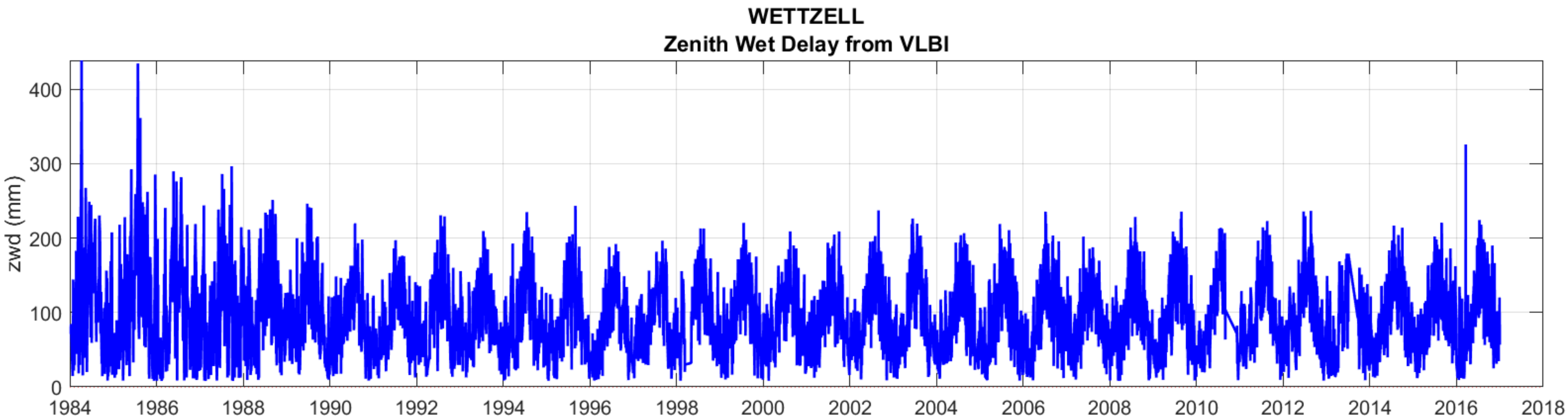
Atmospheric turbulence

- Correlations can be used in
  - analysis (a priori correlation)
  - simulations (e.g. VGOS)



Climate studies

- Zenith wet delays at Wettzell (Landskron, 2018)




Questions?